



FROM FREGE TO WITTGENSTEIN

Perspectives on Early
Analytic Philosophy

EDITED BY
Erich H. Reck



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to Wittgenstein

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OXFORD
UNIVERSITY PRESS

2002

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Oxford New York

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Chennai Dar es Salaam Delhi Florence Hong Kong Istanbul Karachi
Kolkata Kuala Lumpur Madrid Melbourne Mexico City Mumbai Nairobi
Paris São Paulo Singapore Taipei Tokyo Toronto Warsaw

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Published by Oxford University Press, Inc.
198 Madison Avenue, New York, New York 10016

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Library of Congress Cataloging-in-Publication Data
From Frege to Wittgenstein : perspectives on early
analytic philosophy / edited by Erich H. Reck.

p. cm.

Includes bibliographical references and index.

ISBN 0-19-513326-9

1. Analysis (Philosophy) 2. Frege, Gottlob, 1848–1925.
3. Wittgenstein, Ludwig, 1889–1951.

I. Reck, Erich H., 1959–
B808.5 .F78 2001
146'.4—dc21 00-058910

1 3 5 7 9 8 6 4 2

Printed in the United States of America
on acid-free paper

To the memory of my mother,

Anna Reck

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Acknowledgments

The starting point for this volume was a small conference, entitled “Frege, Russell, Wittgenstein: Shaping the Analytic Tradition,” that took place at the University of California at Riverside, Department of Philosophy, in the spring of 1998. Three of the essays in the volume were presented, in earlier versions, at that conference. All of the other essays were solicited shortly thereafter. I am indebted to the Department of Philosophy and the University of California for making it possible for me to organize the conference. I am also grateful to the Center for Ideas and Society, University of California at Riverside, and to the Center for Philosophy of Science and the Department of Philosophy, University of Pittsburgh, for allowing me to work on this volume while being their guest during the autumn quarter 1998 and the academic year 1999–2000, respectively, as well as to the University of California for a Regents’ Faculty Fellowship and Faculty Development Award during the latter period. Finally, I would like to thank Dillon Emerick and Koki Funato for their editorial assistance, and Sally Allen Ness for her help and support overall.

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Preface

Analytic philosophy, after proceeding with little regard for its own history initially, has gained a new historical self-consciousness. Thus a considerable amount of work, both historically informed and philosophically subtle, is being done now on its origins and development. This is especially true for the emergence of analytic philosophy as a separate tradition, that is, for what is now known as “early analytic philosophy” (roughly 1880–1930). It seems that today, at the beginning of the twenty-first century, we finally have enough historical distance to look at this period with new eyes. As a consequence, we no longer see the towering figures in it—Frege, Moore, Russell, Wittgenstein, and Carnap, among others—exactly as our contemporaries. Instead, we are realizing more and more that their interests and approaches were shaped by their own time, in ways we still need to understand better.

At the same time, the new interest in early analytic philosophy is not just historical. It also derives from the extraordinary depth and fruitfulness of the work done during this period, especially on logical, semantic, and metaphysical issues. It is not hard to see that the various implications of this work have still not been fully explored. Moreover, there seems to exist a kind of identity crisis in analytic philosophy today, a loss of certainty about what philosophy really is, or should be. As a result, the boundaries between various philosophical schools—analytic philosophy, pragmatism, phenomenology, hermeneutics, and so on—are getting blurred more and more. In this situation a reflection back on early analytic philosophy is of special interest, not only to understand better when and how this particular tradition started, but also because the nature of “analytic” inquiry was at the center of the discussions then.

This collection of essays is a contribution to the general reexamination of early analytic philosophy and of its continuing relevance. In addition, it has a specific,

distinctive focus: the works of Gottlob Frege and of Ludwig Wittgenstein (mostly the early Wittgenstein), as well as various ties between them. As is generally acknowledged, both Frege and Wittgenstein have had a big impact on twentieth-century philosophy, especially within the analytic tradition. But what exactly this impact consisted in and how precisely to understand their views, including the rich interrelations between them, have become two of the most actively and fruitfully debated questions in the recent literature—as this collection hopes to establish further. In several of the essays Bertrand Russell's ideas and influence are also discussed along the way. They play only a secondary role in the volume as a whole, though, not because they are less important—Russell played at least as big a role in the development of analytic philosophy as did Frege and Wittgenstein, no doubt—but because they have already been explored extensively elsewhere.

The essays have been arranged in four parts, based on the main topics addressed in them. The parts concern, respectively (I) historical background and general themes; (II) specific topics in Frege's philosophy; (III) comparisons of Frege's, partly also Russell's, views to those of the early Wittgenstein; and (IV) further discussions of Wittgenstein's early philosophy. It should be noted that there is considerable overlap between these four parts, especially III and IV, both of which contain mostly comparative essays. Thus several of the essays could also have been placed in a different group (especially those by Gerrard, Diamond, Proops, and Conant). Some biographical information about each of the authors is appended.

*Pittsburgh, Pennsylvania
April 2000*

E. H. R.

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PART I

BACKGROUND AND GENERAL THEMES

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Wittgenstein's "Great Debt" to Frege

Biographical Traces and Philosophical Themes

Erich H. Reck

You write of a great debt of thanks to me. I know nothing
of such a debt. Each of us has gained, I think, in our intel-
lectual exchange.

Frege to Wittgenstein, April 9, 1918

Ludwig Wittgenstein is, no doubt, one of the most original and independent philosophers of the twentieth century. Yet even he developed his ideas not in complete isolation, but as influenced by, or in response to, other thinkers. His relation to Bertrand Russell, his early teacher and mentor in Cambridge, has been documented in detail.¹ Clearly Russell's views, especially those from before World War I, had a significant influence on Wittgenstein. Most important, Russell brought various philosophical questions and problems to the attention of the young Wittgenstein, and he encouraged him to work on them along Russellian lines. Later on, of course, Wittgenstein became more and more critical of Russell, distancing himself not only from Russell's particular views, but also from a Russellian approach to philosophy more generally.

Wittgenstein's relation to Gottlob Frege has also not gone unnoticed. So far it has, however, not attracted the same amount of general attention as that to Russell. Given Russell's prominence as a philosopher and as a public figure, given also the wealth of information we have about him, including about his ties to Wittgenstein, it is not hard to explain why. The relation to Frege has, then, typically been presented as subsidiary to that to Russell and not explored much in itself.² In addition, Wittgenstein's and Frege's positions have often been taken to be so far apart that Wittgenstein could only be understood as being a hostile critic of Frege, it seemed. His debt to Frege has thus been taken to be rather minimal, except in the negative sense of Wittgenstein finding various targets for his criticisms in Frege's work.³ On the other hand, recently a number of interpreters have started to look in a new way at this side of Wittgenstein's background. To some degree this is happening because new source material on it has become available (especially Frege, 1989), more because Frege's views and their influence on twentieth-century

philosophy are undergoing a general reappraisal.⁴ As a result, a different picture of Wittgenstein's relation to Frege is starting to emerge.

In this essay I want to contribute to this new line of research. More particularly, I want to discuss a variety of biographical traces and philosophical themes that connect Wittgenstein to Frege. My discussion will start with Frege's relatively direct impact on the young Wittgenstein; but I will also consider the more indirect influence Frege had on the middle and later Wittgenstein, an influence often ignored or denied in the literature. My goal cannot, of course, be to deal with every aspect of Wittgenstein's relation to Frege in detail; that would require a book-length treatment. What I will do, instead, is to develop a general framework or setting, a stage, so to speak, for more detailed and comprehensive research later. This will involve collecting all the information we have about Wittgenstein's personal relation to Frege (since they did, in fact, know each other personally), as well as various other bits and pieces concerning his continued engagement with and admiration for Frege's works (which actually went on until the very end of Wittgenstein's life). I will present these pieces as part of a condensed retelling of Wittgenstein's life. The overall conclusion will be that Wittgenstein's ties to Frege, both personal and philosophical, were stronger than is often assumed—so strong that it makes sense to speak of Wittgenstein's “great debt” to Frege, as he himself does.⁵

I. Wittgenstein's Path into Philosophy and His First Encounter with Frege

As is well known, Wittgenstein (1889–1951) grew up in a very wealthy and cultured family in fin-de-siècle Vienna. His early intellectual influences included Schopenhauer's philosophy (recommended to him by his sister Hermine), the writings of the cultural critic Karl Kraus and the architect Adolf Loos, as well as the scientific and philosophical writings of Ludwig Boltzmann and Heinrich Hertz. Science and engineering became, then, what Wittgenstein decided to pursue as a student. To do so he first went to Berlin (1906–8) and then to Manchester (1908–11).

While Wittgenstein was in Manchester, a decisive shift of interests occurred, from engineering and science to mathematics and philosophy. Here is how Rush Rhees, later one of Wittgenstein's students, friends, and literary executors, describes this shift:

When [Wittgenstein] left the Technical University of Berlin he went to the College of Technology in Manchester (I think this was on the advice of his father), still to study engineering; and in particular certain problems in aero-engineering. This was in 1908, and he continued in Manchester until the end of 1911.... It has been suggested—by W. Mays (“Wittgenstein's Manchester Period,” *Guardian*, 24 March 1961, p. 10)—that the mathematical problems in the designing of air screws led Wittgenstein's interests more and more into mathematics on its own account. Wittgenstein himself told me that while he was working in the Engineering Laboratory, he and two others doing research there began to meet for one evening each week to discuss questions about mathematics, or “the foundations of mathematics.” (As they went on, they may have met more often—I do not

remember.) At one of these meetings Wittgenstein said he wished there were a book devoted to these questions, and one of the others said, "Oh there is, a book called *The Principles of Mathematics*, by Russell; it came out a few years ago." Wittgenstein told me that this was the first he had heard of Russell; and that this was what led him to write to Russell and to ask if he might come and see him. I believe it was from *The Principles of Mathematics* that Wittgenstein learned of Frege. Whether he then went to see Frege before he went to see Russell, or sometime after, is disputed—but I cannot see that it matters. (Rhees 1984a, pp. 213–14)⁶

Two facts of particular importance to us come to light in this report: first, it was Russell's work on the foundations of mathematics, in particular his book *The Principles of Mathematics*, that drew Wittgenstein into philosophy. Second, it was through Russell's writings, most likely the appendix to *Principles* called "The Logical and Arithmetical Doctrines of Frege," that Wittgenstein first became aware of Frege's work.⁷

What is the "dispute" Rhees refers to at the end of the preceding quotation? It concerns the question of when exactly Wittgenstein met Frege, in particular whether it was before he met Russell or afterward. The relevant background here is the following: in his obituary on Wittgenstein in the journal *Mind*, written shortly after Wittgenstein's death in 1951, Russell describes how the two first met:

When I made the acquaintance of Wittgenstein, he told me that he had been intending to become an engineer, and with that end in view had gone to Manchester. In the course of his studies in engineering he had become interested in mathematics, and in the course of his studies in mathematics he had become interested in the principles of mathematics. He asked people in Manchester (so he told me) whether there was such a subject, and whether anyone worked on it. They told him that there was such a subject and that he could find out more about it by coming to me at Cambridge, which he accordingly did. (Russell, 1951, p. 297)

A little later (after a somewhat theatrical anecdote about their first meeting in Cambridge), Russell goes on to say about Wittgenstein: "He made very rapid progress in mathematical logic, and soon he knew all that I had to teach. He did not, I think, know Frege personally at that time, but he read him and greatly admired him" (p. 298). What Russell states here about Wittgenstein's "great admiration" for Frege is confirmed by Rhees's and other reports (more on that below). What is in doubt, however, is Russell's claim that Wittgenstein "did not know Frege personally at that time" (i.e., 1911).

As Rhees indicates, Russell is contradicted on this point by various people. These include G. H. von Wright, another of Wittgenstein's later students, friends, and literary executors; von Wright describes what happened this way: "Having decided to give up his studies in engineering, Wittgenstein first went to Jena in Germany to discuss his plans with Frege. It was apparently Frege who advised Wittgenstein to go to Cambridge and study with Russell. He followed the advice" (von Wright, 1958, p. 5). Von Wright adds in a footnote: "This is how Wittgenstein related the matter to me. His account is confirmed by notes made by his sister Hermine. Russell seems, therefore, to be mistaken when in his memorial article in *Mind*, n.s. LX (1951) he says that Wittgenstein had not known

Frege before he came to Cambridge” But perhaps Rhees is correct that it “does not matter” who is right here, at least not very much.⁸ What is clear is this: Either Wittgenstein went directly to Russell, after having read his *Principles*, or he first went to see Frege in Jena, so as to be sent by him to Russell in Cambridge. The year is 1911, in any case. We should note that Wittgenstein was twenty-two years old, Frege already sixty-three, while Russell was thirty-nine. Frege’s “advice” to Wittgenstein to go to Cambridge may thus have been based on the fact that Russell was in his prime as a logician at this point, while Frege himself was already close to retirement.

Actually, even before Wittgenstein visited either Frege or Russell in 1911, there is evidence for another incident that deserves our attention. Namely, Wittgenstein seems to have been in more indirect contact with Russell two years earlier. The evidence for this contact is not completely definite, and it is only mentioned in passing in some of the literature.⁹ But here is what seems to have happened: after discovering the foundations of mathematics as a discipline, Wittgenstein read Russell’s *Principles of Mathematics* (published in 1903), and probably also at least parts of Frege’s *Grundgesetze der Arithmetik* (1893/1903), while still in Manchester. What caught his special attention was the antinomy Russell had found in Frege’s and similar systems. Russell describes this antinomy, in explicit connection with Frege’s *Grundgesetze*, in his first appendix to *Principles* (see above). In a second appendix, called “The Doctrine of Types,” he sketches a tentative solution; and at the end of it he adds cautiously: “What the complete solution of the difficulty may be, I have not succeeded in discovering; but as it affects the very foundations of reasoning, I earnestly commend the study of it to the attention of all students of logic” (Russell, 1903, p. 528).

Apparently remarks such as these were all the “bait” it took to hook Wittgenstein on logic.¹⁰ In other words, he started to see himself now, more and more, as the “student of logic” to which Russell’s remark was addressed. Even more strikingly, he sat down at once, it seems, to attempt his own solution to Russell’s antinomy. This new solution he then—already in 1909—sent to P. E. B. Jourdain, another logician active in Britain at this point, as well as a friend of Russell’s.¹¹

Unfortunately, the manuscript Wittgenstein sent to Jourdain does not, as far as we know, exist any more. On the other hand, a bit of evidence exists that Jourdain discussed Wittgenstein’s proposal explicitly with Russell himself. Thus Jourdain writes in his notebook on April 20, 1909, shortly after a visit by Russell: “Russell said that the views I gave in a reply to Wittgenstein (who had ‘solved’ Russell’s contradiction) agree with his own. These views are: The difficulty seems to me to be as follows. In certain cases (e.g., Burali-Forti’s case, Russell’s ‘class’ . . . , Epimenides’ remark) we get what seems to be meaningless *limiting cases* of statements which are not meaningless” (Grattan-Guinness, 1977, p. 114).¹² We can infer, then, that Jourdain sent Wittgenstein a negative response to his proposal, one with which Russell basically agreed. Perhaps as a consequence of this early rebuff, Wittgenstein, instead of embracing philosophy immediately, stayed in engineering a bit longer to do research on aircraft engines and propellers.

Wittgenstein was, nevertheless, hooked on doing philosophy now. While still involved in aeronautics research in Manchester, he kept working on a paper on the foundations of mathematics. This paper was either a descendant of the one he had sent to Jourdain or a new one; we do not know exactly. His sister Hermine

later gives the following vivid description of Wittgenstein's state of mind during this period:

At this time, or shortly after, philosophy, or rather reflections on philosophical problems, suddenly became such an obsession with him, and took hold of him so completely against his will, that he suffered terribly, feeling torn between conflicting vocations. This was the first of several transformations he was to undergo in his life, and it shook his whole being. He was working at the time on a piece of philosophical writing, and decided finally to show the plan of the work to a Professor Frege in Jena, who was concerned with similar problems. (H. Wittgenstein, 1984, p. 2)

And Wittgenstein soon acted on his "decision": he actually went to visit Frege in Jena. His sister goes on to describe their first meeting, at the end of the summer of 1911, as follows (thereby also contradicting Russell's report, quoted above):

During this time Ludwig was in a constant, indescribable, almost pathological state of agitation, and I was very much afraid that Frege, whom I knew to be an old man, would not have the patience or understanding to go into the matter in the way which the seriousness of the situation demanded. Consequently I was in a state of great worry and anxiety during Ludwig's trip to visit Frege, but it went much better than I had thought it would. Frege encouraged Ludwig in his philosophical quest and advised him to go to Cambridge to study under Professor Russell, and this he did. (p. 2)

Wittgenstein himself described this first meeting with Frege not only to his sister, but also later—often in vivid and memorable terms—to a number of students and friends. This indicates that he saw it as an important, influential event in his life. M. O. C. Drury, for example, reports one of these descriptions as follows:

Wittgenstein: I remember that when I first went to visit Frege I had a very clear idea in my mind as to what he would look like. I rang the bell and a man opened the door; I told him I had come to see Professor Frege. "I am Professor Frege," the man said. To which I could only reply, "impossible!" At this first meeting with Frege my own ideas were so unclear that he was able to wipe the floor with me. (Drury, 1984, p. 110)

It appears that Wittgenstein was, in particular, quite taken by Frege's "wiping the floor with him," as that aspect of their first meeting is repeated almost verbatim by several other people to whom he told the story.¹³

A brief aside on Frege in this connection: it is clear that Frege was an ingenious, pathbreaking logician and philosopher. As a person, however, he seems to have been much less admirable: bitter and withdrawn, lacking charisma and charm; he also held unsettlingly conservative, nationalistic, and anti-semitic political views, at least late in his life.¹⁴ To illuminate another, less well known side of his personality, it is worth quoting from one more of the reports of Wittgenstein's first meeting with him. Here is how Peter Geach, one of Wittgenstein's last students, recounts the story:

Wittgenstein's story of his relations with Frege was as follows: "I wrote to Frege, putting forward some objections to his theories, and waited

anxiously for a reply. To my great pleasure, Frege wrote and asked me to come and see him.

When I arrived I saw a row of boys' school caps and heard the noise of boys playing in the garden. Frege, I learned later, had had a sad married life—his children had died young, and then his wife; he had an adopted son, to whom I believe he was a kind and good father.

I was shown into Frege's study. Frege was a small, neat man with a pointed beard, who bounced around the room when he talked. He absolutely wiped the floor with me, and I felt very depressed; but at the end he said 'You must come again,' so I cheered up." (Geach, 1961, pp. 129–30)

Wittgenstein himself was, of course, a rather eccentric human being, but he was not known to be a bad judge of character. Thus it is noteworthy how this report displays not only his affection for Frege as a philosopher, but also his sympathetic appreciation of him as a father and family man.¹⁵

II. Student Years in Cambridge: Further Visits and Correspondence with Frege

On February 1, 1912, Wittgenstein was admitted as a student at Trinity College, Cambridge. From then until the beginning of World War I, in 1914, he worked there as a research student with Russell. The focus of Wittgenstein's work was on questions about logic and language. More particularly, he was starting to think through what a proper "theory of symbolism" would have to look like—a theory that not only made the basis of linguistic representation clear, but also led to an understanding of the special nature and status of logic. Wittgenstein's interactions with Russell were intense, as we know from Russell's various reports. As a result, he quickly became Russell's protégé. Russell even shocked Wittgenstein's sister Hermine, on a visit to Cambridge at the end of Ludwig's first year there, with the following remark: "We expect the next big step in philosophy to be taken by your brother" (H. Wittgenstein, 1984, p. 2).

Wittgenstein also stayed in touch with Frege during this period. In fact, he visited him several more times, both in Jena and in Brunshaupten (a small town in Mecklenburg where Frege sometimes spent his summer vacation), before the outbreak of the war. During these meetings they got to know and value each other more. In fact, Wittgenstein's sister Hermine continues her report (see above) as follows: "By the way, I have to mention that Ludwig, who had become such close friends with Professor Frege before the war . . . several times . . . spent a few days with him" (H. Wittgenstein, 1984, p. 5). Similarly, Geach's report of what Wittgenstein told him continues as follows:

I [Wittgenstein] had several discussions with him [Frege] after that [first meeting]. Frege would never talk about anything but logic and mathematics; if I started on some other subject, he would say something polite and then plunge back into logic and mathematics. He once showed me an obituary on a colleague who, it was said, never used a word without knowing what it meant; he expressed astonishment that a man should be praised for this. (Geach, 1961, p. 129)

I will come back later both to Frege's uncompromising seriousness and intensity in their discussions, as reported in Geach's report, and to the "close friendship" that developed between Frege and Wittgenstein, as noted by Hermine Wittgenstein. But let me first make a general observation as well as raise an initial question about their face-to-face meetings.

The observation concerns the number and dates of these meetings. Actually, when exactly Wittgenstein's and Frege's additional meetings took place, as well as how many of them there were, is not completely clear. We do have good evidence that, after their first meeting in 1911, Wittgenstein visited Frege at least two more times: in the fall of 1912 and in the fall of 1913.¹⁶ But it may have been more often, as both Hermine Wittgenstein and Geach emphasize that there were "several" more meetings.

What, in general, was it that Frege and Wittgenstein talked about during their meetings? From Geach's report we can infer that it was almost exclusively questions about "logic and mathematics." But what is meant by that? Clearly "logic" must be understood fairly broadly here, so as to include parts of what we would call "philosophy of logic" and "philosophy of language" today—especially what I called "a proper theory of symbolism" above. Thus, after one of his visits Wittgenstein sent the following report to Russell (in a letter from December 26, 1912): "I had a long discussion with Frege about our theory of symbolism of which, I think, he roughly understands the general outline. He said he would think the matter over" (Wittgenstein, 1961, p. 120). "Mathematics", on the other hand, must be understood primarily as "foundations of mathematics" or "principles of mathematics," in the sense these terms had assumed in Frege's and Russell's work. Besides Russell's antinomy, which we saw Wittgenstein address already in his contact with Jourdain, another prominent issue in that area is the following: How are we supposed to think about the nature of numbers? Geach's report of his conversations with Wittgenstein concludes, in fact, with a corresponding remark: "The last time I [Wittgenstein] saw Frege, as we were waiting at the station for my train, I said to him 'Don't you ever find *any* difficulty in your theory that numbers are objects?' He replied 'Sometimes I *seem* to see a difficulty—but then again I *don't* see it.'" (Geach, 1961, p. 130). Again, when exactly this last meeting between the two took place (in 1913 or later) is not completely clear.¹⁷ What is relatively clear, though, is that Wittgenstein's conversations with Frege were continuous in content with his work with Russell in Cambridge, with respect to both "logic" (including "theory of symbolism") and "mathematics."

Wittgenstein's personal contact with Frege before World War I did not just take the form of a few visits, as discussed so far; nor did it end with his last such visit, which probably took place in the fall of 1913. There was also a prolonged, fairly substantive correspondence by mail between the two. This correspondence started in 1911, when Wittgenstein asked Frege for permission to visit him in Jena for the first time; and it continued at least until 1920, the year in which Wittgenstein, after having come back from prison camp in Italy, left philosophy to become an elementary school teacher in Lower Austria.¹⁸ Unfortunately, only a small part of this correspondence is preserved, in particular for the period before World War I.

Actually, until a few years ago it looked as if all that was left of this correspondence were some brief summary notes taken by Heinrich Scholz.¹⁹ Not only the content of these notes, but also their origins and some surrounding facts are of interest to us. In the mid-1930s, Scholz, a well-known logician at the time, began to

search for and then to collect Frege's scientific correspondence and his unpublished manuscripts, that is, his scientific *Nachlaß*. This *Nachlaß* was supposed to become part of an archive for the history of logic at the University of Münster, Germany. Scholz had significant success in his search. In particular, he was able to secure an earlier, substantive collection of letters to Frege, a collection made available to the historian of science Ludwig Darmstädter by Frege himself. Scholz succeeded also in obtaining from Bertrand Russell the originals of Frege's letters to Russell; similarly in some other cases, for example, those of Husserl, Hilbert, and Löwenheim.

Crucially for us, Scholz contacted Wittgenstein in this connection, too. In this case, however, he was less successful. Wittgenstein's response to Scholz was this: "While I do own a few cards and letters from Frege, they have a purely personal and no philosophical content. For a collection of Frege's writings they have no value *at all* [*keinerlei Wert*]; they do, however, have a sentimental value [*Erinnerungswert*] for me. The thought is repugnant to me to make them available for a public collection" (Scholz, 1976, p. 265, my translation). It is not insignificant that Wittgenstein found Frege's mail to him to have some personal, "sentimental value." At the same time, his judgment that the corresponding cards and letters have "no value *at all*" for a collection of Frege's scientific writings is rather questionable (in particular now that some of them have been rediscovered and, several years after Wittgenstein's death, been made available; more on that below).

Scholz must have been disappointed about Wittgenstein's negative response. In spite of it, he soon had the following in his possession: eight letters, three post-cards, and four cards from the front (*Feldpostkarten*) from Wittgenstein to Frege; four letters, one letter-card (*Briefkarte*), and one card from Wittgenstein's sister Hermine to Frege; and two letters and one card from Frege to Wittgenstein—twenty-four documents altogether, thus a significant part of the Frege-Wittgenstein correspondence. In the process of cataloging them, Scholz recorded the dates of these documents: all from 1913 to 1919 (although in some cases their date is conjectural). He also added brief annotations concerning their contents. The first five of these annotations read as follows:

- XLV/1 Wittgenstein to Frege, 22.10.1913: Wittgenstein asks Frege for "permission for a visit."
- XLV/2 Frege to Wittgenstein, date unknown: Frege answers XLV/1 and accuses Wittgenstein of putting "too much weight on reference [*Bezeichnung*]."
- XLV/3 Wittgenstein to Frege, 29.11.1913: Content of this letter: "Important arguments against Frege's theory of truth. In particular against the determination of reference [*Bedeutungsfestsetzung*] for functions."
- XLV/4 Wittgenstein to Frege, date unknown: This letter, in which Wittgenstein announces his visit (cf. XLV/1), is an answer to XLV/2 and concerns "the establishment of a set of basic concepts for logic and the desiderata for such a set."
- XLV/5 Frege to Wittgenstein, date unknown: The letter continues the verbal conversation (XLV/4). (Scholz, 1976, pp. 265–66, my translation)

These five are all of Scholz's entries for letters and cards written before World War I. The next entry, dated 25.8.1915, concerns the beginning of Frege's and Wittgenstein's exchange about what will become the *Tractatus Logico-Philosophicus*.

Scholz's short notes provide us with some interesting glances into Frege and Wittgenstein's correspondence. They confirm the occurrence, partly also the dates, of some of the meetings between Frege and Wittgenstein during that period. They also document, again, that Frege and Wittgenstein were engaged in debates over various "logical and mathematical" themes, in particular, questions about functions, reference, truth, and, more generally, "the basic concepts for logic." One would, of course, like to find out more about these debates—one would like to look at the letters and cards themselves. Unfortunately, this is not possible any more. Most of Scholz's archival collection was destroyed during World War II when the library of the University of Münster, where he had deposited it for safekeeping, went up in flames as the result of Allied bombings. All we are left with, then, are Scholz's notes, except in a few cases in which people elsewhere kept copies of letters and cards sent by Frege to them.²⁰

Wittgenstein, it turns out, kept the mail he received from Frege. Moreover, several decades after his death, in the 1980s, it was rediscovered by accident in Vienna, as part of a larger collection of mail to Wittgenstein from various authors during 1914–31. This part or side of the Frege-Wittgenstein correspondence is now kept in the Brenner-Archiv in Innsbruck, Austria. In 1989 it was made available to the general public, as "Gottlob Frege: Briefe an Ludwig Wittgenstein."²¹ It consists of twenty-one documents—six letters and fifteen cards from Frege to Wittgenstein. In this case the dates of the documents range from 1914 to 1920. They are, thus, from the time when Wittgenstein had enlisted in the Austrian army until the time when he was in an Italian prison camp after the end of the war.²²

But before turning to this period—Wittgenstein's time as a soldier in World War I—we should look at two additional sources of information in connection with his relation to Frege before the war. The first consists of some letters exchanged between Frege and P. E. B. Jourdain, whom we have already encountered above, during 1913–14. Jourdain had earlier, probably again under Russell's influence, discussed Frege's ideas in some of his own publications. A correspondence between the two logicians ensued. This correspondence, unlike that between Frege and Wittgenstein, is fully preserved, and it contains two pieces that are of particular interest to us. In a letter from Jourdain to Frege, dated March 29, 1913, we can read:

Dear Prof. Frege,

In your last letter to me you spoke about working on the theory of irrational numbers. Do you mean that you are writing a third volume of the *Grundgesetze der Arithmetik*? Wittgenstein and I were rather disturbed to think that you might be doing so, because the theory of irrational numbers—unless you have got quite a new theory of them—would seem to require that the contradiction has been previously avoided; and the part dealing with irrational numbers on the new basis has been splendidly worked out by Russell and Whitehead in their *Principia Mathematica*. . . .

Yours very sincerely

Philip E. B. Jourdain

(Frege, 1980, pp. 76–77)

Two aspects of this passage (besides the surprising remark about Frege's supposed work on irrational numbers) are particularly noteworthy for us. First, Wittgenstein

is mentioned in it as a friend or close associate of Jourdain's. Apparently either their previous contact or Wittgenstein's subsequent association with Russell or both had led to further, amicable contacts between the two. Second, Jourdain refers to Wittgenstein in a way that assumes Frege would immediately know who he was.

Jourdain's next letter to Frege, dated January 15, 1914, is even more interesting:

Dear Prof. Frege,

Would you be kind enough to give me permission to translate part of your 'Grundgesetze' for 'The Monist'. I was thinking of the more popular parts (Bd. I, S. vi–xxvi, 1–8, 51–52; Bd. II, S. 69–80). If you will give me your permission, Wittgenstein has kindly offered to do the translation, & then I would send it on to you. . . .

Yours very sincerely

Philip E. B. Jourdain

(Frege, 1980, pp. 77–79)

Frege's prompt response, in a letter from January 28, 1914, is this:

Dear Mr. Jourdain,

I am very glad to give you permission to translate parts of my *Basic Laws* for the *Monist*. From your letter it seems to me that Wittgenstein is back in Cambridge. I had lengthy conversations with him before Christmas, and I would like to write to him so that something fruitful will come out of them, but I do not know where he is. . . .

Yours sincerely

G. Frege

(Frege, 1980, p. 81–84)

This exchange of letters reveals three things: First, Wittgenstein was interested and invested enough in Frege's *Grundgesetze* to offer to work on a translation of it. Second, Frege trusted Wittgenstein enough in connection with such a translation to give Jourdain permission to pursue the project. Third and most strikingly, the earlier conversations Frege had had with Wittgenstein were so important for him that he wanted to see that "something fruitful [would] come out of them."²³ Parts of Frege's *Grundgesetze* did, in fact, appear in the *Monist* in an English translation during the following years, but Wittgenstein seemed not to have been involved in the translation project in the end.²⁴ Then again, something fruitful did come out of his "lengthy conversations" with Frege, as we will see soon.

The second additional source of information concerning Wittgenstein's ties to Frege before the war consists of Wittgenstein's *Notebooks*, especially the part called "Notes on Logic" from 1913. In it we can observe the young Wittgenstein working hard to formulate his own insights—gained in dialogue with Russell, Frege, and their writings—about the "logical and mathematical" issues brought up above. Frege is mentioned explicitly five times in these notes. In each case Wittgenstein addresses and criticizes some particular Fregean view; he objects to (i) Frege's thesis that propositions are names, (ii) Frege's positing of truth values as objects, (iii) Frege's analysis of "is true" in connection with the content of propositions, (iv) Frege's appeal to laws of deduction to justify inferences, and (v) Frege's use of the assertion sign.²⁵ These objections are all significant—they lead to the core

of Wittgenstein's own thoughts about logic during this time. It would, thus, be worth analyzing each one of them in detail. Because my goal in this essay is Wittgenstein's relation to Frege in general, let me, instead, move ahead in his life.²⁶

III. World War I: Continuing Correspondence and Growing Friendship

We have arrived in the year 1914, one that brings more drastic change to Wittgenstein's life. With the beginning of World War I, he gives up his life as a student in Cambridge, enrolls in the Austrian army as a volunteer, and is soon engaged in active combat on the eastern front of the war. As Wittgenstein is confronted with the possibility of death almost every day, his thoughts turn increasingly to ethical and religious questions. This shift is reflected in his *Notebooks 1914–1916*.²⁷ At the same time, he keeps thinking and writing about logical issues, with the goal of publishing the results in a book. Correspondingly, the name of Frege comes up again several times in the *Notebooks*, similarly to the earlier "Notes on Logic." The particular contexts now are (i) the issue of sentences with and without sense; (ii) numbers, especially their definition and designation; and (iii) Frege's view that every assertion involves a certain content, an "assumption," that can be considered independently. Clearly the focus is, thus, again on "logic and mathematics." Also, each of those entries would, once more, be worth analyzing in detail.²⁸

Since their rediscovery, Frege's letters and cards to Wittgenstein, as contained in the Brenner-Archiv, have provided a second source of information about the two men's relation during World War I. They are, as it turns out, a valuable, sometimes surprising, source, revealing a significant strengthening of Frege and Wittgenstein's ties, on both a personal and an intellectual level.

Many of these letters and cards contain simply good wishes and patriotic encouragement from Frege to the "heroic soldier" Wittgenstein. Along these lines, Frege conveys his hope for a quick victory of the German and Austrian armies, his hope for a subsequent lasting peace, and his hope that Wittgenstein may stay alive and in good spirits during these hard times. But Frege also comments repeatedly on his desire to continue their scientific conversations from before the war. Thus he writes in a card from November 28, 1915: "I am glad that you still find the time and energy for scientific work. . . . I hope we will see each other again, in good health, after the war" (Frege, 1989, p. 10).²⁹ In a card from October 28, 1916, he suggests: "Couldn't you try to find the time, every once in a while, to put down your thoughts on paper, even if in a disconnected and unordered form, and send them to me? I would then keep your letters and attempt to answer them. That way a scientific interchange between us could perhaps be established after all, as a small substitute for our verbal exchanges" (p. 13). And in a letter from December 12, 1918, he adds: "I hope it will be granted to me to get to know your views better in a verbal exchange. . . . I still remember fondly our walks in Jena and Brunshaupten" (p. 18). It is possible that such remarks were just meant to lift Wittgenstein's spirits while he was on the front. More likely, it seems to me, is that Frege genuinely valued his earlier intellectual exchanges with Wittgenstein.

In a number of Frege's letters and cards to Wittgenstein there is another aspect worth mentioning, namely, the way their "friendship" is emphasized more and

more by Frege.³⁰ This emphasis is remarkable, I think, and not just if we take into account the age difference between the two men: Frege is forty-nine years older than Wittgenstein. Then again, it is in consonance with Hermine Wittgenstein's observation that her brother Ludwig "became such close friends with Professor Frege" during this period (see above). It is also not unintelligible, especially if one pays attention to remarks such as the following by Frege: "In long conversations with you I have come to know a man who has, like me, searched for the truth" (Frege, 1989, p. 21). Probably Frege's and Wittgenstein's deepening friendship was based on several factors, including the following: Wittgenstein's eagerness and persistence to engage in the logical investigations Frege valued so highly; his willingness to seek out Frege for conversations about corresponding issues; probably also Wittgenstein's patriotic (as Frege saw it) choice to volunteer as a soldier and his subsequent bravery in the war; conversely, Wittgenstein's general admiration for Frege's work; the strong impression Frege's intensity and seriousness made on him during their conversations; and Wittgenstein's gratitude for Frege's encouragement of his own logical work. We can find more evidence for the first three of these factors, concerning Frege's side, in his letters and cards to Wittgenstein. On Wittgenstein's side further evidence can be found in the various reports by students and friends about his lasting admiration and affection for Frege, such as those by Geach, Drury, and Hermine Wittgenstein quoted above.³¹

The longest and philosophically most substantive of Frege's letters to Wittgenstein in the Brenner collection are the last four, from the period between June 1919 and April 1920. These letters concern Frege's reaction to the *Tractatus*, which Wittgenstein sent to Frege immediately after finishing it. We will turn to this reaction shortly (in a separate section). However, there is another, earlier letter that is perhaps even more remarkable, especially as far as their personal relationship goes. It is dated April 9, 1918, and consists of a response to a letter by Wittgenstein from March 25, 1918. It falls, thus, in the period in which Wittgenstein was trying hard to finish a draft of the *Tractatus*, the book that was supposed to contain all his philosophical insights. Let me quote Frege's letter in full, as it is surprising in a number of ways:

Dear Mr. Wittgenstein:

You cannot imagine how astonished I was on reading your friendly letter from 25.III.18. You write of a great debt of thanks [*grosse Dankesschuld*] to me. I know nothing of such a debt. Each of us has gained, I think, in our intellectual exchange. If I have furthered you more in your endeavors than I realized, I am glad; in particular because I know that these endeavors, in their lofty flight, rise high above the world of base selfishness. What you have gained in our exchange will, I hope, move humanity ahead a bit in the right direction. If the words I have exchanged with you thus live on in their effects, that is a comforting prospect for me. May you, dear friend, be granted to experience some of these effects. What remains for me is to accept, with heartfelt thanks, what, in a noble impulse, you have given me as a present [*was Sie mir in edelster Regung zugesucht haben*], as I assume it was meant by you that way.

With friendly greetings,

Yours, G. Frege

PS: I haven't received a notification from Jena yet; however, the Niederösterreichische Escompte-Gesellschaft has notified me of the crediting [Akkreditierung] of the bank in Thüringen. (Frege, 1989, p. 16)

As revealed by this letter, Wittgenstein felt enormously grateful—he felt “a great debt of thanks”—to Frege in connection with his recent intellectual endeavors. That is to say, he appreciated their earlier meetings and their correspondence even more than did Frege. But not only that; it seems that Wittgenstein also sent Frege—who was not particularly well paid as a professor at Jena, as well as about to retire—a significant amount of money (see the postscript). It is possible that this money was part of the inheritance Wittgenstein was in the process of giving away more generally. In any case, Frege accepted it, while also gracefully stressing the mutual benefit of their intellectual exchanges.³²

One more observation about the period 1914–18: The letter from Frege just quoted is from 1918, close to the end of World War I. The two years preceding it, 1916–18, were the time of the composition of Wittgenstein's *Tractatus*, or better of its compilation and construction out of earlier notes. Wittgenstein worked on his book both at the eastern front and back in Vienna, during a few short leaves from the front. These leaves also gave him a chance to try to arrange some further meetings with Frege, this time not in Germany, but in Austria. Thus both in the winter of 1916 and in the spring of 1917, he invited Frege to come and visit him in Vienna. In both cases Frege declined, as the trip would either have been too strenuous or otherwise impossible for him.³³ Frege was, we should note, almost seventy years old by then, as well as not in great health. In the fall of 1919 Wittgenstein, just back from the war and from prison camp, then planned to visit Frege again in Jena.³⁴ But apparently that visit also did not take place. The reason is not clear: maybe Frege, retired since the fall of 1918, was not in Jena much any more; or Wittgenstein simply changed his mind, for example, in connection with his new plans to become a schoolteacher—or perhaps he found Frege's recent reactions to the *Tractatus* less than encouraging.

IV. Wittgenstein's *Tractatus*: High Hopes and Deep Disappointments

Wittgenstein finished a complete draft of the *Tractatus* in the summer of 1918, probably in July and August, while on leave from the front in Vienna and Salzburg.³⁵ Shortly after that he informed Frege about it, who congratulated him promptly in response (actually several times, most emphatically in letters from September 12 and October 15, 1918).³⁶ Frege also wished Wittgenstein to be able to publish his work soon, and he expressed an interest in seeing a copy of the book himself, remarking, “I would be very pleased if you sent me a copy” (Frege, 1989, p. 18). Earlier, Frege had already sent him a very encouraging and inviting note in response to his announcement, in May 1918, that he was almost done with his work. Dated June 1, 1918, this note reads as follows:

Many thanks for your card from 10.V. I am glad that you have come to some conclusion concerning your work. I hope it will be possible for you to put down on paper everything you have come up with, so that it

doesn't get lost. Perhaps I, too, will gain from it while travelling in the difficult terrain in which I am toiling. I am, of course, always ready to learn more and to let myself be led back onto the right path if I have gotten lost. And the paths you have followed always promise some gain for me, even if I am not able to follow you in the essentials. Good luck for further vigorous work!

With warm regards,

Yours, G. Frege

(Frege, 1989, p. 17).

Frege's expressed willingness to follow Wittgenstein down "his paths" is noteworthy here. It shows, again, how seriously he took Wittgenstein as an interlocutor (although his qualification about "not being able to follow in the essentials" would prove prophetic). Later, in a letter from October 15, he also promised to reciprocate by sending Wittgenstein a new work of his own: his article "Der Gedanke," which had just come out.

In terms of his work on logic, things now looked good for Wittgenstein. Not only was he done with the book he had worked on for several years, under extremely difficult conditions, but a revival of his meetings and conversations with Frege, cherished by both of them, seemed imminent. On the other hand, Austria's and Germany's war efforts were proving futile. Soon Wittgenstein found himself captured and put into prison camp in Italy; first in Como, from early November 1918 on, then in Monte Cassino, until late August 1919. Nevertheless, in June 1919, while still in prison camp in Monte Cassino, he managed (with the help of his sister, several of his friends from Cambridge, and the Red Cross) to send the two copies of the *Tractatus* he had with him to the two people he wanted most to read it: Frege and Russell.³⁷ After that he eagerly awaited their response.

Before looking at Frege's response directly, let us briefly consider the following question in the background: What, in general, is the relation of the *Tractatus* to Frege's work? As we just saw, it was shortly before its completion in 1918 that Wittgenstein talked about the "great debt of thanks" he owed to Frege. Similarly, in the preface to the *Tractatus* Wittgenstein emphasizes: "[I] am indebted to Frege's great works and to the writings of my friend Mr. Bertrand Russell for much of the stimulation of my thought" (Wittgenstein, 1963, p. 3). Whether or not Wittgenstein intended to slight Russell here by calling only Frege's works "great," it is clear that he saw the *Tractatus* as closely related to both Frege's and Russell's works. It is also clear that he had high hopes of impressing both of them with it. Actually, Wittgenstein's philosophical aspirations were extremely high; as he writes in the preface to the work: "I . . . believe myself to have found, on all essential points, the final solution to the problems [of philosophy]" (p. 4). The "problems" here include, presumably, all the questions concerning "logic and mathematics," or concerning a "proper theory of symbolism," that had come up in his earlier conversations with both Frege and Russell; or at least those among these questions that he saw as left unanswered in Frege's and Russell's works.

A brief look at the *Tractatus* itself confirms the importance of Frege's ideas for the book. Apart from the preface, Frege's name is mentioned in eighteen of its entries. (In that respect he is second only to Russell, whose name comes up twenty-eight times.) The topics addressed in these entries, in their order of appearance, are the following: Frege's thesis that sentences are composite names (3.143), gen-

eral Fregean ideas about the compositionality of propositions (3.318), the notions of logical grammar and of a conceptual notation (3.325), the use of truth values (4.063), the need, or lack thereof, for functions and classes (4.1272), the notion of a formal concept (4.1273), the concept of truth and the sense of negation (4.431), Frege's judgment stroke (4.442), Frege's more particular views about compositionality (5.02), the role, or lack thereof, of laws of inference in logic (5.132), the rejection of logical objects and logical constants, both in Frege's and in Russell's sense (5.4), the undefinability of primitive signs of logic (5.24), Frege's use of definitions for that purpose (5.451), sentences without sense (5.4733), Frege's and Russell's treatment of generality (5.521), Frege's appeal to self-evidence for basic laws of logic (6.1271), and Frege's treatment of equations (6.232).

As this list shows, in the majority of cases in which Frege is mentioned by name in the *Tractatus*, Wittgenstein is critical of his views. But this is not always the case (e.g., 3.318), and sometimes he partly agrees, partly disagrees with Frege (e.g., 3.325, 4.431). In addition, there are a number of more indirect references to Frege in the *Tractatus*, some of which are clearly not critical at all. An interesting example occurs in 3.3: "Only propositions have sense; only in the nexus of a proposition [im Satzzusammenhang] does a name have meaning" (Wittgenstein, 1963, p. 14). The implicit reference here is to Frege's so-called context principle: "It is only in a proposition that words have any meaning" (Frege, 1950, p. 73; cf. also pp. x, 71, 116). As this example shows, Wittgenstein does adopt some theses, or themes, from Frege so as to build them into his own approach.³⁸ Other, more indirect, examples of Frege's positive influence on the *Tractatus* include the notions of logical form and of formal concept, the saying-showing distinction, and the discussion of sense versus nonsense in the *Tractatus*. With respect to these Wittgenstein is, arguably, building on Frege's distinction between concepts and objects, Frege's remarks on elucidation, and similar sources.³⁹ Still, often Wittgenstein does develop his own ideas in opposition to Frege's (similarly for Russell), as my earlier examples illustrated. Then again, in those cases, too, Fregean ideas are not insignificant for Wittgenstein, at least as starting points or foils.

After sending a copy of the *Tractatus* to Frege, Wittgenstein was, as indicated above, eagerly awaiting a response from Frege. Soon, however, his hopes in this connection were disappointed. Expressions of this disappointment can be found in his correspondence with Russell. Thus in a letter to him from August 19, 1919, Wittgenstein exclaims: "[Frege] wrote to me a week ago and I gather he doesn't understand a word of [the *Tractatus*]. So my only hope is to see you soon and explain all to you, for it is VERY hard not to be understood by a single soul!" (Wittgenstein, 1995, p. 124, emphasis in original). Similarly, in another letter to Russell, dated October 6, 1919, he writes in exasperation: "I am in correspondence with Frege. He doesn't understand a word of my work and I am already quite exhausted from all my explanations" (p. 131). Russell, of course, also did not understand Wittgenstein's ideas properly, as would become clear soon. Russell's more positive reaction to the *Tractatus* overall would, on the other hand, prove helpful in publishing the work later.

Wittgenstein's disappointment and exasperation about Frege, as reflected in his letters to Russell, were based on the four letters he received from Frege in 1919–20. The first of these is dated June 28, 1919—more than a year after Frege's quite

encouraging comment about “hoping to gain from” Wittgenstein’s work and about being willing to “follow him in his paths.” This letter, much longer than all the previous ones from Frege, starts as follows: “You have, I am sure, been waiting for a response from me for quite a while, as well as for a remark by me about your treatise which you have sent me” (Frege, 1989, p. 19). After that Frege goes on to make several “remarks” about the *Tractatus*, that is, he presents Wittgenstein with his immediate reaction to it. He continues to do so in letters from September 16, 1919, September 30, 1919, and April 3, 1920.

What, then, is Frege’s reaction to the *Tractatus* in these letters? Overall it is quite critical and negative. His most basic criticism is this: the book it is “hard to understand,” its “sense is not clear enough,” and it “needs to be explained more” (Frege, 1989, p. 19). In fact, Frege gets stuck already on the first few pages of the *Tractatus*:

Right at the beginning I come across the expression ‘to be the case’ [*der Fall sein*] and ‘fact’ [*Tatsache*] and I suspect that being the case and being a fact are the same. The world is everything that is the case and the world is the collection of facts. Is not every fact the case and is not that which is the case a fact? Is it not the same if I say, *A* is a fact, as if I say, *A* is the case? Why then this double expression? . . . Now comes a third expression: ‘What is the case, a fact, is the existence of states of affairs (*Sachverhalte*).’ I take this to mean that every fact is the existence of a state of affairs, so that another fact is the existence of another state of affairs. Couldn’t one delete the words ‘existence of’ and say ‘Every fact is a state of affairs, every other fact is another state of affairs?’ Could one perhaps also say ‘Every state of affairs is the existence of a fact?’ . . . You see, from the very beginning I find myself entangled in doubt as to what to say, and so make no proper headway. (pp. 19–20)⁴⁰

Later Frege reiterates his incomprehension of these notions and theses by asking: Is the statement “The world is everything that is the case” a definition; is it instead a statement of fact, that is, a “recognition judgment [*Wiedererkennungsurteil*]”; or does Wittgenstein think there is a third alternative (p. 26)? These are all very basic questions, and without answers to them Frege does not seem to be able to make it further into the book.

And there is more. Frege also criticizes Wittgenstein’s general way of presenting his ideas. He observes that the *Tractatus* consists mainly of various claims presented without either a real justification or a proper motivation; he comments: “I would like to see a question asked at the beginning, a puzzle for which it is pleasing to find the solution. . . . I miss a proper introduction in which a goal is presented” (pp. 23–24). On a related theme, Frege finds Wittgenstein’s remark in the preface concerning the general purpose of the book strange. In particular, to say, as Wittgenstein does, that “its purpose would be achieved if it gave pleasure to one person who read and understood it” (Wittgenstein, 1963, p. 3), blurs, in Frege’s eyes, the distinction between “aesthetic achievement [*künstlerische Leistung*]” and “scientific achievement [*wissenschaftliche Leistung*]” in an unacceptable way (Frege, 1989, p. 21). Overall Wittgenstein must have been devastated. Not only did Frege display a disappointing lack of understanding for

the content of his work (even for its early parts on logic, not just the later parts on ethics and the mystical); he also did not appreciate its style, an aspect to which Wittgenstein had devoted so much attention and which he considered almost as crucial as its content.

As mentioned above, Frege responded to the receipt of the *Tractatus* not just by commenting on it, but also by sending Wittgenstein his own article “Der Gedanke.” From a few asides in Frege’s four letters of 1919–20, we can infer that Wittgenstein, in turn, sent Frege comments on that article. And he, too, was critical in his response, although it is hard to tell what exactly those criticisms were. Some of them must have had to do with Frege’s own criticism of idealism in “Der Gedanke”; thus Frege writes: “I have just noticed, from an earlier letter of yours, that you recognize some deep, true core in idealism” (p. 26). Similarly earlier: “I would like to know which sources of idealism you think I haven’t grasped” (p. 24). There is here, I would suggest, another noteworthy connection to the *Tractatus* in play. The “deep, true core in idealism” which Frege quotes Wittgenstein as recognizing probably has to do with what the *Tractatus* says about this issue, or about the related issue of “solipsism,” namely:

- 5.62 This remark provides the key to the problem, how much truth there is in solipsism. For what a solipsist *means* is quite correct; only it cannot be *said*, but makes itself manifest. The world is *my* world: this is manifest in the fact that the limits of *language* (of that language which alone I understand) mean the limit of *my* world.
- 5.621 The world and life are one.
- 5.63 I am my world. (The microcosm.) (Wittgenstein, 1963, p. 57)

How exactly these Tractarian theses connect with Frege’s criticism of idealism in “Der Gedanke” would, again, be very interesting to explore further. One also wishes, of course, that Wittgenstein’s related responses to Frege, as collected by Scholz, had not been lost; they might have shed some light on this cryptic and difficult side of the *Tractatus*. Moreover, Wittgenstein’s corresponding letters probably contained some interesting further comments on Frege’s “Der Gedanke.” (I will come back to this issue very briefly later.)

In retrospect, clearly neither Frege nor Wittgenstein reacted very positively to the other’s work from this period, and neither of them concealed his negative reaction in their correspondence. This makes one wonder whether it was this clash of views that prevented Wittgenstein from visiting Frege in Jena once more after coming back from prison camp in August 1919. Similarly, one wonders whether it was this clash that caused the end of their correspondence in April 1920. We should not be too hasty here, though. Note, for example, that in his criticism of the *Tractatus* Frege is actually less harsh and polemical than in his criticism of almost every other thinker in his writings (from Husserl to Dedekind and Cantor, not to mention Kerry and Schubert).⁴¹ In particular, his tone always remains respectful, and he does not dismiss Wittgenstein’s ideas simply because he does not understand them. In the letter from September 16, 1919, Frege even goes out of his way to emphasize his continuing “hope for understanding.” He adds: “I combine it with the hope that you will, at some point, stand up for [*eintreten für*] the insights I believe I have achieved in logic. Before that you would, however,

have to be won over to them. For that reason a further exchange of opinions with you is desirable for me" (Frege, 1989, p. 21). Later in the same letter Frege remarks that it is important to help each other "see with the eyes of the other." Finally, he emphasizes (as already quoted above): "In long conversations with you I have come to know you as a man who, like me, has searched for the truth, if sometimes on different paths" (p. 21). Thus, even while fundamentally failing to understand the *Tractatus*, Frege kept holding Wittgenstein in very high esteem. Wittgenstein, in turn, did not give up his admiration and affection for Frege either, as will become clear soon.

One final piece of information is worth considering in connection with Frege and the *Tractatus*, to round off our discussion. It involves Wittgenstein's attempts to publish his book. After finishing a complete manuscript, in the summer of 1918, Wittgenstein sent this manuscript to two Viennese publishing companies: first the company of Jehoda and Siegel, the publishers of Karl Kraus's works (this actually happened before he became a prisoner of war in November 1918); then to the company of Braunmüller, the publishers of Otto Weininger's works (in that case after coming back from prison camp to Vienna late in August 1919). But neither of these publishers was willing to accept the *Tractatus*, at least not in a form acceptable to Wittgenstein. (Braunmüller offered to publish it if Wittgenstein paid for the paper and the printing costs himself, which he steadfastly refused.) At this point Wittgenstein turned to Frege for help. In particular, he inquired if Frege was willing to approach the publisher of his own recent writings, including "Der Gedanke," in this connection: the journal *Philosophie des Deutschen Idealismus*.

Despite all his explicit criticisms of the *Tractatus*, as documented above, Frege's reaction to Wittgenstein's inquiry was not wholly negative. In fact, he offered to write a letter on his behalf to the philosopher Bruno Bauch, who was on the editorial board of the journal in question. Bauch was, in addition, a colleague of Frege's in Jena at this point, as well as someone with whom he had good collegial relations. The offer to write such a letter was thus promising. Moreover, Frege indicated to Wittgenstein that in it he would be willing to present him as a "thinker to be, by all means, taken seriously" (Frege, 1989, p. 23). At the same time, Frege made clear that he would not be able to say much in praise of the *Tractatus* itself, because he did not understand its content well enough. Finally, he cautioned Wittgenstein that his influence on the matter was limited. In the end the whole attempt came to nothing. It seemed to be the length and form of Wittgenstein's work that prevented the editors, not surprisingly, from including it in such a journal. And Wittgenstein was, needless to say, not willing to change those aspects, not even at Frege's urging.

Several more failed attempts at publishing the *Tractatus* followed. Thus, the Austrian journal *Brenner*, edited by Ludwig von Ficker (to whom Wittgenstein had also given a considerable amount of money recently, for distribution among contributors to the journal such as Trakl and Rilke), the Insel Verlag and the Reclam Verlag in Germany, as well as Cambridge University Press in England rejected it. When the *Tractatus* finally did get published, it was largely due to Russell's help. It first appeared in German, in 1921, in the *Annalen der Natur- und Kulturgeschichte*, edited by Wilhelm Ostwald; then in 1922, also in an English translation (by C. K. Ogden, with substantial help from Frank Ramsey) published by Routledge and Kegan Paul.⁴²

V. Wittgenstein's Middle Period: Continuing Engagement with Frege's Work

In August 1919 Wittgenstein came back to Vienna from prison camp in Italy. In December of the same year, he and Russell met in The Hague, Netherlands, to talk about the *Tractatus* and its publication. In the meantime, Frege had retired, in 1918, and in 1918–19 two of his last publications appeared, “Der Gedanke” and “Die Verneinung.” (Frege’s very last publication, a related article called “Gedankengefüge,” would appear in 1923.) Frege and Wittgenstein were still in correspondence by then, until April 1920. At that point their correspondence stopped, at least as far as we can tell. It is also at that point that another of Wittgenstein’s life-altering “transformations” occurred, as his sister Hermine called them: not only did he give up his whole inheritance, including a large amount of money; he also did not go back to Cambridge, but became an elementary school teacher in Lower Austria instead. For Wittgenstein the decision not to go back to Cambridge was a conscious choice to leave philosophy behind. His new career as a schoolteacher started in the fall of 1920, in the small Austrian village of Trattenbach.

It is natural to assume that these events mark the end of Wittgenstein’s relation to Frege. In one sense this is obviously correct (especially after Frege’s death in 1925): the two did not see each other in person any more, nor did their correspondence by mail continue. In another sense, however, Wittgenstein’s relation to Frege did not break off at this point: he kept being engaged with Frege’s writings and ideas. This engagement continued, in fact, until the very end of Wittgenstein’s life in 1951.

My discussion of Wittgenstein’s connection to Frege so far has been relatively comprehensive, that is, I have tried to utilize all the major pieces of evidence we have concerning it up to the year 1920. For the time after 1920 I must be more eclectic, both because of constraints on the length of this essay and because much more research is needed in this connection. My discussion will, nevertheless, be representative in at least this sense: it will touch on all the major remaining periods in Wittgenstein’s life. In the rest of this section I will consider several incidents involving Frege’s work from Wittgenstein’s middle period (after the *Tractatus* into the 1930s); in the next section I will consider several more from his later period (through the 1940s until 1951), including some from the last two to three years of his life.

But first back once more, briefly, to the time before 1920. The earliest traces of Wittgenstein’s ongoing engagement with Frege’s works are from when he was still in prison camp in Monte Cassino. Thus, in a letter to Paul Engelmann, dated May 24, 1919, Wittgenstein asks his Viennese friend for the following favor: “Now a request: please send me, safely and quickly if you can, Frege’s *Grundgesetze*” (Engelmann, 1967, p. 17). Note that this event occurred after Wittgenstein had finished a draft of the *Tractatus* and shortly before sending it off to Frege and Russell. Why did Wittgenstein want a copy of *Grundgesetze der Arithmetik* at this point? It could be that he merely wanted to check some references to Frege’s work in the *Tractatus*. But the *Tractatus* is not a very scholarly work; the references in it are scarce and mostly implicit. More likely, then, is that Wittgenstein wanted to keep reading Frege, the logician he still admired very much. Additional evidence for this continuing

admiration can be found in a report from Franz Parak, another of Wittgenstein's friends during this period. In his report Parak writes about various conversations he had with Wittgenstein, some of them concerning philosophical issues, while they were both in Monte Cassino. Frege is mentioned in this connection as a person "whom [Wittgenstein] admired [so] much [*den er so verehrte*]."⁴³

This pattern continues after 1920, after Wittgenstein became a schoolteacher in Trattenbach. In a letter from October 31, 1920, he writes again to Engelmann:

Would you do me the following great favor! Will you kindly send me the two volumes of Frege, *Grundgesetze der Arithmetik*. REGISTERED AND EXPRESS to the following address: Miss Anna Knaur, c/o Faber, Heinrichsthal, near Lettowitz, Moravia. This lady will not herself study logic, but will bring me the book unread. As she will leave on the 10th, the matter is very pressing. When you come for Christmas, you shall get the Frege back (Engelmann, 1967, pp. 39–40)

Apparently Wittgenstein had left his copy of *Grundgesetze* in Vienna, but he did not want to be without Frege's work during his time in Trattenbach.

Wittgenstein did not stay in Trattenbach for long, as the Austrian school board moved him around. His next station as an elementary school teacher was Puchberg, another small village in Lower Austria, where he started to teach in 1922. Soon after his arrival in Puchberg, Wittgenstein was contacted by Frank Ramsey, then a young and promising logician in Cambridge. Wittgenstein—who knew of Ramsey's involvement in Ogden's English translation of the *Tractatus*—agreed to several meetings with him (both in Puchberg and in Vienna, during some school vacations). The main topic of discussion in their meetings was the *Tractatus*. But not only that; as in Parak's case, Wittgenstein also impressed the greatness of Frege's works on Ramsey.⁴⁴

At this point Wittgenstein seemed to have been particularly interested in Frege's criticism of formalist views in mathematics, as contained both in *Grundgesetze der Arithmetik* (in connection with Frege's discussion of various views about the real numbers) and in another, smaller text called "Die Zahlen des Herrn H. Schubert." Ramsey witnesses this interest, as well as the more general impact his discussions with Wittgenstein had on himself, in a letter to him from November 11, 1923:

[I] have been reading miscellaneous things, a little Relativity and a little Kant, and Frege. I do agree that Frege is wonderful; I enjoyed his critique of the theory of irrationals in the *Grundgesetze* enormously. I should like to read 'Über die Zahlen des Herrn H. Schubert' but haven't yet found a copy. (Wittgenstein, 1995, pp. 190–91)

In another letter from December 27, 1923, Ramsey writes: "*I think Frege is more read now*; two great mathematicians *Hilbert and Weyl* have been writing on the *foundations of mathematics* and *pay compliments* to Frege, appear in fact to have appreciated him to some extent. His unpopularity would naturally go as the generation he criticized does" (p. 194, emphasis in the original). In this second passage Ramsey seems to be responding to complaints by Wittgenstein that Frege is not appreciated enough by their contemporaries. That is, thus, apparently another point emphasized by Wittgenstein in their conversations.

Wittgenstein's time as a schoolteacher ended, unsuccessfully and unhappily, in 1926. He moved back to Vienna at this point, where he stayed until 1929. During this period in Vienna he was involved in various activities: he worked as, among other things, a gardener, an architect, a sculptor, and a photographer. He was also, more and more, drawn back into philosophy. In particular, Moritz Schlick, the head of the Vienna Circle, succeeded in contacting him in 1927, after some failed attempts earlier. Schlick, like various other members of his circle, was very impressed by the *Tractatus* and wanted to meet its author. Schlick's initiative led to several meetings, from 1927 until 1932 (when Wittgenstein was already back in Cambridge, but still visiting Vienna regularly). In these meetings Wittgenstein discussed both his own book and some further texts with various members of the circle (Schlick, Carnap, Waismann, and others).⁴⁵ One of the further texts the group covered, quite likely on Wittgenstein's suggestion, was again Frege's *Grundgesetze der Arithmetik* (especially its second volume).⁴⁶

Wittgenstein's discussions of *Grundgesetze* with the Vienna Circle were, it turns out, a continuation of his earlier discussions with Ramsey. One main topic in them was again Frege's criticism of formalist views about mathematics. But the discussions also extended further, to issues such as the following: the role of contradictions in mathematics, the use of the infinite, and the notion of a calculus. The explicit appeal to Frege's *Grundgesetze* in connection with these issues shows that Wittgenstein still found it, if not congenial, then at least a valuable and fruitful starting point for further investigations. At the same time, it becomes clear that he wanted to move beyond Frege in crucial ways at this point—in some sense even to defend formalist ideas, or some distant descendants of them, against his criticisms. A particular focal point for Wittgenstein's thoughts during this period was the comparison of the role of number words, or words more generally, with the use of pieces in a chess game. It seems, in fact, that further reflections on this parallel played a crucial role in Wittgenstein's move away from his earlier Tractarian views about language and toward his later conception of language games.⁴⁷ If this is true, Frege's writings are again tied to a central Wittgensteinian idea, now one from his middle and later periods.

This general thesis is confirmed if we move on to what is probably the best known and most influential of Wittgenstein's writings from his middle period: the *Blue Book* (1933–34). Frege's name does not occur often in this text, actually only once; but it does so at a crucial point. Wittgenstein writes:

Frege ridiculed the formalist conception of mathematics by saying that the formalists confused the unimportant thing, the sign, with the important, the meaning. Surely, one wishes to say, mathematics does not treat of dashes on a bit of paper. Frege's idea could be expressed thus: the propositions of mathematics, if they were just complexes of dashes, would be dead and utterly uninteresting, whereas they obviously have a kind of life. And the same, of course, could be said of any proposition: Without a sense, or without a thought, a proposition would be an utterly dead and trivial thing. And further it seems clear that no adding of inorganic signs can make the propositions live. And the conclusion which one draws from this is that what must be added to the dead sign in order to make a live proposition is something immaterial, with properties different from all mere signs.

But if we had to name anything which is the life of the sign, we should have to say that it is its *use*. (Wittgenstein, 1958, p. 4)

Clearly the reference to Frege and formalism here is continuous with Wittgenstein's conversations with Ramsey and the Vienna Circle. Also, the conclusion at the end points, once more, toward Wittgenstein's later language game conception.

The years from 1929 to the mid-1930s—partly still in conversation with Schlick's circle in Vienna, partly back in Cambridge, where the conversations were with Ramsey, Sraffa (a new friend), and others—form a period in which Wittgenstein kept thinking intensely both about mathematics and about language. A further important question occupying him then was: Which general method or methods are appropriate for philosophical inquiry? He was more and more convinced that the method used in the *Tractatus* was fundamentally inadequate. In addition, Wittgenstein was led to reflect more generally on the most important influences on his own thoughts so far. A result of those reflections is the following interesting entry in his notebooks from the early 1930s (probably 1931):

I don't believe I have ever invented a line of thinking. I have always taken one from someone else. I have simply straightaway seized on it with enthusiasm for my work of clarification. That is how Boltzmann, Hertz, Schopenhauer, Frege, Russell, Kraus, Loos, Weininger, Spengler and Sraffa have influenced me. (Wittgenstein, 1980, p. 19)

Clearly Wittgenstein is too depreciating of his own originality in the first half of this passage. The second half, on the other hand, is a further piece of evidence for his continuing awareness and acknowledgment of Frege's strong influence on him—Frege is presented as one of a few thinkers whose work he “seized on with enthusiasm.”

A second notebook entry from the early 1930s points toward a more particular aspect of Frege's influence. Wittgenstein writes: “The style of my sentences is extraordinarily strongly influenced by Frege. And if I wanted to, I could establish this influence where at first sight no one would see it” (Wittgenstein, 1970, §712).⁴⁸ This remark is at first quite surprising because Wittgenstein's style—both in the *Tractatus* and in his middle and later works—is not clearly or obviously similar to Frege's. Moreover we saw above that Frege was rather dissatisfied with the way Wittgenstein presented his ideas in the *Tractatus*. The key here is, I would suggest, not to interpret “style” too narrowly. What is probably included for Wittgenstein under that heading are attributes such as the following: seriousness, commitment, and intensity of tone; the way in which philosophical questions are addressed head-on; and the importance of precise, suggestive images and metaphors, as well as of illuminating comparisons. All of these attributes can be found both in Frege's and in Wittgenstein's works, if not exactly in the same way. And if we compare, say, Russell's writings we find a much looser, sometimes almost frivolous, discussion of philosophy in them that stands in real contrast to Frege's and Wittgenstein's works. Finally, what may be included under “style” as well are aspects of philosophical method in some general sense, such as these: Frege's technique of taking problematic philosophical positions (including formalism) really and seriously at their word, so as to think them through until their absurdity becomes apparent; and his ability of looking at logical and philosophical is-

sue with new eyes, of putting aside all preconceived ideas and starting afresh in connection with them. Such aspects, too, can be found both in Frege's and in Wittgenstein's works.⁴⁹

VI. Wittgenstein's Later Years: Readings and Respect until the End

It would be worthwhile, I think, to examine further Wittgenstein's notebooks, lectures, and conversations with friends from his transitional period (including the late 1930s, as well as the early 1940s) with respect to their relation to Frege's work. But let me jump ahead to his later work now, in particular to *Philosophical Investigations* (finished 1945–49), its most mature presentation.

In the *Investigations*, too, Frege's name comes up several times—four times, to be precise. In three cases Wittgenstein is clearly critical of Frege, in the first two of those in connection with issues we have encountered before: (i) he criticizes Frege's use of the assertion sign (§22); (ii) he remarks on Frege's claim that every assertion contains an “assumption” (n. 11); and (iii) he addresses Frege's requirement that concepts have sharp boundaries (§71). The fourth appeal to Frege, in §49, is not critical:

For naming and describing do not stand on the same level: naming is a preparation for description. Naming is so far not a move in the language-game—any more than putting a piece in its place on the board is a move in chess. We may say: *nothing* has so far been done, when a thing has been named. It has not even *got* a name except in the language-game. This is what Frege meant too, when he said that a word had meaning only as part of a sentence. (Wittgenstein, 1953, p. 24)

Interestingly, Wittgenstein's explicit agreement with Frege in this passage is, just as in the *Tractatus*, with Frege's context principle.⁵⁰

Wittgenstein's appeal to the context principle has, in fact, more deep-seated and systematic roots than the brief, unexplained reference to it in *Investigations* §49 may suggest. Thus further, similar references occur throughout Wittgenstein's writings, from the *Tractatus* to some of his very last notebooks. To recognize them as such let us recall, first, Frege's original formulation of the principle:

[I]t is only in a proposition that words have any meaning. (1884).

Wittgenstein echoes it—in what sounds like variations on a theme—as follows:

Only in the context of the sentence does a name have meaning. (1918)

A name has meaning, a sentence has sense, in the calculus to which it belongs. (1932–34)

Only in the practice of a language can a word have meaning. (1943–44)

Only in the stream of life do words have their meaning. (1948–49)⁵¹

Clearly many of Wittgenstein's philosophical views change significantly, even dramatically, from the *Tractatus* to his last writings. Similarly, his appeals to the context principle—the ways in which it is formulated and absorbed into Wittgenstein's own approach—change.⁵² That being so, it is striking that he finds

something at the core of this principle not only important, but also congenial, from early to late.

Wittgenstein's *Philosophical Investigations* (essentially finished in 1949) was his last major work. It is, however, not the last piece of evidence for us to consider. There are three more such pieces, all three involving not Wittgenstein's writings, but his interactions with students and friends. First, in the summer of 1949, Wittgenstein went on a trip to the United States, at the invitation of Norman Malcolm. In July and August of that year he was in Ithaca, New York, where he had various conversations with Malcolm and his colleagues at Cornell University. More particularly, Malcolm and the others engaged Wittgenstein in several reading groups. Participants in these groups, besides Wittgenstein and Malcolm, were Max Black, Oets Bouwsma, Stuart Brown, Willis Doney, and John Nelson. Later both Malcolm and Bouwsma reported on various aspects of these meetings.⁵³ Both tell us that Wittgenstein did not just want to talk about his own work with them, but also about some other texts—including some of Frege's writings. In this case it was especially Frege's article “Über Sinn und Bedeutung” he wanted to discuss. Finally, both Malcolm and Bouwsma relate that Wittgenstein's attitude toward this text was not, as one might expect, that toward an antiquated, long-refuted work. Rather he still, or again, was very much engaged in its content; he still found it worthwhile, even necessary, to actively separate what was in his view right in it from what was wrong.

Thus even very late in his life, Wittgenstein wanted his students to read Frege's works, and not just as an example of someone who was definitely or completely refuted. A second episode, from 1950, points in the same direction. In the fall of this year, Wittgenstein went on another trip, this time to Norway. He was accompanied by Ben Richards, another of his students and then his close friend. Earlier in 1950 one of Frege's works, *Die Grundlagen der Arithmetik*, had finally come out in an English translation, by J. L. Austin (Frege, 1950). Wittgenstein and Richards took this translation with them to Norway; it, like “Über Sinn und Bedeutung” in Ithaca, was to serve as the basis of joint readings and discussions for them. Again Wittgenstein wanted to treat it, we may suppose, not just as one text among many others from the history of philosophy, but as a text that still had real relevance. However, their trip had to be cut short, as first Richards fell ill and then Wittgenstein became unwell himself. In fact, Wittgenstein's health was deteriorating rapidly from this point on, as he was suffering from cancer.⁵⁴

The last episode I want to mention is from the years 1950–51. It brings us back to G. H. von Wright and Peter Geach. The background here is this: Geach had just been offered his first academic job, and Wittgenstein, one of whose last students Geach had been, took an active interest in how he was doing. Geach, it turns out, had picked Frege's philosophy as the topic for his very first philosophical lectures. On this choice, as well as on the success of Geach's lectures, Wittgenstein remarks in a letter to von Wright dated February 12, 1950: “I am very glad indeed to hear that Geache's [sic] lectures are good. Frege was just the right food for him!” (Wittgenstein, 1993, p. 475). Geach himself, in a later reflection on this period, comments more generally: “Shortly before Wittgenstein's death I often talked to him about Frege; he was pleased at my taking Frege seriously, and gave me much help and advice. I am grateful to learn, from a recently published letter of his to von Wright, that he welcomed my appointment to give

a course of lectures on Frege in the Moral Science Faculty at Cambridge" (Geach, 1991, p. 14).

Geach's interest in Frege's writings actually went further than his lectures. He and Max Black had also embarked on a project, parallel to J. L. Austin's, of translating a number of these writings into English for the first time. Their work would soon result in the collection *Translations from the Philosophical Writings of Gottlob Frege* (Frege, 1952), which would stimulate considerable interest in Frege's work in the English-speaking world. Later Geach would add *Logical Investigations* (Frege, 1977), another collection of translations of Frege's writings. Wittgenstein, who knew of the work on the first of these collections, supported it strongly. Thus Geach reports later:

Frege needs no introduction; but readers may be interested in the remarks Wittgenstein made to me about this work in the last months of his life. He took a good deal of interest in the plan Max Black and I had for a little book of Frege translations; and it was through him that I was able to locate some rare works of Frege—the review of Husserl's *Philosophie der Arithmetik* and the essays 'Was ist eine Function?' and 'Die Verneinung'—in the Cambridge University Library. (Geach, 1977, p. vii)

He then adds a remark that brings us right back to Wittgenstein's correspondence with Frege from 1919–20:

[Wittgenstein] advised me to translate 'Die Verneinung,' but not 'Der Gedanke': that, he considered, was an inferior work—it attacked idealism on its weak side, whereas a worthwhile criticism of idealism would attack it just where it was strongest. Wittgenstein told me he had made this point to Frege in correspondence: Frege could not understand—for him, idealism was the enemy he had long fought, and of course you attack your enemy on his weak side. (p. viii)

In fact, Geach and Black did follow Wittgenstein's advice; they decided not to include a translation of "Der Gedanke" in *Translations from the Philosophical Writings of Gottlob Frege* (Frege, 1952), their "little book of Frege translations." Later a translation of the article was, however, added in *Logical Investigations* (Frege, 1977).

Toward the end of his report about Wittgenstein, in another remark that brings us back to other by now familiar themes, Geach writes: "Of his great debt to Frege Wittgenstein remained conscious to the end of his life. A few days before his death he said to me 'How I wish I could have written like Frege!'" (Geach, 1977 p. viii).⁵⁵ The last few days of Wittgenstein's life were in the spring of 1951. On April 29 of that year—twenty-six years after Frege—he died.

VII. Conclusion: The Nature of Wittgenstein's "Great Debt" to Frege

In my introduction I noted that Wittgenstein's relation to Frege is often understood to be that of a hostile critic, thus Wittgenstein's debt to Frege as being rather minimal. This is still a widespread view, although it is being challenged increas-

ingly. In this essay I have amassed evidence—from Wittgenstein’s first encounter with philosophy and with Frege to remarks about him made during the last days of Wittgenstein’s life—that the ties between these two thinkers were actually quite strong, both personally and intellectually. Having become more aware of that evidence, one may be tempted to move over far in the other direction now; one may even want to go as far as Peter Geach when he claims: “[T]he most important single influence on Wittgenstein was Frege, and that . . . is true of the late as well as the early work, where the influence is so much closer to the surface” (Geach et al., 1970, pp. 5–6).⁵⁶ Or at least one may be willing to grant that significant parts or aspects of Wittgenstein’s views have their roots in Frege’s work.⁵⁷

It seems to me that there is considerable plausibility to such claims, even to Geach’s stronger one. To look back briefly at the evidence discussed in this essay: Wittgenstein’s path into philosophy—his first excitement about it, his initial attempts at writing about philosophical issues, his decision to study with Russell, and so on—was clearly influenced strongly by his encounter with Frege and his works. After that, while a student in Cambridge, then also while a soldier in World War I, he kept in touch with Frege, both in person and through correspondence. At the end of this period, shortly before finishing the *Tractatus*, Wittgenstein himself stressed his “great debt” to Frege. Similarly, in the preface to the *Tractatus* Frege’s works are mentioned as a major stimulus; and in the main text his name comes up repeatedly. All of this indicates that Frege had a deep influence (whatever the details) on Wittgenstein’s first main work.⁵⁸ Later, after the *Tractatus* and after Frege’s death, Wittgenstein remained actively engaged with Frege’s writings, as illustrated by his conversations with Ramsey, Schlick, Malcolm, Geach and others. Moreover, in his notebooks he kept acknowledging Frege as a major influence on his general approach and “style.” Finally, some particular themes in Wittgenstein’s middle and later writings illustrate further Frege’s lasting impact, in particular: the sustained reflections on Frege’s criticism of formalist views, which led Wittgenstein (at least in part) to his notion of language game; and the continuing, positive appeals to Frege’s context principle, extending from the *Tractatus* through his middle writings to his very last notebooks.⁵⁹ In sum, Frege and his works appear, indeed, to have been a crucial influence on Wittgenstein, both early and late—perhaps even “the most important single influence.”

Then again, it is easy to go too far in this direction as well. There were, after all, other major influences on Wittgenstein, starting with his Viennese upbringing, his reading of figures such as Schopenhauer, Boltzmann, and Hertz, and especially Russell’s initial mentorship in Cambridge. As to the latter, we have seen that Wittgenstein was led to Frege’s work initially via Russell’s; and his first philosophical writings had as their main topic Russell’s antinomy. Furthermore, Russell’s works kept having an important influence on Wittgenstein, too.⁶⁰ This is true even if, in contrast to Frege’s case, Wittgenstein came to dislike Russell more and more personally, did not suggest reading his works with students later on, and used Russellian ideas almost exclusively as a foil in the end. As to Frege’s influence, it is also not clear whether Wittgenstein ever really or fully understood some of his main ideas, for example, his distinction between sense and reference or his conception of truth. Arguably, his understanding of them was distorted, especially early on, by his more intimate knowledge of Russell’s corresponding views.⁶¹ In addition, like most great philosophers Wittgenstein never simply

adopted an idea from someone else. Rather, he always had to think it through afresh for himself, with the result of adopting a digested, modified variant, if anything. Finally, it is undeniable that in a number of crucial respects Wittgenstein's views are opposed to Frege's. This is already clear from many of the explicit references to Frege in the *Tractatus*; and it does not diminish in Wittgenstein's later work, as the corresponding references in the *Investigations* indicate.

So what is the overall conclusion? How should we assess Wittgenstein's relation to Frege in general? Before answering that question directly, let me add two further cautionary clarifications, now of a more metatheoretic nature. First, there are not just two extreme poles—minimal debt or all-pervasive influence—that are available with respect to assessing this relation as a whole. In fact, there is much leeway in between, much room for more complex, more nuanced stances. W. W. Bartley, in his commentary on Wittgenstein, presents such a more nuanced stance when he writes:

Some persons think of influence in absolute terms, as if to be influenced by someone is slavishly to accept his entire view of the world. . . . But there is another kind of influence which consists in the critical and passionate confrontation with a point of view, in the course of which one absorbs the point of view—i.e., one is able to understand and think in terms of it, and seriously tries it out; withdraws from it whatever is of value—whether theoretical, practical, or in conception of problems; and rejects what is false or useless, and whatever, for any reason, one is incapable of absorbing. In the process one may tremendously clarify and enrich the original idea. Wittgenstein was influenced by Frege in this positive way. (Bartley, 1973, pp. 112–13)

This passage is, in my view, clearly pointing in the right direction. In fact, Wittgenstein's relation to Frege was, if anything, even less "absolute" than indicated—it was not just a matter of "passionate confrontation," "understanding," and "thinking things through further," but also, at least sometimes, of misunderstanding and distortion (on both sides; recall Frege's reaction to the *Tractatus*). In other words, when considered in all its details, their relation was multiform, complex, and rich.

Second, it is questionable whether attempting to rank or quantify Frege's influence on Wittgenstein in any precise way is all that fruitful at this point. It should be clear by now that Frege's role in Wittgenstein's life was quite significant. Perhaps it makes sense, in addition, to argue that Russell's influence was more pervasive and important early on, while Frege's went deeper and lasted longer, at least as far as positive influence is concerned. The comparisons between Frege's and Russell's roles scattered throughout this essay point in that direction. But which one was "the most important single influence" on Wittgenstein overall: Frege's or Russell's (or someone else's)? Well, how could we decide?⁶² Also, what would be gained by making such a decision, except perhaps for propaganda or publicity purposes?⁶³ It seems more important, or more constructive, to recognize both Frege's and Russell's influence as very significant, and then to reflect further on the precise nature of each.

One way of reflecting further on the nature of Wittgenstein's relation to Frege is to compare it not only to his relation to Russell, but also to other close relations between great thinkers. Peter Geach brings up one such comparison when

he writes: “Of each of them [Wittgenstein and Frege] I would use the words Aristotle used of Plato: a man whom the base have no right even to praise. Hearing Wittgenstein on Frege was like hearing Aristotle on Plato” (Geach, 1991, p. 14). Cora Diamond has recently suggested another, more elaborate, parallel:

We might compare Wittgenstein’s relation to Frege with Frege’s own relation to Kant. Frege’s conception of arithmetic developed in great part as a critical response to Kant, but he wanted it to be quite clear that his criticisms were not those of a petty fault-finding spirit vis-à-vis “a genius to whom we must all look up with grateful awe” (*Grundlagen*, p. 101). In Frege’s criticism of Kant one can see his sense that the pursuit of issues raised by Kant must be of the greatest value. He attempted to hold on to Kant’s insights, sharpening them when he could, and removing what he took to be extraneous or in tension with Kant’s most fruitful ideas. It is precisely that combination of great respect and deeply serious criticism, criticism the seriousness of which is itself expressive of respect, which we find mirrored in Wittgenstein’s relation to Frege. (Diamond, forthcoming, first paragraph)

She adds: “Throughout his life, Wittgenstein was enormously influenced by Frege. Frege’s writings shaped, to a great extent, the problems Wittgenstein confronted in his own thought—and not just the problems, but also methods of approach, and ideas about what could count as a satisfactory solution” (next paragraph). I find both of these comparisons helpful, especially insofar as both point toward the great respect in which Wittgenstein held Frege. Diamond’s claim that Wittgenstein’s whole outlook on philosophy—its problems, its methods, and so forth—was strongly influenced by Frege also accords well with the general evidence presented in this essay.

This brings me to my own conclusion. Most striking to me about this evidence is the extent to which Wittgenstein himself emphasizes his respect, admiration, even reverence for Frege. To repeat, not only does he write to Frege in 1918 about the “great debt of thanks” he owes him; he also praises his “great works” repeatedly in public, from the preface of the *Tractatus* to his last conversations with Geach in 1951; and he keeps emphasizing Frege’s influence in notes to himself, for example, when he lists him as one of a few thinkers whose ideas he “seized on with enthusiasm.” Similarly, not only does Wittgenstein keep being actively engaged with Frege’s writings after Frege’s death; he also suggests to various of his students and friends, from Frank Ramsey in 1922 to Ben Richards in 1950, to read these writings together, thus impressing their importance on them as well. On a more personal level, a cordial friendship between Frege and Wittgenstein develops during the ten years in which they are in direct contact, a friendship again based on mutual respect. Overall, Wittgenstein is deeply impressed by qualities such as the following in Frege’s work and personality: seriousness and integrity, single-mindedness and intensity, focus and clarity. Frege’s “style” as praised and emulated by Wittgenstein is, thus, not just a matter of literary surface; it extends to deeper, more substantive aspects of method and content, including themes such as the context principle.

Suppose such an overall assessment, based largely on biographical considerations, is correct. What should we conclude from it, especially as far as further interpretive and philosophical work is concerned? From Wittgenstein’s deep, life-

long admiration for Frege it follows clearly, it seems to me, that he would have disdained any broad-brushed, unsympathetic criticisms of Frege's works, as can sometimes be found in the Wittgenstein literature today. Similarly, he would have rejected any suggestion that there is nothing to be gained in studying Frege's works, in themselves or in relation to his own works. In fact, I would go further than that; I am convinced that there lies great promise in exploring this relation more, including the various ways, however complicated, in which Frege's ideas did influence Wittgenstein's positively. To do so may, in particular, help us in interpreting and assessing some difficult passages in Wittgenstein's works, both early and late; perhaps also in Frege's. Paying special attention to Frege's continued influence on Wittgenstein may, finally, help us in understanding better the dynamics of Wittgenstein's views, that is, their development from early to late.⁶⁴

Notes

I would like to thank Cora Diamond and Wolfgang Kienzler for making available to me some of their unpublished work; Ray Monk for some helpful hints concerning sources; Leonard Linsky for his infectious enthusiasm about both Frege and Wittgenstein; as well as Stuart Glennan, Mike Price, Larry Wright, and especially Sally Allen Ness for intellectual, moral, and other support in connection with this essay.

1. For lucid general accounts, see McGuinness, 1988; Monk, 1990, 1996.

2. Wittgenstein's Viennese background, including his relation to thinkers such as Kraus, Loos, Weininger, etc., has also received detailed, book-length treatments; see Janik and Toulmin, 1973; Janik and Veigl, 1998. In these, too, his relation to Frege assumes a relatively minor role.

3. See especially Baker and Hacker, 1980; Hacker, 1986. The same basic assumption also informs, at least to a certain degree, various other commentaries on Wittgenstein, including Monk, 1990. For some early dissenters, see Anscombe, 1963; Geach, 1961; Geach et al., 1970; partly also Dummett, 1981.

4. For recent examples (books and collections) of what I have in mind here, see Weiner, 1990; the first few chapters of Diamond, 1991b; Carl, 1994; the articles in Schirn, 1996; and Ricketts, forthcoming; those on Frege in Tait, 1997; and Floyd and Shieh, forthcoming; as well as many of the articles in this volume; for Frege's philosophy of mathematics, cf. Demopoulos, 1995.

5. For my general point of view concerning Frege's influence on (not only the early, but also the later) Wittgenstein, cf. Reck, 1997. For other recent studies that make Wittgenstein's relation to Frege their focus, cf. Diamond, forthcoming; and, in a different way, Kienzler, 1997. I will come back to Diamond's and Kienzler's work later.

6. For Wittgenstein's Manchester period, see Mays, 1961; as well as Mays, 1955, 1967.

7. See Appendix A in Russell, 1903, pp. 501–22. Conversely, Frege mentions Russell in an appendix to Frege, 1893/1903. Brian McGuinness discusses the possibility of Wittgenstein having encountered Frege's or Russell's writings even earlier, perhaps as early as during his student days in Berlin; see McGuinness, 1988, pp. 54–76. But the evidence for that is meager, as well as in conflict with Rhees's and similar reports.

8. Most commentators believe that Russell is mistaken on this point, i.e., that Wittgenstein first went to visit Frege and was sent to Russell by him; cf. Monk, 1990, p. 36; McGuinness, 1988, pp. 73–76 (who is relatively cautious in this respect); earlier also Bartley, 1973, p. 14.

9. It is brought up in McGuinness, 1988, p. 74, and discussed briefly in Monk, 1990, pp. 30–33. P. E. B. Jourdain, from whom we have the main evidence here (see below), calls it "a Wittgenstein curiosity"; see Grattan-Guinness, 1977, pp. 112–18, also p. 139.

10. See Monk, 1990, p. 32.

11. Ray Monk comments: "That Wittgenstein sent his solution to Jourdain rather than to Russell or Frege perhaps indicates some degree of tentativeness. He presumably came across Jourdain's name in a 1905 issue of the *Philosophical Magazine*, which contains an article by Jourdain on the foundations of mathematics and an article by his professor at Manchester, Horace Lamb" (Monk, 1990, p. 33).

12. Ivo Grattan-Guinness, the author of *Dear Russell—Dear Jourdain*, in which Jourdain's notebook entry is published, comments: "[T]he legitimacy of [this] passage seems undisputable; but the details of the relationship between Wittgenstein, Jourdain, and Russell are not clear. Jourdain's reference to a 'reply' to Wittgenstein indicates that the contact was with him rather than with Russell, and probably in a written rather than a verbal form. Wittgenstein was at Manchester University or abroad at this time . . . and may have seen Jourdain's address at the bottom of one of his papers. If he did write, then unfortunately Jourdain did not put the correspondence in his surviving notebooks. No direct contact with Russell is implied, but the subject matter is Russell's paradox, and it seems impossible that Jourdain would not have mentioned to Russell by name this new correspondent who had views on 'solving' his paradox" (Grattan-Guinness, 1977, p. 115).

13. Cf. Rhee, 1984a, p. 214 (immediately after the passage quoted above); also the report by Peter Geach to be quoted below.

14. For the former, see, e.g., Carnap, 1963, esp. pp. 4–6 (although Carnap's personal impression of Frege seems not to have been entirely negative); for the latter, see the preface of Dummett, 1973, in particular p. xii; more extensively Gabriel and Kienzler, 1994; as well as the now available original source (Frege, 1994).

15. For more on Frege's caring relation to his adopted son, Alfred, see Kreiser, 1997.

16. See here Frege's letter to Russell from December 26, 1912, in which he reports the 1912 visit (Wittgenstein, 1961, p. 120, partly quoted below); then Wittgenstein's letter to Frege from October 22, 1913 (briefly summarized by Heinrich Scholz), in which he asks for permission to visit him again (Frege, 1980, p. 265), as well as Frege's letter to Jourdain from January 28, 1914, in which a meeting in the fall of 1913 is confirmed (p. 129, again quoted later in the text).

17. The question of date is of some interest for Frege interpretation. As Geach's report shows, Frege had not entirely given up on his view that numbers are objects by this time. But what exactly his position was is not entirely clear; cf. here Frege, 1996 (i.e., Carnap's notes from Frege's lectures on logic from roughly the same period), and the corresponding remarks in the preface by Gottfried Gabriel.

18. For the 1911 date, see again the report of Wittgenstein's sister Hermine quoted above; for the 1920 date, see the last item in Frege, 1989, a letter from Frege to Wittgenstein dated June 30, 1920.

19. See Scholz, 1976; and the editors' introduction to Frege, 1980; a translation of the introduction to Frege, 1976, for background information; cf. also Veraart, 1976; and, more recently, Hill, 1995.

20. Recently there have been speculations that perhaps important parts of Scholz's archival material were not stored in the university library after all, thus that the Frege *Nachlaß*, or parts of it, may have survived. A new search for it has, however, not yet brought anything to light; cf. Wehmeier and Schmidt am Busch, 2000.

21. Frege, 1989; for some background to this discovery, see the preface by A. Janik (pp. 5–7).

22. A few additional notes by Wittgenstein in connection with receiving these letters and cards from Frege have also surfaced recently; see Wittgenstein, 1992a, e.g., the entry from October 30, 1914 (p. 37, cf. also n. 51), in which Wittgenstein records the receipt of a "very nice note [*sehr liebe Karte*] from Frege."

23. In an earlier draft of his letter from January 28, 1914, Frege writes: "I had lengthy conversations with [Wittgenstein] before Christmas, and I wanted to write him a letter

about them in order to spin out the thread further [*um den Faden etwas weiter zu spinnen*]" (Frege, 1980, p. 79, translations emended).

24. As the editors of *Philosophical and Mathematical Correspondence* remark: "With Frege's permission, a translation of parts of GGA I appeared in the *Monist* 25 (1915), pp. 484–94, 26 (1916), pp. 182–99, and 27 (1917), pp. 114–27, but Jourdain collaborated on the translation with Johann Stachelroth, and it is not known how much of the translation is due to Jourdain" (Frege, 1980, second footnote).

25. See Wittgenstein, 1961, pp. 97, 99, 100, again 100, and 103, respectively.

26. Detailed analyses of some of these objections can be found in the essays by Diamond, Goldfarb, Proops, Ricketts, Sluga, and several of the other contributors to this volume.

27. In Wittgenstein's parallel *Geheime Tagebücher 1914–1916* (Wittgenstein, 1992a), the focus is even more on ethical and religious issues.

28. See Wittgenstein, 1961, pp. 2, 10, and 29, respectively. For analyses of (i) and (iii) see the essays in this volume by Conant, Diamond, Goldfarb, and Ostrow; for (ii), see Floyd, this volume.

29. All the passages from Frege, 1989, are quoted in my own English translations.

30. See the end of the letters from April 9, September 12, and October 15, 1918, but also earlier.

31. As far as the impact of Frege's intensity and seriousness on Wittgenstein is concerned, see also the following report by Peter Geach about his own conversations with Wittgenstein later on (around 1950): "I cannot now remember at what time Wittgenstein began to ask me to go for walks with him. Those walks were rewarding but very tiring; on a walk Wittgenstein never relaxed mentally for a moment, and required the same degree of concentration from me; attempts at light conversations were immediately quashed, and careless talk about philosophy was ruthlessly and devastatingly exposed" (Geach, 1991, p. 13). Whether Wittgenstein was consciously emulating Frege in this respect or not, the similarity is surely striking.

32. This aspect of Wittgenstein's relation to Frege has not been noted much in the literature so far. Lothar Kreiser, in the only reference to it of which I am aware, suggests a connection between Frege's retirement in the fall of 1918, the subsequent sale of his house in Jena, and a "gift from Ludwig Wittgenstein (early in 1918) which made it possible for him to move back to his home in Mecklenburg" (Kreiser, 1997, p. 77).

33. See the letters from April 21, 1916, and June 30, 1917 (partly also that from September 16, 1917), in Frege, 1989.

34. Here I am relying on item XLV/23 in Frege, 1976: "Wittgenstein to Frege, undated. Wittgenstein announces a visit to Frege" (p. 268, my translation). The corresponding letter, itself lost, is placed (tentatively?) between a letter from September and one from December 1919.

35. Cf., e.g., Nedo, 1993, p. 21.

36. Frege, 1989, pp. 17–18.

37. See again Nedo, 1993, p. 22.

38. For further discussions of this particular theme, see Kremer, 1998; and Reck, 1997.

39. For more on these themes, see Geach, 1976; Diamond, 1991b, chaps. 2–6; Diamond, 1991a; Conant, 1989; Conant, this volume; for Frege's side, see also Weiner, 1990, chap. 6; forthcoming.

40. Here I have largely adopted the translation in Monk, 1990, p. 163.

41. For what is probably Frege's most polemical and ironical attack on any writer, see Frege, 1899.

42. Cf. Nedo, 1993, pp. 21–25, for a fuller history of Wittgenstein's attempts to publish his work.

43. Parak, 1992, p. 154 fn. 2.

44. For general information about this period, including Ramsey's visits, see the editorial appendix to Engelmann, 1967, pp. 144–46. For Ramsey's own reports, see the quotations below. In addition, compare the discussion in Kienzler, 1997, chap. 1, which alerted me to some of these traces.

45. For a general, brief chronology of this period, see Nedo, 1993, pp. 26–29. Wittgenstein's meetings with the Vienna Circle are documented directly in Waismann, 1967.

46. See especially the meetings on December 30, 1930, and January 1, 1931, in which parts of *Grundgesetze*, vol. 2, are the topic of discussion (Waismann, 1967, pp. 130–36, 150–51). I was again led to these and some related sources by the discussion in Kienzler, 1997.

47. This claim is defended in detail in Kienzler, 1997. I am also indebted to unpublished work by Wolfgang Kienzler here, as well as to some corresponding conversations. Two qualifications, though: First, Kienzler does not claim that it was Frege's criticism of formalism alone that led Wittgenstein in this direction; he also points to the importance of his concurrent studies of Plato, Augustine, Spengler, Frazer, etc. Second, Kienzler emphasizes the opposition between Wittgenstein and Frege much more than I do; his general thesis is that Wittgenstein wanted to go “with Frege beyond Frege [*mit Frege über Frege hinaus*].” I will come back to such general evaluations in the last section of this paper.

48. While this remark appears in print in *Zettel*, which otherwise contains mostly notes from the 1940s and 1950s, it actually goes back to notebooks from the early 1930s (see especially MS 112, 20/211).

49. I am indebted both to Kienzler, 1997, and to Diamond, forthcoming, with respect to this last paragraph.

50. The same agreement also underlies, somewhat more implicitly, the beginning of §10: “Now what do the words of this language [of the builders] *signify*?—What is supposed to shew what they signify, if not the kind of use they have?” For further discussion of these passages, see Reck, 1997.

51. See Frege, 1950, p. 73, also pp. x, 71, 116; then Wittgenstein, 1963, p. 14; 1974, p. 63; 1978, p. 344; 1982, p. 118. There are further, more implicit, echoes of the context principle in other works by Wittgenstein.

52. I plan to explore some of the changes in Wittgenstein's understanding of the context principle in future publications. In Reck, 1997, my goal was to establish the continuities and similarities first.

53. See Malcolm, 1966, p. 86; Bouwsma, 1986, pp. xi, 49–51; cf. also Monk, 1990, p. 552.

54. For more on this trip to Norway, see Monk, 1990, p. 574. Monk writes: “He [Ben Richards] and Wittgenstein spent much time reading and discussing Frege's work.”

55. See also Geach, 1991, p. 14: “The very last time I saw Wittgenstein we were talking about Frege; taking the book in his hands, he said slowly ‘How I envy Frege. I wish I could have written like that.’”

56. The introduction to Geach et al., 1970, is not signed specifically by Geach. But it seems fair to attribute this claim, if not primarily, then at least also to him, as one of the editors of that work.

57. In Dummett, 1973, the basic claim is that many of Wittgenstein's most fruitful ideas have their roots in Frege's work.

58. Recently it has also been claimed that Frege's “*Der Gedanke*” was influenced by Wittgenstein's early ideas in turn. This claim is more audacious, I think; but see Sluga, this volume, for an interesting defense.

59. Again, the first of these is discussed more in Kienzler, 1997, the second in Reck, 1997. There are others such themes one could mention. For example, Frege's views about the objectivity of logic seem to be tied directly to Wittgenstein's rule-following considerations in his later work; note, in particular, that Frege's main example of an “objective fact” in logic is how one number follows another in a series (Frege, 1950, p. 93). I plan to explore this connection further in a future publication.

60. And not just Russell's works from before World War I; cf., e.g., Wittgenstein's reactions to *The Analysis of Mind* (1921), both in his *Blue Book* and in *Philosophical Investigations*, as discussed in the appendix to Kripke, 1982. I owe this reminder to Leonard Linsky.

61. See again Kremer, 1998, and the essays in this volume by Diamond, Goldfarb, Proops, and Ricketts. In Macbeth, this volume, too, the claim is that Wittgenstein did not really understand some of Frege's views.

62. One numerical fact in this connection, for what it's worth: if one searches Wittgenstein, 1992b, the CD-ROM edition of Wittgenstein's published works, the name 'Frege' (and its cognates: 'Frege's,' 'Frege's,' etc.) comes up 91 times. This is second only to the name 'Russell,' which comes up 201 times.

63. I suspect that the strong emphasis on Frege's primacy by Peter Geach et al. (see above) was meant at least partly to serve such purposes. More particularly, it was meant to challenge the status Russell's works had, into the 1950s and 1960s, as the undisputed center of analytic philosophy, including as an influence on Wittgenstein. Frege's ideas were only starting to gain prominence at that point (at least in the English-speaking world). The latter has, of course, changed significantly since then.

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Frege, Lotze, and the Continental Roots of Early Analytic Philosophy

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The title of my essay implies the thesis that at least *early* analytic philosophy has its roots in the tradition of continental philosophy, especially in the philosophy of Hermann Lotze. Indeed, this is the thesis I want to argue for. The thesis itself is not really new. As far as Frege is concerned, it has been presented before by Hans Sluga in various papers as well as in his book on Frege. This book was the starting point of a controversy between Sluga and Michael Dummett about a crucial point of understanding Frege.¹ I do not want to go into the details, but, in developing my own position, it might be helpful to say a few words about this controversy. I agree with Dummett that some of Sluga's interpretations of Frege are not correct and that Sluga was misled by such interpretations in his evaluation of Frege as a philosopher. On the other hand, I agree with Sluga's general picture of Frege's philosophical background. I do not accept all of the details, but I think his thesis that Frege's efforts are part of the Neo-Kantian tradition is correct. Neo-Kantianism is to be understood as an alternative to German speculation in the tradition of Hegel, on the one hand, and to British empiricism in the tradition of Hume and Mill, on the other hand. Hermann Lotze, Frege's teacher at the University of Göttingen, can be regarded as the founder of Neo-Kantianism.

Agreeing with Sluga on these points does not imply accepting his bold assertion that Frege turns out to be a transcendental idealist. We should realize that to be a Neo-Kantian does not mean to be a Kantian in all respects. The Neo-Kantians worked in the spirit of Kant's philosophy, but they very often, and sometimes fundamentally, disagreed with the letter of his works. For example, most of them did not accept the thing in itself. What most of the Neo-Kantians shared with Kant was his apriorism. This apriorism links them together in their fight against every kind of naturalism with respect to the foundation of science (including logic, ethics, and aesthetics). But neither was it on the question of idealism or realism where

they agreed with one another. So it seems to me that Dummett's and Sluga's controversy about whether Frege was a realist or a transcendental idealist is not posed very well. Concerning Frege's philosophical background, other questions are much more central. This becomes clearer when we compare Frege with the Neo-Kantians *in detail*. And it is *here* where one can find a deficiency in Sluga's argument: Although his thesis that Frege belongs to the Neo-Kantian tradition seems to be correct, he does not really *show* that Frege was a Neo-Kantian. His thesis might be true, but he did not prove it. The evidence he presents is too thin. Insofar as this is the case, we have to concede to critics (like Dummett) that they are right not to be convinced.

What I want to do is to make Sluga's thesis more defensible, and to present some more historical evidence for it. Mainly, I will compare some of Frege's views with those of the two Neo-Kantians Otto Liebmann (1840–1912) and Wilhelm Windelband (1848–1915). What seems to me important here is the fact that both were influenced by Hermann Lotze, especially Windelband, who wrote his doctoral thesis at the University of Göttingen under Lotze 1870, shortly before Frege moved from Jena to Göttingen. Lotze was indeed a central figure in the whole intellectual scene before the unfortunate separation of continental and analytic philosophy. Especially the University of Jena became a center of Lotze studies.² Therefore, to understand Frege and the roots of analytic philosophy, we have to go back to this scene. It will turn out that Lotze's influence was not restricted to Germany; indirect reactions can even be found in the logical atomism of Russell and Wittgenstein.

I. The Philosophy of Hermann Lotze

It was J. Passmore who called Lotze the most “pillaged” philosopher of the nineteenth century.³ If we ask about Lotze's importance, we have to take into account especially his *Logic*, which impressed a whole generation of academic philosophers. In this connection we have to realize that though Lotze's *Mikrokosmos* was much more popular because it includes a complete Weltanschauung, its influence was restricted to popular philosophy, whereas academic philosophy was much more interested in Lotze's *System of Philosophy* (*System der Philosophie*). This *System* includes as its first part the *Logic*, as its second part the *Metaphysics*. Its third part on ethics, aesthetics, and philosophy of religion did not appear because of Lotze's sudden death in 1881.⁴

There is at least one Lotzean concept that unites continental and early analytic philosophy, namely, the concept of validity. Of course, the *concept* itself, that is, the distinction between the genetic point of view of *psychological* explanation and the foundationalist point of view of *logical* validity can already be found in Kant and Herbart and even in Leibniz, but it was Lotze who took up this tradition and established the distinction by introducing the term “validity” (*Geltung*). He prepared the logical discussion of nineteenth-century philosophy to argue against naturalistic tendencies that reduced thinking to processes of ideas (*Vorstellungsverläufe*). In doing so Lotze provided later philosophers like Frege, Windelband, and Husserl with antipsychologist arguments. On the other hand, we have to keep in mind that antipsychologism is not opposed to psychol-

ogy. Lotze belongs to the classic authors of psychology as well, but he was, at the same time, completely aware of the categorical difference between psychology and logic.

The concept of validity became the leading concept in the logical and epistemological doctrines up to the thirties of our century, until *Lebensphilosophie* and existential hermeneutics succeeded to dominate German philosophy. Although the central role of the concept of validity is not restricted to the German-speaking philosophical world, it is this German tradition about which I mainly want to talk. Concerning logic and epistemology, we may divide this tradition into two lines, namely, the Neo-Kantian and the phenomenological line. Both lines go back to Lotze. To be more correct, for there are two schools of Neo-Kantianism, it is the so-called southwest German school of Neo-Kantianism that is influenced directly by Lotze: besides Wilhelm Windelband and Otto Liebmann, there are Heinrich Rickert, Bruno Bauch, Emil Lask, and the early Martin Heidegger. The sociologist Max Weber also came into contact with this tradition via Rickert. The members of the so-called Marburg school seem to have been influenced only indirectly. Concerning the phenomenological tradition, we have to note that Franz Brentano was in contact with Lotze, and that his disciples Carl Stumpf, who was the teacher of Edmund Husserl, and Anton Marty were students of Lotze, too.

The most influential part of Lotze's *Logic* was the epistemology in the third book. This book includes in its second chapter "The World of Ideas" Lotze's reconstruction of Plato's theory of ideas in terms of the concept of validity. Let us have a more detailed look at this chapter. Lotze tries to defend Plato against the old Aristotelian accusation that he (Plato) had hypostasized ideas into real existing things. His argument runs as follows:

While Plato by . . . describing the Ideas, takes security for their independent validity, he has at the same time abundantly provided against the confusion of the validity thus implied with that wholly distinct reality of Existence which could only be ascribed to a durable thing. When he places the home of the Ideas in a super-celestial world, a world of pure intelligence . . . , when again more than this he expressly describes them as having no local habitation, such language makes it abundantly clear to any one who understands the mind of Greek Antiquity, that they do *not* belong to what we call the real world. To the Greek that which is not in Space is not at all, and when Plato relegates the Ideas to a home which is not in space, he is not trying to hypostasize that which we call their mere validity into any kind of real existence, but on the contrary he is plainly seeking to guard altogether against any such attempt being made. (Lotze, 1980, §318; references are to numbers of paragraphs)

We do not have to establish that this is a correct interpretation of Plato. From a logical point of view I myself like it very much, but we have to keep in mind that the non-conceptual, contemplative aspect of intellectual intuition (*intellektuelle Anschauung*) in Plato is neglected by Lotze. Anyway, what this interpretation achieved was the reunion of Platonic and Kantian philosophy in an epistemological position that might be, and in fact was, called "transcendental Platonism." I think that the position of Frege and some Neo-Kantians (like Windelband, Rickert, and Bauch) can be described exactly in this way. Transcendental Platonism is

Platonistic because it accepts contents of thinking (thoughts) that are independent of the individual thinking subjects, and it is transcendental (as opposed to transcendent) because the independence is not thought of as an ontological one of existence, but a logical one of being valid.

To put this idea into the form of a transcendental argument, we may give the following explanation: logic starts with making a “distinction of value” between “truth and untruth” (§II). True and untrue, or false, cannot appear as properties of *processes* of thinking, but only of *contents* of thinking. To talk about truth and falsehood necessarily presupposes—as a *conditio sine qua non*, that is, as a “condition of possibility” in the Kantian sense—that we have first grasped the same cognitive content and are discussing the same thought. To take this consequence seriously, we have to accept that a thought cannot be a psychological item, because such a view would imply that different individual subjects are not able to participate in the same cognitive content or thought.

The independence of thoughts thus means nothing more than stating the following categorical fact: An item which we want to value as true or false, that is, an item that is meant as the “bearer” of a truth-value, cannot have individual psychological existence. On the other hand, this does not imply that we have to search for some kind of *existence* different from psychological existence. Such an attempt is out of place, and insisting on such an attempt is a category mistake. Cognitive content does not exist; it is valid (or not valid). To be *valid* does not imply *to be*. To give a short and handy characterization of this position, we might explain it by converting the well-known Quinean slogan “no entity without identity” into the additional slogan “but identity without entity.” It is a transcendental condition of talking about truth and falsity that the bearer of a truth-value remains identical, but it is not necessary to accept this bearer as an ontological entity. Lotze’s conception of validity “as a form of Reality [*Wirklichkeit*]” presupposes “the eternally self-identical significance of Ideas,” but it does not include the “Being or Existence [*Sein*]” of these Ideas and their conceptual content (§317). Reality (*Wirklichkeit*) could come in here only on the level of psychological events or on the level of logical validity. To give a more complete account, let us take a look at a famous and often-quoted passage:

For we call a thing Real which is, in contradistinction to another which is not; an event Real which occurs or has occurred, in contradistinction to that which does not occur; a relation Real which obtains, as opposed to one which does not obtain; lastly we call a proposition Really true which holds or is valid as opposed to one of which the validity is still doubtful. (§316)

This quotation implies the following categorical distinctions:

Reality (*Wirklichkeit*)

of things (*Dinge*): they are (*or exist*)

of events (*Ereignisse*): they happen (*or occur*)

of relations (*Verhältnisse*): they obtain

of propositions (*Sätze*): they are valid (*or hold*).

These modes of reality are conceived of as independent from each other; it is not possible to explain one by reducing it to any other. What is missing in this scheme is the “self-identical” content of propositions themselves. Using Lotze’s own ter-

minology, which is not completely coherent here, we might say that he accepts for thoughts as the meanings of propositions the status of *objectivity* (§3), which is simply the negation of subjectivity, namely, the independence of these contents from singular subjects which might conceive these contents. The self-identical contents in themselves are objective but not real. They can get reality either in psychological realization as events or in being “really true,” that is, valid.

It should be clear that Lotze here prepared the categorical basis for the separation of logical investigations from psychological ones. The self-identical content of propositions later appeared, for instance, as *Gedanke* (Frege), *Sinngebilde* (Rickert), *Objektiv* (Meinong), *ideal identischer Inhalt* (Husserl), or *logischer Inhalt* (Heidegger). Concerning Lotze himself, we have to add that he was not completely clear about the fact that the content has to be accepted as the same even if it is not valid or true. “The conception of Validity,” Lotze explains, “at once excludes the substance of the valid assertion from the reality of actual being and implies its independence of human thought” (§316). The English translation here is a little bit artificial. The expression “the substance of the valid assertion” corresponds to nothing else but “valid content [*geltender Inhalt*]” in the German original. What is problematic here is the implicit restriction to *valid* assertions or contents, for the self-identity of the content does not depend on its validity. A content that is not valid has to be the same, too, otherwise we could not apply the categorical predicate “invalidity” to it. The transcendental condition of self-identity holds not only for valid but also for invalid contents. This point was made clear by Frege in his article “On Negation,” as well as in the further development of logical value theory.

This development may be divided into two strands. First there is the southwest German school of Neo-Kantianism, which extended Lotze’s conception of validity (*Geltung*) by building on its basis a comprehensive value theory that included the normative disciplines of logic, ethics, and aesthetics with their differentiation in regions of values. Considered from a historical point of view, this universal value theory was a philosophical response to Nietzsche’s nihilistic *Umsturz der Werte*. Neo-Kantianism tried to substitute Nietzsche’s will to power (*Wille zur Macht*) by a Kantian will to value (*Wille zum Wert*). Windelband used the phrase “will to truth [*Wille zur Wahrheit*].” In contemporary philosophy we find a revival of this debate in J. Habermas’s defense of a discourse-theoretical variant of the Neo-Kantian value-theoretical program against Nietzschean postmodernism. Habermas’s distinction between different claims of validity (*Geltungsansprüche*) goes back to Neo-Kantian concept, via the sociologist Max Weber.

Besides this Neo-Kantian development of Lotzean ideas we have already mentioned the phenomenological tradition, which, at least in its beginning, was mainly concerned with logical (and ontological) questions. It was the phenomenologist Husserl who, following Fregean insights, made the Lotzean distinction between objective (but not real) contents of thought and real events of thought the starting point of his logical investigations. The very early Heidegger, who was a disciple of both Husserl and the Neo-Kantian Rickert, formulated this starting point in a way that shows very clearly the decisive influence of Lotze’s distinctions on the antipsychologist program. He wrote in 1912:

Fundamental to the recognition that psychologism is nonsensical and theoretically barren remains the distinction between mental act and

logical content, between the real, temporal process of thinking and the ideal, atemporal, identical sense, in short the distinction between what “is” and what “is valid.” The pure, self-subsistent sense in question is the topic of logic, and with that subject matter, the character of an empirical discipline is taken away from logic from the outset. (Heidegger, 1978, p. 22)⁵

I think this statement is very interesting because Heidegger, who at that time called Lotze’s *Logic* the “fundamental book [*Grundbuch*]” of modern logic (p. 23, fn. 9), later became (under the influence of *Lebensphilosophie*) the most radical critic of his own tradition. When we read in *Sein und Zeit* his polemics against the value theory of his time, which culminates in calling “validity” a “word-idol [*Wortgötze*]” (pp. 155–56), we should remember Heidegger’s beginnings. It seems to me that Lotze’s Platonistic departure from a restriction to existing things was at least one necessary step into the direction of Heidegger’s critique of the ontology of *Vorhandenheit*. This would explain why even in later years he recommended Lotze’s *Logic* to beginners in philosophy. An astonished-looking student, G. Picht, was informed by Heidegger that beginners should realize what hard work his (Heidegger’s) own thinking had had to go through.⁶ If we take this statement by Heidegger seriously, it seems to imply a general advice: Before reading Heidegger, first study Lotze’s *Logic* or, at least, do not read Heidegger without studying Lotze! Now I want to show that for a better understanding of Frege, we should read Lotze at least *after* studying Frege.

II. Frege and the Neo-Kantians

As we have already seen, Lotze used the word “logic” in the broader sense of the nineteenth century, thus as including epistemology. To show how Lotze’s conception of validity has influenced logic and epistemology of continental and analytic philosophy, we should now consider some aspects that both Neo-Kantians and Frege took over from Lotze. I will start with Otto Liebmann, the originator of the Neo-Kantian slogan “back to Kant.”

Liebmann was a colleague of Frege’s at the University of Jena from 1882 to 1911. In 1900 Frege was involved in a discussion and correspondence with the son of Otto Liebmann, Heinrich Liebmann (later professor of mathematics at the University of Heidelberg), about Hilbert’s *Grundlagen der Geometrie*. My comparison of Frege and Otto Liebmann concerns Frege’s *Grundlagen der Arithmetik* and Liebmann’s *Zur Analysis der Wirklichkeit*.

The fact that Frege refers to Kant directly in *Grundlagen* might be the reason that it has been overlooked that Frege’s views are similar to those of Liebmann. And indeed Frege does not even mention his elder contemporary, that is, Liebmann’s name appears nowhere in Frege’s works. (Hermann Lotze and Wilhelm Windelband met the same fate.) It might be interesting here to mention that Frege had indeed read Liebmann’s *Analysis der Wirklichkeit*. Frege borrowed Liebmann’s work from the Jena University library when he was writing his *Grundlagen*.⁷ This fact should serve to remind us that the absence of names does not imply the absence of influence or of agreement. Now let us get into the texts of Liebmann and Frege.

First of all, we find agreement between them on the conception of the a priori. Against empiricism and its overvaluation of induction, Liebmann maintains that

induction is impossible without “general fundamental truths [*allgemeingültige Fundamentalwahrheiten*]” (Liebmann, 1900, p. 208). In the same vein, Frege declares in his *Grundlagen*:

If we recognize the existence of general truths at all, we must also admit the existence of such primitive laws, since from mere individual facts nothing follows, unless it be on the strength of law. Induction itself depends on the general proposition that the inductive method can establish the truth of a law, or at least some probability for it. If we deny this, induction becomes nothing more than a psychological phenomenon, a procedure which induces men to believe in the truth of a proposition, without affording the slightest justification for so believing.

(Frege, 1953, p. 4, footnote)

We can find the source of Liebmann’s and Frege’s view of induction in Lotze’s *Logic*:

It is clear therefore that the attempt to derive the entire body of general knowledge from experience, that is to say from a mere summing up of particular perceptions, breaks down. We have invariably to help ourselves out by assuming at one point or another some one of those self-evident principles, some principle to which when once its content has been thought we at once concede with intuitive confidence that universal validity to which it makes claim. (Lotze, 1980, §330, last section)

The quotation from Frege above refers to the Lotzean distinction between questions of genesis (*Genese*) and questions of validity (*Geltung*). Epistemology is not concerned with the genetic-psychological question of how it is that we accept some propositions as true. Rather, epistemology deals with the question of the validity of these propositions, that is, with the question of the justification of a true proposition. Like Kant, Lotze and the Neo-Kantians conceded that from a genetic point of view all knowledge might have its origin in experience. But they deny that this shows that all knowledge is empirical. When we seek to justify the foundation of knowledge, we have to accept propositions that are nonempirical a priori truths. In this sense Frege points out in *Grundlagen*:

By this I do not mean in the least to deny that without sense impressions we should be as stupid as stones, and should know nothing either of numbers or of anything else; but this psychological proposition is not of the slightest concern to us here. Because of the ever-present danger of confusing two fundamentally different questions, I make this point once more. (Frege, 1953, p. 115, footnote)

It is the same confusion that Liebmann addresses by distinguishing psychological laws from laws of knowledge (*Erkenntnisgesetze*). For him the psychological laws are natural laws of the changing content of the mind, whereas the laws of knowledge are norms that must be followed if we want to reach the truth (Liebmann, 1900, pp. 251–52).

So far we have considered the general basic consensus between Frege and the Neo-Kantians in the tradition of Hermann Lotze. A more specific agreement between Liebmann and Frege emerges when we look at their views about mathematics. Here they both disagree with Kant in a significant respect: the status of arith-

metic in relation to geometry. As is well known, Frege agrees with Kant's view that geometry is an a priori synthetic science. Frege's reason is that insight into the validity of the geometrical axioms is impossible without intuition. But unlike Kant, Frege wants to demonstrate in his *Grundlagen* that arithmetic is a branch of logic and therefore an *analytic* a priori science. Frege tries to make this so called logicism of the *Grundlagen* plausible by considering the differences between the domain that is "governed" by the truths of arithmetic and that governed by the truths of geometry (Frege, 1953, §14). He states that the domain of geometry is (in opposition to the temporally intuitable) "all that is spatially intuitable [*das räumlich Anschauliche*]". This includes the actual as well as the fictitious. This assignment of domains amounts to a restriction of geometry in relation to arithmetic. Frege argues: "The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but everything thinkable" (Frege, 1953, p. 21). For Frege these considerations were helpful in making the logicist program plausible *before* carrying it out. So he concludes with the suggestive question: "Should not the laws of number, then, be connected very intimately with the laws of thought?" Now, Frege starts his attempt to draw arithmetic into the domain of nonintuitive conceptual (pure logical) thinking with a short treatment of non-Euclidean geometry. Frege states that a non-Euclidean space cannot be intuited, but can be thought. Among his arguments is the following: to assume the negation of an axiom of Euclidean geometry does *not* involve thought in self-contradiction; whereas assuming the negation of any basic law of arithmetic does. Frege's treatment of non-Euclidean geometry seems to be inspired directly by Otto Liebmann, who defended non-Euclidean geometry from a Kantian (!) point of view. Liebmann distinguishes between "logical necessity" and "necessity of intuition." He says that the negation of an intuitively necessary proposition is not a contradiction, it is merely not intuitable. Pointing out that he agrees with Kant essentially, Liebmann maintains that the axioms of (Euclidean) geometry are nonlogical necessities that are nevertheless unavoidable for beings with intuitive capacities like ours; in this sense they are a priori intuitions. And because they are a priori *intuitions*, they are subjective. Of course, Kant would not have agreed with this last point.

I think Frege agrees with all points of this reformulation of Kant, even with the last one, namely, the view that the Euclidean axioms are subjective. This can be seen in Frege's definition of objectivity, which includes independence of intuition (Frege, 1953, §26, last sentence). Dummett has argued against Sluga that Frege's view of geometry is not really Kantian.⁸ In a way, Dummett is right here, and Sluga is perhaps too vague. It turns out that Frege was not a Kantian, but a Neo-Kantian.

Liebmann's discussion of geometry concludes with some views on the relationship between arithmetic (*Größenlehre*), logic, and geometry which we have already found in Frege's *Grundlagen*. For Liebmann, the "extension or domain of validity" of arithmetic and logic is broader than that of Euclidean geometry. Whereas the latter is valid only for an intelligence with the same type of intuition, arithmetic and logic are valid for all intelligent beings whatever (Liebmann, 1900, p. 254). Consequently, both Frege and Liebmann come to the same judgment about actual and possible beings whose laws of logic and arithmetic (not of

geometry) would contradict ours. One would have no choice but to regard them as “mad [*verrückt*]” (Liebmann, 1900, p. 253; Frege, 1962, vol. 1, p. xvi).

We can conclude our comparison between Liebmann and Frege with an amusing example of an implicit agreement. This agreement consists in some polemic remarks against German physiological materialism (represented, e.g., by Vogt, Moleschott, Büchner). From a Neo-Kantian point of view, this position is a good example of confusing the question of validity with questions of genesis as a consequence of confusing the laws of thought with its natural physiological conditions. Liebmann asks polemically: “What have the protein, potash and phosphorous in the brain-substance [...] got to do with logic?” (Liebmann, 1900, p. 540). And in the “Introduction” to the *Grundlagen*, Frege exclaims ironically: “Otherwise, in proving Pythagoras’ theorem we should be reduced to allowing for the phosphorous content of the human brain” (Frege, 1953, p. vi). Without mentioning the name, Liebmann and Frege here obviously (obviously at least for their readers in the nineteenth century) refer to a slogan of Jacob Moleschott, who wrote in his *Der Kreislauf des Lebens*: “No phosphorous no thought.”

Looking for further circumstantial evidence for Frege’s connection to Neo-Kantianism, we now come to Windelband. Several of Liebmann’s and Frege’s Neo-Kantian positions also appear in Windelband’s writings. For instance, we can find the distinction between genesis and validity (Windelband, 1915, vol. 1, p. 24) and the view that induction is impossible without nonempirical presuppositions which must, without proof, be acknowledged as general laws (vol. 2, p. 107), or, as Frege puts it in the *Grundlagen*, “which neither need nor admit of proof [*die selber eines Beweises weder fähig noch bedürftig sind*]” (Frege, 1953, §3). By the way, this formulation is an acknowledged quotation from Lotze (Lotze, 1874, §200). It goes back to Leibniz and corresponds to the conception of axioms in Aristotle’s *Analytica posteriora*. What is said about general laws holds in particular of the laws of logic, which both Windelband and Frege consider to be unprovable presuppositions of *all* thought (*Denken*).

Nevertheless, Windelband demands, and Frege develops, an argument for why we must accept the laws of logic. As Windelband and Frege stress, this argument itself cannot be a logical one (i.e., it cannot give a logical reason). Following Windelband’s presentation, it is a teleological one of the following form: *if* we want to fulfill the purpose of thought, that is, truth, we are forced to accept the laws of logic (Windelband, 1915, vol. 2, p. 109). Frege refers to this kind of transcendental argumentation, which he delegates to epistemology,⁹ when he says: “We are compelled to make judgments by our own nature and by external circumstances; and if we do so, we cannot reject this law—of Identity, for example; we must acknowledge it unless we wish to reduce our thought to confusion and finally renounce all judgment whatever” (Frege, 1982, p. 15). Though Frege “neither dispute[s] nor support[s] this view,” he in fact accepts it: “This impossibility of our rejecting the law in question hinders us not at all in supposing beings who do reject it; where it hinders us is in supposing that these beings are right in so doing, it hinders us in having doubts whether we or they are right. At least this is true of myself” (p. 15).

The basis of Windelband’s transcendental-teleological argumentation is his theory of values. Windelband is the founder of the value-theoretical *Südwestdeutsche*

school of Neo-Kantianism. He used the term “truth-value” (*Wahrheitswert*) before Frege (Windelband, 1915, vol. 1, p. 32). It should be added that Liebmann (1900, pp. 252–53), too, considers truth as “value.” Also, once more the trail goes back to Lotze, who speaks of the “value-difference” (*Wertunterschied*) between truth and untruth (Lotze, 1874, p. 4). Frege was not concerned with all of the values treated by value theory, but only with the value “true.” Yet, in the opening passage of “Der Gedanke” he refers to the same triad of values as Windelband: “Just as the word ‘beautiful’ points the way for aesthetics and ‘good’ for ethics, so does ‘true’ for logic.” Moreover, Frege states an “affinity” of logic with ethics (Frege, 1979, p. 4) and thus follows the Windelbandian connection of the teleological and the value-theoretical aspects of truth: “Like ethics, logic can also be called a normative science. How must I think in order to reach the goal, truth?” (p. 128).

Seen against the background of the Neo-Kantian value-theoretical tradition, even Frege’s problematic connection between truth-value and *Bedeutung*, that the *Bedeutung* of a sentence is its truth-value, becomes more plausible.¹⁰ Finally let us have a look at the indirect reception of Lotze’s philosophy in logical atomism (Russell, Wittgenstein).

III. Monism, Logical Atomism, and the Fregean Context Principle

So far we have considered the influence of Lotze’s *Logic*. Lotze’s *Metaphysics* was influential in some respects, too, but more in the English Neo-Hegelian tradition than in the German Neo-Kantian tradition. To give some hints concerning the British reception of Lotze’s philosophy, we have to note that the English translations of Lotze’s *Logic* and *Metaphysics* were prepared by the Neo-Hegelians, especially by B. Bosanquet, who was the editor of both books (the translation was initiated by T. H. Green). The Neo-Hegelians mainly agreed with Lotze on his ontological holism, which consists of the thesis that the being of things (*das Sein der Dinge*) means standing-in-relation (*in Beziehung stehen*). They took this conception as the basis of their holistic monism; compare, for instance, Bradley’s view that “reality is not made up of separate objects with relations among them” (Hylton, 1990, p. 54).

It is interesting and amusing to look at the reception of this holistic thesis in British philosophy. Holism was criticized as a result of Hegelian monistic idealism by Bertrand Russell from a logical atomistic point of view, or, to put it the other way around, Russell tried to overcome the Hegelianism of his own time by means of logical atomistic arguments against holistic implications of Hegelianism. Following the *modus tollens*, a theory that implies a wrong thesis is itself wrong. As Russell declares in *The Philosophy of Logical Atomism*: “The logic which I shall advocate is atomistic, as opposed to the monistic logic of the people who more or less follow Hegel” (Russell, 1972, p. 32). Compare also later his emphasis concerning his logic as opposed to monistic logic:

The acquaintance with the simpler is presupposed in the understanding of the more complex, but the logic that I should wish to combat maintains that in order thoroughly to know any one thing, you must know all its

relations and all its qualities, all the propositions in fact in which that thing is mentioned; and you deduce of course from that that the world is an interdependent whole. It is on a basis of that sort that the logic of monism develops. (Russell, 1972, p. 59)

Although Russell is arguing here against the Hegelianism of his time, the position described is, in some respects, similar to that of Lotze, who, as a disciple of C. H. Weiße, was in contact with Hegelianism. The irony is now that analytic philosophy, following the anti-Hegelian tradition, which identifies the birth of analytic philosophy with Russell's break with Hegelianism, had to rediscover holism via the Fregean context principle, "nach der Bedeutung der Wörter muss im Satzzusammenhange, nicht in ihrer Vereinzelung gefragt werden" (Frege, 1986, p. 10). The context principle appears as a semantic version of a metaphysical Hegelian principle that Frege took over from his teacher Lotze, while restricting it to propositions. Frege did not defend a holism outside of propositions, that is, he did not hold a coherence theory of truth.

It is the same in the case of Wittgenstein (in the *Tractatus*). Here the formulation of the context principle is the following: "Only propositions have sense; only in the nexus of a proposition does a name have meaning" (Wittgenstein, 1966, 3.3). In Wittgenstein's ontological way of speaking, an object can occur only within a state of affairs and cannot exist on its own (2.0121). The independence of things is only a relative one: "Things are independent in so far as they occur in all *possible* situations, but this form of independence is a form of connexion with states of affairs, a form of dependence. (It is impossible for words to appear in two different roles: by themselves, and in propositions.)" (2.0122). As a consequence, Wittgenstein acknowledges that objects do have internal properties and that these properties are essential ones: "If I am to know an object, though I need not know its external properties, I must know all its internal properties" (2.01231).

Wittgenstein does not go as far as Bradley and Lotze. For Lotze the possibility of understanding the world-process (*Weltlauf*) is grounded in thoroughgoing (*durchgängigen*) relations that connect all objects with one another (*welche alle Dinge miteinander verknüpfen*) (Lotze, 1872, p. 483). Wittgenstein does not defend the holistic chain of all beings, but he accepts the chain of beings *inside* a state of affairs: "In a state of affairs objects fit into one another like the links of a chain" (Wittgenstein, 1966, 2.02). So Wittgenstein's holistic internalism is restricted to states of affairs and thus to elementary propositions. But we have to realize that his logical atomism works only on the higher level of complex propositions.

Wittgenstein's view that things have only a relative independence appears almost in the same formulation already in Lotze's writings. This becomes quite clear if we compare the German originals:

Das Ding ist selbständige, insofern es in allen *möglichen* Sachlagen vorkommen kann, aber diese Form der Selbständigkeit ist eine Form des Zusammenhangs mit dem Sachverhalt, eine Form der Unselbständigkeit. (Es ist unmöglich, daß Worte in zwei verschiedenen Weisen auftreten, allein und im Satz.) (Wittgenstein, 1966, 2.0122)

Allerdings müssen die Dinge *sein*, um sich aufeinander zu beziehen können; aber dies noch beziehungslos gedachte Sein, das wir uns als Grund der Möglichkeit des bezogenen vorstellen, ist nicht eine für sich

vorkommende Wirklichkeit, aus der die Dinge in gegenseitige Beziehungen treten, und in welche sie sich aus allen Beziehungen zurückziehen könnten; vielmehr besteht es nur latent in den Formen des bezogenen Seins, unabtrennbar von diesen [. . .]. (Lotze, 1872, pp. 483–84; cf. p. 473)¹¹

If we take into account the reception of ideas, we can see that even Wittgenstein's use of the context principle is indirectly connected with Hegel-Lotzean holism. Against this background it is less astonishing to find common views between English Neo-Hegelians and Frege.¹² Both learned from Lotze or at least from the widespread discussion about Lotze's philosophy. In the case of the context principle, we have a good example of how it can happen that historical ignorance forces one to discover old ideas in a new way. At least we see that in the *history* of ideas holism might be an adequate approach: everything seems to be connected with everything, even such things as continental and analytic philosophy.

Notes

I am grateful to Erich Reck for correcting my English in this essay.

1. Sluga, 1980; Dummett, 1981.

2. Cf. Kreiser, 1984, p. 23.

3. Passmore, 1966, p. 49.

4. For more historical and biographical details, see Pester, 1997.

5. Translated by Erich Reck; in the original German: "Grundlegend für die Erkenntnis der Widersinnigkeit und theoretischen Unfruchtbarkeit des Psychologismus bleibt die Unterscheidung von psychischem Akt und logischem Inhalt, von realem in der Zeit verlaufenden Denkgeschehen und dem idealen außerzeitlichen identischen Sinn, kurz die Unterscheidung dessen, was 'ist,' von dem, was 'gilt.' Dieser reine, in sich Bestand habende Sinn ist Gegenstand der Logik, und damit wird ihr von Anfang an der Charakter einer empirischen Disziplin genommen."

6. Picht, 1977, p. 201.

7. Kreiser, 1984, pp. 25–26.

8. Dummett, 1982.

9. Cf. Gabriel, 1996.

10. Gabriel, 1984.

11. Cf. Lotze, 1884, §§13–14.

12. Manser, 1984, pp. 307–8.

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3

One Wittgenstein?

Steven Gerrard

Many of us were raised to believe that there were two great philosophers named “Wittgenstein”: one had the first name “Early,” the other was baptized “Later.” Some of us even claimed to have discovered a hidden brother with the unusual name of “Middle” (Gerrard, 1991b). The heresy of this essay will be to outline a reading of the *Tractatus* that will give us one Wittgenstein.¹

Every heresy, of course, requires a dogma to sin against, so I will begin with a sketch of the traditional story. It will be useful for my purposes to tell the orthodox tale of the differences between the Early and Later Wittgensteins via reference to three notions: *metaphysical realism*, *metaphysics*, and finally, *realism* (sans qualifier).

I. Metaphysical Realism

Hilary Putnam introduced the term “metaphysical realism.” According to his original definition, on that perspective: “The world consists of some fixed totality of mind-independent objects. There is exactly one true and complete description of ‘the way the world is.’ Truth involves some sort of correspondence relation between words or thought-signs and external things and sets of things” (Putnam, 1981, p. 49). Putnam continued: “I shall call this perspective the *externalist* perspective, because its favorite point of view is a God’s Eye point of view.”

The *Tractatus* is traditionally seen as the prime instance of metaphysical realism, indeed, metaphysical realism in its most sophisticated form.² (Just as the Augustinian Picture of the *Philosophical Investigations* is sometimes seen as metaphysical realism in its most primitive form; see Goldfarb, 1983.) Following is a typical way of telling the *Tractatus’* story, at enough of a level of generality to call it the “standard story.”

The scope of the *Tractatus* is wide indeed: Wittgenstein wrote on August 8, 1916, that “[m]y work has extended from the foundations of logic to the essence [Wesen] of the world” (Wittgenstein, 1979a). A central problem, however, can be put in one question: What is the connection between language and the world? The standard story gives the following elegant solution. Language and the world mirror each other. A distinction on one side always has a correlate distinction on the other. The world consists of simple objects that fit together to make facts. Language consists of simple names that fit together to make propositions. The way in which names can join together to constitute propositions has a form, and the way in which objects can join together to constitute facts has a form. *And the forms are the same.* Meaning is grounded and guaranteed by the identity of the possible structures of language and the world.³ At this elementary level, the standard story fits perfectly with metaphysical realism’s defining characteristics.

The standard story takes as full voice and uncomplicated the features of Tractarian objects characterized in the 2.0s:

- 2.02 Objects are simple. . . .
- 2.021 Objects make up the substance of the world. . . .
- 2.0271 Objects are what is unalterable and subsistent; their configuration is what is changing and unstable.

It directly follows from the standard story’s innocent interpretation of these passages that Tractarian objects are mind independent (for if they were dependent on anything they would not be simple and unalterable) and form a fixed totality.⁴

If anything approaches universal agreement among commentators, it is that the Later Wittgenstein combats metaphysical realism. Some misinterpretations have confused Wittgenstein’s opposition to *metaphysical* realism with an opposition to *realism* sans qualifier, thus reading Wittgenstein as an antirealist. Some of the history of Wittgenstein interpretation is encapsulated in this passage from a 1984 work by Baker and Hacker:

It has become fashionable in the last decade, under the influence of Michael Dummett, to view Wittgenstein’s development from the *Tractatus* to the *Philosophical Investigations* as a transformation of a realist into an anti-realist, a truth-conditional theory into an assertability-conditions theory. This has been an unfortunate influence, forcing Wittgenstein into a Procrustean bed, rather than looking carefully to see what he says. Kripke, like others, falls victim to this distorted way of looking at Wittgenstein. (Baker and Hacker, 1984, pp. 46–47)

To the “like others,” Baker and Hacker accurately (and refreshingly) footnote: “Ourselves (in the past) included.”⁵ The last decade, however, has seen a new thrust in the interpretation of the Later Wittgenstein. I am thinking of articles by (scrupulously listed in alphabetical order): James Conant, Cora Diamond, Juliet Floyd, Warren Goldfarb, John McDowell, Edward Minar and Hilary Putnam. These recent critiques argue, among others things, that far from being an antirealist, the Later Wittgenstein is challenging the picture in which both metaphysical realism and antirealism make sense. Although there are certainly significant differences among those commentators, there is enough of a similarity to call their view by one name: the “No-Nonsense Interpretation.” Briefly for now, here is why I have

chosen that name: No-Nonsense for the *Tractatus* because there is no chickening out, no gesturing. No-Nonsense for the *Investigations* because once something is exposed as nonsense, there is no ghost left over, no mourning a lost security, a lost foundation, which cannot even be formulated. (There is also the snide hint and allusion, which all true believers share, that it is only *we* who are directly and plainly reading the text; it is only *we* who are seeing what is really there.)

My chief heresy is this: the one Wittgenstein argued against metaphysical realism both *Early* and *Late*, the Early Wittgenstein is much more like this recent—No-Nonsense—interpretation of the Later Wittgenstein, and this can deepen our understanding of the Later view.

II. Metaphysics

All this talk of metaphysical realism leads us naturally to *metaphysics*. Again, there is almost universal agreement that the Later Wittgenstein was, whatever else, another figure in the time-honored philosophical challenge to metaphysics.⁶ The standard story sees the *Investigations* as the confessions of a recovering metaphysician. The interpretation of the Early Wittgenstein is more complicated. The cluster of interrelated controversies here is how to interpret (1) Wittgenstein's early attitude toward metaphysics; (2) the say/show distinction; and (3) the throwing away of the ladder. I will take them up in reverse order. The standard story is that the *Tractatus* wants to throw away the ladder and keep it, too, thus endorsing a mystical metaphysics of silence, which is, of course, a kind of metaphysics.

The chief exegetical problem of the *Tractatus* is to deal with the ladder and the apparent self-destruction of the text. Why isn't the *Tractatus* just another clever—but failed—metaphysical attempt to eliminate metaphysics; another theoretical argument against arguments and theories?

The *Tractatus* begins:

- 1 The world is all that is the case.
- 1.1 The world is the totality of facts, not of things.

This certainly sounds like full-voice ontological metaphysics, delivered with the creation myth intonations of Genesis. (Talk about a God's Eye point of view!) We then get seventy pages that, with the compactness and elegance of poetry, appear to unveil the essence of the world, language, thought, logic, science, ethics, aesthetics, philosophy, mysticism, truth, the self; until the very end, when the branch on which we are sitting seems to be sawed off (see Wittgenstein, 1953, § 55), the ladder which we have climbed is thrown (or is it kicked?) away:

- 6.54 My propositions serve as elucidations in the following way: anyone who understands me eventually recognizes them as nonsensical, when he has used them—as steps—to climb up beyond them. (He must, so to speak, throw away the ladder after he has climbed up it.)

He must overcome [or defeat] these propositions, and then he will see the world aright.

- 7 Whereof one cannot speak, thereof one must be silent.⁷

To this, Wittgenstein's dear friend Frank Ramsey famously replied: "If you can't say it you can't say it, and you can't whistle it either." Comedians know it is fatal to try to explain humor, but let me explain Ramsey's quip by a reasonably long detour to the say/show distinction. I will give one version of the standard say/show distinction story (which, of course, I am setting up in order to strike down).

In Wittgenstein's "Notes Dictated to G. E. Moore in Norway," dated April 1914, we have: "In any ordinary proposition, e.g., 'Moore good,' this *shews* and does not say that 'Moore' is to the left of 'good'; and *here what* is shewn can be *said* by another proposition. But this only applies to that *part* of what is shewn which is arbitrary. The *logical* properties which it shews are not arbitrary, and that it has these cannot be said in any proposition" (Wittgenstein, 1979a, p. 111). One way—the wrong way—of taking this, is as Tarski's hierarchy of languages with a metaphysical punch line. We will take "Moore good" to be part of the object language [L_o], and using English as the metalanguage [L_1], we can meaningfully say that the fact that "Moore" is to the left of "good" means that "Moore is good", and that this is an arbitrary, conventional feature of L_o . In this view, there are truths that cannot be said in L_o , but can be shown in L_o and said in L_1 . According to the standard story, the metaphysical punch is that although parts of language are arbitrary, not all aspects of language can be. Continuing the story, the *Tractatus* holds that the nonarbitrary parts of all natural languages and all artificial languages, indeed, of all possible languages, are the same. What is the same is the logical form. Thus, in a sense there is only one language, and that language is the subject of the *Tractatus*. English, German, the *Begriffsschrift*, and the language of thought are simply different systems of notation for the same language, different systems of which some are more revealing of logical form than others. Thus, there is a limit to the hierarchy of meta-languages, because *any* language, no matter how meta, must have these characteristics.⁸

This brings us to the heart of traditional Tractarian metaphysical semantics in *Tractatus* 4.12:

Propositions can represent the whole of reality, but they cannot represent what they must have in common with reality in order to be able to represent it—logical form.

In order to be able to represent logical form, we should have to be able to station ourselves with propositions somewhere outside logic, that is to say outside the world.

This is an externalist perspective, a God's Eye point of view, with a vengeance. And this leads to *Tractatus* 4.1212: "What *can* be shown, *cannot* be said."

All this is still part of explaining Ramsey's joke. In "Throwing Away the Ladder," the most significant article on Wittgenstein in many years, Cora Diamond describes two approaches to interpreting Wittgenstein's ladder and the connected cluster of issues. One can "chicken out" or "not chicken out." *Chickening out* is to gesture, to whistle, where one's theory tells you something can't be said. To chicken out, Diamond writes, is to read the *Tractatus* so that "you think that, after the ladder is thrown away, you are left holding on to some truths about reality, while at the same time denying that you are actually *saying* anything about reality" (Diamond, 1991, p. 182). To use my example, to chicken out is to treat

ascending the ladder as ascending a Tarski-like hierarchy of meta-languages. When one gets to the top, one recognizes that the whole hierarchy has something in common, something that it shares with reality, something we will call “logical form.” It would be a violation of the rules⁹ to say what this logical form consists in, but the sentences of the *Tractatus* gesture at this form of reality.¹⁰ We know what we want to say, and what we want to say is perfectly analogous with what we can say, but there is no way to legitimately say it. And this is not an accidental feature, but a necessary feature of language. We are left with a phantom top rung. Like Kant’s noumenal world, we know it is there, but can have no knowledge of it. (This interpretation of the *Tractatus* comfortably goes along with seeing the work as a kind of linguistic Kantianism.) “To chicken out,” Diamond writes, “is to pretend to throw away the ladder while standing firmly, or as firmly as one can, on it” (Diamond, 1991, p. 194).

Thus, the traditional story nails saying/showing and the ladder together in such a way that we are left with a mystical metaphysics of silence. There is metaphysics, all right, but it can not be properly said. And now Ramsey’s joke is that whistling in the graveyard of metaphysics is chickening out.

What, then, would not chickening out be? “To read Wittgenstein himself as not chickening out,” Diamond writes, “is to say that it is not, not really, his view that there are features of reality that cannot be put into words but show themselves. What is his view is that that way of talking may be useful or even for a time essential, but it is in the end to be let go of and honestly taken to be real nonsense, plain nonsense, which we are not in the end to think of as corresponding to an ineffable truth” (Diamond, 1991, p. 181). Not chickening out, on the interpretation I am advancing, is to see everything in the *Tractatus* (including the say/show distinction) as a stage in a dialectic.

A comparison of the *Notebooks* and the *Tractatus* will make my point clearer. Wittgenstein, I claim, did not originally intend to throw the ladder away.¹¹ In his 1913 *Notes on Logic*, when he turns to his understanding of philosophy, Wittgenstein begins with a (now) familiar tone:

In philosophy there are no deductions: it is purely descriptive.

Philosophy gives no pictures of reality.

Philosophy can neither confirm nor confute scientific investigation.

(Wittgenstein, 1979a, p. 106)

But then he continues: “Philosophy consists of logic and metaphysics: logic is its basis.” And still further down Wittgenstein writes: “Philosophy is the doctrine of the logical form of scientific propositions (not only primitive propositions).” Thus, in 1913 Wittgenstein included metaphysics as part of philosophy. The view of philosophy that survives in the *Tractatus*, however, does not appear in the *Notebooks* until the end of 1916. On December 2, 1916, Wittgenstein finally writes the draft of *Tractatus* 6.53, rejecting metaphysics: “and then whenever someone else wanted to say something metaphysical, to demonstrate to him that he had failed to give a meaning [Bedeutung] to certain signs in his propositions” (Wittgenstein, 1979a, p. 91).

We can see Wittgenstein’s pre-*Tractatus* conception of philosophy on the very first page of the *Notebooks* (September 1914). Here Wittgenstein asks, “Does a complete analysis [of sentences] exist? And if not: then what is the task of phi-

losophy??!” (Wittgenstein, 1979a, p. 2). On the next page, Wittgenstein asks a basic ontological question: “Is a point in our visual field a *simple object*, a *thing*?” He then writes: “Up to now I have always regarded such questions as the real philosophical ones: and so for sure they are in some sense—but once more what evidence could settle a question of this sort at all?” This is not quite a smoking gun, but here we do see Wittgenstein beginning to move away from a conception of philosophy that includes metaphysics and ontology to a conception where metaphysics is that which must be eliminated as nonsense.

What we have is an effort to get metaphysical realism precisely right, the gradual awareness of difficulties, and then its rejection. What was written full voice in the *Notebooks*, by the time of the completion of the *Tractatus*, becomes steps in a *reductio ad absurdum*. The *Tractatus*, on this view, becomes a dialectical work, with the metaphysical pronouncements being spoken with the voice of the Later Wittgenstein’s interlocutor.

III. The Dialectic of the *Tractatus*

Now is the time to provide some rough ground¹² for the slippery word “dialectic,” and my slippery claim that the *Tractatus* is a dialectical work. With some artificiality, I want to read the *Tractatus* as a retelling of some of the early history of analytic philosophy, in particular the history of analytic ontology. Quine’s quoted motto for *Word and Object* is “Ontology recapitulates philology.” My motto for the *Tractatus* is “Tractarian ontology recapitulates analytic history.”

The revolutionary year in the history of English analytic philosophy was 1903, with the publication of a pair of insurrectionist tracts: G. E. Moore’s *Principia Ethica* and Bertrand Russell’s *Principles of Mathematics*. A young engineering student named Ludwig Wittgenstein read Russell’s book and decided to study philosophy. Section 49 of Russell’s *Principles* contains a dense two-paragraph argument. I will reconstruct it as follows:

“Socrates is wise,” we are all told, and “wisdom is a virtue” was one of his teachings. Aside from the spelling, what is the difference between “*is wise*” in the first sentence and “*wisdom*” in the second? There is an obvious difference in roles: According to ordinary (as opposed, perhaps, to philosophical) grammar, “*is wise*” is the predicate, while “*wisdom*” is the subject. Is there any other difference? Russell denies there is and says of the opposing claim: “inextricable difficulties will envelop us if we allow such a view.” The essence of the difficulties, Russell writes in a letter to Frege, is that “if there can be something which is not an object, then this fact cannot be stated without contradiction; for in the statement, the something in question becomes an object” (Frege, 1980b, p. 134). Suppose, for example, that a philosopher wishes to deny that something, say *is wise*, can be made into the subject of a sentence. How can he do this, Russell reasons, except by saying “*is wise* cannot be made into the subject of a sentence”?¹³ But then he *has* made it into a subject: the very act of denying that something can be made into a subject shows that that very thing can, indeed, be made into a subject. From this, Russell concludes that “the theory that there are adjectives or attributes or ideal things, or whatever they may be called, which are in some way less substantial, less self-subsistent, less self-identical, than true substances, ap-

pears to be wholly erroneous, and to be easily reduced to a contradiction" (Russell, 1903, p. 46).¹⁴ Stripped of jargon and put into context, the claim is this. F. H. Bradley and other British Idealists, as interpreted by Russell, believed that there is only one thing: the Absolute. Everything else is incomplete, "in some way less substantial." Russell is arguing here that the Idealist's theory cannot be stated without contradiction, and thus must be false.

Using the same strategy in *Principia Ethica*, G. E. Moore argues that the Idealist's claim that "the parts would not be what they are but for the existence of the whole,"¹⁵ cannot be clearly stated without self-contradiction, for once we talk about a part, we are treating it as a separable whole itself.

Note that according to these English founders of analytic philosophy, a theory that cannot be stated must be false. This is precisely what the standard interpretation of the *Tractatus* takes to be mistaken. According to the chickening-out version of the standard story, the theory of the *Tractatus* prohibits the theory of the *Tractatus* from being officially stated, yet the theory itself is true.

Opposing the Idealists' monism, Moore and Russell posited a world consisting of a plurality of, respectively, *concepts* and *terms*. Moore introduces "concepts" by writing:

A proposition is composed not of words, nor yet of thoughts, but of concepts. Concepts are possible objects of thought; but that is no definition of them. It merely states that they may come into relation with a thinker; and in order that they *may* do anything, they must already *be* something. It is indifferent to their nature whether anybody thinks them or not. They are incapable of change; and the relation into which they enter with the knowing subject implies no action or reaction. (Moore, 1899, p. 179)

Acknowledging Moore, Russell echoes this metaphysics in his *Principles of Mathematics*. Russell's chief ontological category, "the widest word in the philosophical vocabulary" is the "term": "Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*" (Russell, 1903, p. 43). And: "Every term is immutable and indestructible. What a term is, it is, and no change can be conceived in it which would not destroy its identity and make another term" (Russell, 1903, p. 44). This is just how the standard story innocently interprets the ontology of the *Tractatus*'s 2.02s (as mentioned previously) and the 1s, especially foreshadowing 1.21: "Each item can be the case or not the case while everything else remains the same."¹⁶ In fact, a version of the standard story reads Tractarian objects as the direct descendants of these Russellian/Mooreian entities.

Even on this level, however, the standard story is in trouble. We do not need a fancy, controversial dialectical reading of the *Tractatus* to see that immediately, in the 2.01s, there is a Fregean correction to the Russellian/Mooreian 1s. *Tractatus* 2.0122 states what the 1903 Moore and Russell explicitly deny:¹⁷ "Things are independent insofar as they can occur in all *possible* situations, but this form of independence is a form of connexion with states of affairs, a form of dependence. (It is impossible for words to appear in two different roles: by themselves, and in propositions.)" I will explain the Fregean correction by going back to Russell's *Principles of Mathematics* argument. Although it was directed against Bradley,

Russell soon became aware he could have written this argument against Frege. (At the time he wrote the *Principles*, Russell had not yet read Frege carefully.) The preface to Frege's *Die Grundlagen der Arithmetik* announces three "fundamental principles." The third is "never to lose sight of the distinction between concept and object" (Frege, 1980a, p. x). It is this principle that Frege believes Russell has violated in the *Principles* argument. In Frege's ontology there are two kinds of entities: complete things (objects) and, contrary to Russell, *essentially incomplete*, "unsaturated" things (functions or concepts). Directly contradicting the claim of Russell's *Principles* argument, Frege writes: "objects and concepts are fundamentally different and cannot stand in for one another" (Frege, 1979, p. 120).

Famously, this leads to the concept horse problem. Although the expression "the planet Neptune" designates a planet, the expression "the concept horse" designates not a concept, but an object. "If I want to speak of a concept," Frege writes, "language, with an almost irresistible force, compels me to use an inappropriate expression which obscures—I might almost say falsifies—the thought" (Frege, 1979, p. 119). In "Concept and Object," Frege asks for "a pinch of salt" and says of any Russell-like account that would deny the essentially predicative or unsaturated nature of the concepts: "This may be done; but anybody who thinks the difficulty is avoided this way is very much mistaken; it is only shifted." Frege goes on to predict precisely how the problem shifted to Russell: "For not all the parts of a thought can be complete; at least one must be 'unsaturated' or predicative; otherwise they would not hold together" (Frege, 1977, p. 193). This has become known as the problem of the unity of the proposition.

We see here two fundamentally different kinds of metaphysics. Peter Hylton calls Russell's view an "object-based metaphysics" and Frege's a "judgement-based metaphysics."¹⁸ In a posthumously published work, Frege is clear about the priority: "So I do not begin with concepts and put them together to form a thought or judgement; I come by the parts of a thought by analysing the thought" (Frege, 1979, p. 253). In another posthumous work Frege goes into a little more detail: "Instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicated, we do the opposite and arrive at a concept by splitting up the content of possible judgement" (p. 17). Speaking of signs for properties and relations, Frege goes on to use an image that could come straight out of the *Tractatus*'s 2.01s: "I could compare this with the behavior of the atom: we suppose an atom never to be found on its own, but only combined with others, moving out of one combination only in order to enter immediately into another" (p. 17). An object-based metaphysics creates the problem of the unity of the proposition, while a judgment-based metaphysics avoids the problem from the beginning.

The wrong interpretation of Frege (and this is how Russell seems to have read Frege) construes Frege as having an object-based metaphysics. Licensed by Frege's use of metaphor, let me explain this by the following image: there are certain entities out there (let us picture them as jigsaw pieces). Some are whole, and others have holes in them. The complete pieces fit into the incomplete ones, and thus we get thoughts or judgments. We then use language to describe the parts and the whole. Complete linguistic expressions stand for complete entities (objects), and incomplete linguistic entities stand for incomplete entities (functions). But then

we have a problem. The only way to talk *about* something is to use a complete expression (as subject under an older logical model, as argument under the new). This means we can not talk about anything incomplete. Thus the sentences “no concept is an object,” “an incomplete thing is not complete,” “the concept horse is a concept” are false: We can not say what we want to say, we can not describe the world.

The right interpretation of Frege, however, sees Frege as holding a judgment-based metaphysics. Now our image is this. We begin with a block of wood. Then we cut it into jigsaw pieces. Surprise! The pieces fit together. The unity, the whole, the judgment are *prior* to the parts. On what basis do we cut the parts? The *sole* guideline is the uses of inference.¹⁹ It makes no sense to talk about the parts apart from the language of inference. The concept horse is a matter *not* of not being able to talk about something that is already out there. There is no *about* that one can not talk about. Frege had a failure of nerve. He requested “a grain of salt” when there was *nothing* that needed seasoning. The notion of talking about an object apart from language or discourse is alien to Frege and should have been incoherent to him. Unlike the Russell of the Platonic Atomism stage, Frege believed that there is no non-perspectival way of describing the world. Not only does his judgment-based metaphysics show this, but also the notion of *Sinn*. *Sense* is a perspective, a route, to a reference. It is Russell who sought direct access to objects, direct reference unmediated by sense. It is Russell, not Frege, who has the problem of how language hooks on to the world. Frege’s problem is how to analyze our judgments, our inferences, our discourse.²⁰

The punch line is that the Tractarian 2.01s are a judgment-based metaphysics, as the previously quoted 2.0122 should make clear. Tractarian objects much more closely resemble Fregean concepts than they do Russellian terms or Mooreian concepts.

Let me now relate this to the standard story that the *Tractatus* is a work of metaphysical realism. As Warren Goldfarb (1983) points out, serious misreadings arise if we take the criticisms in the *Investigations* to be straightforwardly criticisms of the *Tractatus*, and thus, in a way, interpret the *Tractatus* through the *Investigations'* critique. Here is a crucial example. On some versions of the standard story, the *Tractatus*’s solution to the problem of the connection between language and the world depends, not only on the identity of logical form, but also on the connection between name and object. Further, some of these readings take *that* connection, between name and object, to be forged through ostensive definition: the connection between language and the world runs through my mind, down my pointing arm and finger. On that reading, the *Investigations'* critique of ostensive definition is devastating to the *Tractatus*. Ostensive definition needs a slot, a category—it depends on language, and so cannot serve as the foundation for language. Thus, the characteristic of metaphysical realism that “truth involves some sort of correspondence relation between words or thought-signs and external things and sets of things,” while not being refuted, becomes extremely problematic. The problem with reading the *Tractatus* that way is that it reads the *Tractatus* as if it had been written by Russell, and ignores the Frege correction.

But if we go further, only to stop at Frege’s correction, that is still stopping too soon. We leave behind the naive mentalism of the standard story *Tractatus*, but we still stop at the level of semantic theory: on this modification of the standard

story, at this stage, Wittgenstein is giving us an account, an explanation of how language hooks up to the world—in each case, in the *Tractatus* and in the *Investigations*, on this story, the hooking depends on holistic considerations. My claim is we have not climbed far enough: Wittgenstein was arguing, both in the *Tractatus* and in the *Investigations*, that the problem of hooking is nonsensical.

There is another stage, where the formulation of ontology, much less competing ontologies, is called into question. Here, then, is what I mean by a “dialectical reading” of the *Tractatus*. The *Tractatus* begins with a modified Russellian/Mooreian ontology, where ontology is presented in an unproblematic way. That is the first stage. Then there is a Fregean corrective to that simple ontology, substituting an object-based metaphysics with a judgment-based metaphysics.²¹ Finally, however, the very notion of ontology itself is given up as nonsensical—not whistling, gesturing, chickening out nonsensical, but pure, unadulterated all mimsy were the borogoves nonsensical. Each stage is a corrective of the previous one. The last stage is where neither the questions nor the answers of ontology can be formulated, and thus:

6.5 When the answer cannot be put into words, neither can the question be put into words.

The riddle does not exist.

At that point we have thrown the ladder away.

IV. Realism sans Qualifier

I hope to make things clearer by exploring our last promised notion: *realism* (sans qualifier). One way of examining what might be meant by that phrase is to ask: Metaphysical realism versus *what?* Diamond, as the title of her book indicates, contrasts metaphysical realism with the *realistic spirit*. Putnam has also used the phrase *the realistic spirit* (Putnam, 1990, p. 42). As his view has evolved, Putnam has changed the name of his position: It was first called (although not, originally, by Putnam) “internal realism,” then “pragmatic realism,” and most recently, simply “realism with a small ‘r.’”²²

In his “Wittgenstein and the ‘Skeptical Paradoxes,’” W. W. Tait (1986) provides a nice example of the contrast between metaphysical realism and the realistic spirit. Responding to Kripke’s claim that the Later Wittgenstein is conceding solipsism, Tait first quotes *Investigations* 402: “For *this* is what disputes between Idealists, Solipsists and Realists look like. The one party attack the normal form of expression as if they were attacking a statement; the others defend it, as if they were stating facts recognized by every reasonable human being.” (It is striking how similar this is to the *Tractatus*.)²³ Tait goes on to say: “Now this is hardly conceding solipsism. Kripke is certainly right that Wittgenstein is criticizing the Realist here, too (142). But the Realist he is criticizing is one who defends our form of expression as though he were stating a fact, just as the Solipsist attacks that ‘statement of fact.’” In other words, the Realist that Wittgenstein is attacking is a *metaphysical* realist.

Tait then goes on to characterize a form of this metaphysical realism: “In both cases, there is the presumption that there are ‘facts’ which transcend the language

in which we express them and which adjudicate the correctness of our forms of expression.”

Now Tait gives a characterization of what I have been calling “realism sans qualifier”:

But there is another sense of “realism,” which finds its expression in #10: “Now what do the words of this language signify?—What is supposed to show what they signify, if not the kind of use that they have?” I think this is one of the most important passages in PI and that it is a refinement of Frege’s context principle. That a word ‘*X*’ is a name is shown by the role that it plays in our language. “What does it name?” is a question asked in our language and can be answered only there: “The word ‘*X*’ names *X*”—or something equivalent, possibly involving gestures such as pointing in the case of sensible objects. Terms referring to sensible objects and sensations do play the role of names in our language. And the assertions that sensible objects and sensations exist are unassailable—so long as we remember that these are assertions *in* our language and do not think of them as somehow mysteriously transcending our language and serving as justification for our normal form of expression.

Thus, the solipsist’s argument depends on just the picture of language that Wittgenstein is arguing, correctly I think, has no coherent application. (Tait, 1986, p. 488)

Another example, similar to Tait’s, is the connection between language and the world—the very question that is the center of the standard story of the *Tractatus* as well as the center of metaphysical realism. The early passages of the *Investigations* are partly an effort to undermine the legitimacy of the question. The inquest reaches something of a climax at *Investigations* 37, where the simple question is asked: “What is the relation between name and thing named?” To this we expect a *philosophical* answer, but instead we are charged to look at the details of various language games: “Well, what *is* it? Look at language-game (2) or at another one: there you can see the sort of thing this relation consists in. This relation may also consist, among many other things, in the fact that hearing the name calls before our mind the picture of what is named; and it also consists, among other things, in the name’s being written on the thing named or being pronounced when that thing is pointed at.” Much of the *Investigations* consists of arguments why, contrary to all our metaphysical urges, that is a satisfactory answer.²⁴ “We must learn to understand,” Wittgenstein writes in *Investigations* 52, “what it is that opposes such an examination of details in philosophy.”

Many of the examples of realism sans qualifier concern a grounding for our practices. Depending on the case, the No-Nonsense approach is that careful diagnosis might reveal not that the grounding is impossible, but that the grounding cannot even be described: the thought that there might be such a foundation is pure nonsense. There is a crucial difference between: (1) *x* is impossible; and (2) *x* is nonsense. (1) carries with it a residue; (2) does not. (1) keeps the metaphysics (and in some particular cases, metaphysical realism); (2) is the lower case realism, the realistic spirit. It is not the dismissal of problems, but the recognition of the right sort of problems.

Here is an example apart from Wittgenstein. Phillipa Foot begins her article “Moral Relativism” this way:

Some philosophical questions interest only philosophers: they would never occur to the plain man, and if he hears of them he may very well think that those who spend their time on philosophy must be a trifle mad. There are, however, other problems, no less philosophical and just as important, that are apt to present themselves to any enquiring mind. . . . The thesis of moral relativism is one of these natural philosophical thoughts. (Foot 1982, p. 152)

But yet, we have to see something is fishy with the way the *philosophical* problem of relativism is often formulated. It looks like there is an antinomy.²⁵ Reason is universal/normative, yet all reasoning is situated/factual. The philosophical problem seems to be to reconcile the two, and the stance seems to be standing above our practices, taking the external perspective, peering from the God’s Eye point of view and finding an a priori problem to be solved. Under the No-Nonsense Interpretation, however, the *philosophical* problem requires exposing in detail the picture that makes it seem like a problem. Yet it is crucial to recognize that there remain real practical problems of relativism (conflict between cultures, felt lack of grounding for moral claims, lack of common ground, etc.). In other words, the task is not to make everything easy by making the problem go away, but to get rid of the pseudoproblems so we can see and negotiate the real problems. The problem of standing above and looking down on our practices—looking at the world “from sideways on,” in John McDowell’s phrase²⁶—is an illusion we need Wittgensteinian therapy to rid us of (while, importantly, getting the illusion just right);²⁷ but from the standpoint *within* a practice (the only standpoint we have), there are, of course, real problems. Here there is an analogy with the philosophical problem of Other Minds. Supposing we could dissolve the a priori problem, it certainly does not follow that we would then always understand each other.²⁸ The goal of the realistic spirit is not to make things easy, but to find the right places to get down to work.

And yes: I realize all that sounds too much like a sermon, and that is why I now close with a section on the dangers of this interpretation of Wittgenstein. A chief danger, of course, is sermonizing instead of philosophizing.

V. The Danger of the No-Nonsense Interpretation

The Later Wittgenstein has been accused of a “*quietism*”—a quietism that many see as a rejection of philosophy, indeed, a rejection of seeing *anything* as problematic.²⁹ The reading of the *Tractatus* I am offering should better enable us to understand how, contrary to some immediate appearances, the *Investigations* is antquietistic.

Wittgenstein himself raises the problem, writing in *Zettel*: “Some philosophers (or whatever you like to call them) suffer from what may be called ‘loss of problems.’ Then everything seems quite simple to them, no deep problems seem to exist any more, the world becomes broad and flat and loses all depth, and what

they write becomes immeasurably shallow and trivial" (Wittgenstein, 1967, § 456). (Wittgenstein then adds, in an odd coupling: "Russell and H. G. Wells suffer from this.")

The danger of Wittgenstein's alleged *quietism*, the danger of the No-Nonsense Interpretation, is that by avoiding the Scylla of metaphysics we recoil into the Charybdis of flatness. The challenge for this reading is how to avoid metaphysics without seeing the world as flat. There are real problems within practices, and these problems have depth.

There are two kinds of flatness here, and, conveniently enough for my theme, we can delineate the different kinds by reference to Russell and—in this case—the later Moore.

The flatness that Wittgenstein perceives in Russell is the flatness of scientism. For the Russell of the *Principles of Mathematics*, the world is a puzzle to be solved. It was this Russell who wrote in 1900: "I discovered what appeared to be definitive answers to the problems which had baffled me for years. And in the course of discovering these answers, I was introducing a new mathematical technique, by which regions formerly abandoned to the vagueness of philosophers were conquered for the precision of exact formulae" (Russell, 1967, p. 218). It was this Russell who brought Wittgenstein into philosophy, and it was this Russell that Wittgenstein had to work through in the *Notebooks*. Since, however, scientism is not a danger that present-day Wittgensteinians are likely to fall prey to, I will not stop here, but will go on to the more prevalent danger of the No-Nonsense Interpretation.

Although beginning from similar positions, Russell became extremely suspicious of common sense, whereas Moore came to increasingly rely on it.³⁰ In 1925 Moore published a paper titled "The Defence of Common Sense," and in 1939 he published the infamous "Proof of an External World," where the two key premises involve holding up first one hand, and then another. Moore, ever since, and with justification, has been seen as waving his hands at philosophy. The second, and in the present climate the most dangerous, form of flatness is to read the later Wittgenstein as the commonsense Moore. Common sense served as a *justification* to Moore, as a solution to *philosophical* problems. Some of the commonsense readings of Wittgenstein see, first, a problem that requires solving (say a skepticism about meaning), and then, second, community, tradition, practices, ordinary language, or whatever, as providing the solution by playing a justificatory role.³¹ However, it is precisely Moore and this justificatory role of common sense that Wittgenstein argues *against* in many of his works, up to and including *On Certainty*.

The danger here is simply *dismissing* a philosophical account, an explanation, instead of working our way through it.³² The danger is in waving our hands and thinking we are doing philosophy. In the *Blue Book*, Wittgenstein raises this danger with an implicit criticism of Moore:

Certainly we shouldn't pity him if we didn't believe that he had pains; but is this a philosophical, a metaphysical belief? Does a realist pity me more than an idealist or a solipsist? . . .

Now the answer of the common-sense philosopher—and that, *n.b.*, is not the common-sense man, who is as far from realism as from idealism—the answer of the common-sense philosopher is that surely there is no difficulty in the idea of supposing, thinking, imagining that someone else has what I have. *But the trouble with the realist is always that he does not*

solve but skip the difficulties which adversaries see, though they too don't succeed in solving them. The realist answer, for us, just brings out the difficulty. (Wittgenstein, 1958, pp. 48–49, my emphasis).

(The meaning of “realist” here is that of “metaphysical realist.”)³³

Here is another example of Wittgenstein's *Blue Book* rejection of Moore:

Now when the solipsist says that only his own experiences are real, it is no use answering him: “Why do you tell us this if you don't believe that we really hear it?” Or anyhow, if we give him this answer, we mustn't believe that we have answered his difficulty. There is no common sense answer to a philosophical problem. *One can defend common sense against the attacks of philosophers only by solving their puzzles, i.e. by curing them of the temptation to attack common sense; not by restating the views of common sense.* (Wittgenstein, 1958, pp. 58–59, my emphasis).³⁴

It is part of Wittgenstein's charge of looking “from close to”³⁵ that one can never simply dismiss the metaphysician, one cannot simply *refuse* to give an explanation. When Wittgenstein writes in *Investigations* 464, “My aim is: to teach you to pass from a piece of disguised nonsense to something that is patent nonsense,” he is giving his methodology both early and late.

As is well known, in the *Investigations* Wittgenstein compared philosophy to therapy.³⁶ In his so-called Middle Period, Wittgenstein was more specific, calling forth some of the imagery of psychoanalysis: “A psycho-analysis is successful only if the patient agrees to the explanation offered by the analyst” (Moore, 1993, p. 108); and

One of the most important tasks is to express all false thought processes so characteristically that the reader says, “Yes, that's exactly the way I meant it.” To make a tracing of the physiognomy of every error. . . .

For only if he acknowledges it as such, is it the correct expression.
(Psychoanalysis.)

What the other person acknowledges is the analogy I am proposing to him as the source of his thought. (Wittgenstein, 1993, p., 165)³⁷

My claim is that this is the same method in both the *Investigations* and the *Tractatus*. The *Investigations'* interlocutor, the *Investigations'* voice of temptation, speaks in the dialectical stages of the *Tractatus*.³⁸ That voice must get metaphysical realism absolutely right—it must work through, not dismiss or not wave its hands at—the positions of Moore, Russell, and Frege. Far from being the prime example of metaphysical realism, the *Tractatus* is an attempt at a conclusive refutation, and the refutation consists in bringing the nonsense of an external perspective, a God's Eye point of view, to light.³⁹

Notes

A version of this essay was given at a conference in honor of Professor William W. Tait, “Philosophy of Mathematics, Logic, and Wittgenstein,” the University of Chicago, June 2, 1996.

1. Even heretics have confederates. In addition to the publications referred to in this essay, I have learned much from conversations about this topic with James Conant,

Burton Dreben, Juliet Floyd, Warren Goldfarb, John McDowell, and Hilary Putnam. David Cerbone and Lydia Goehr generously criticized an earlier draft; I am very grateful to them.

2. See, for instance, Saul Kripke's interpretation of the *Tractatus* in Kripke, 1982, pp. 72–73.

3. See Gerrard, 1991a, p. 113. One version of the standard story sees the *Tractatus* as a giant transcendental argument for this isomorphism.

4. Anticipating my later discussion, the standard story takes the 2.02s, which concern the simplicity of objects, apart from the 2.01s, which concern the combinatorial possibilities of objects. The 2.02s are Mooreian/Russellian; the 2.01s, Fregean.

5. See the first edition of Hacker, 1972, p. 104. After discussing Wittgenstein's famous attending of Brouwer's 1928 lecture, Hacker writes that this "suggests we should look at the transformation of Wittgenstein, the theorist of formal semantics, into Wittgenstein, the theorist of communication-intention, as being merely one aspect of a deeper and more general transformation. The convergence and affinities suggest that we view Wittgenstein's later philosophy as a generalized intuitionist theory, and that we view his transformation as being from realism in semantics to constructivism. Such an interpretation of Wittgenstein has been pioneered by M. Dummett in a series of papers, and substantially furthered and given preliminary systematization by G. P. Baker. There are no more fruitful tasks in Wittgensteinian exegesis than the elaboration of this interpretation, and few more important undertakings in philosophical logic than the examination of the nature and implications of a constructivist semantics" (footnote omitted).

6. See Putnam, 1990, p. 19: "I take it as a fact of life that there is a sense in which the task of philosophy is to overcome metaphysics and a sense in which its task is to continue metaphysical discussion."

7. Accepting James Conant's correction of the Pears and McGuinness translation in the last two lines; see Putnam, 1994, pp. xli, lxxiii n. 51.

8. See Wittgenstein, 1979a, pp. 18, 25. "In order to be able to frame a statement at all, we must—in some sense—know how things stand if the statement is true (and that is just what we portray). The proposition *expresses* what I do not know: but what I must know in order to be able to say it at all, *I shew in it*" (p. 18). And: "What can be said can only be said by means of a proposition, and so nothing that is necessary for the understanding of *all* propositions can be said" (p. 25).

9. According to a more detailed version of the standard story, propositions can only picture different combinations of objects. Anything else is beyond the limits of language. (Just as to Kant, causation only applies to phenomena. To apply it to noumena would be beyond the limits of the category.) Which arrangements of objects actually obtain is a contingent matter. Thus, it is impossible for language to picture anything *necessary*. Necessity, however, is treated as a kind of fact—a peculiar kind that can not be pictured, can not be said, can only be *shown*. On the No-Nonsense Interpretation, whatever necessity there is turns out to be trivial, and perfectly stateable. For the last point, see Dreben and Floyd, 1991.

10. There is a similar "gesturing" interpretation of the later Wittgenstein: since officially Wittgenstein rejects philosophical theory, he can only gesture toward constructive philosophy. See McDowell, 1994, pp. 92–93 n. 7, where he accuses Crispin Wright of this interpretation.

11. Here I take it I am agreeing with a suggestion of Putnam's. "Wittgenstein tells us, both in the prefatory remarks and at the end of the *Tractatus*, that his own theory is nonsense (*Unsinn*), and although some have seen this repudiation as weakened, if not altogether canceled, by the Tractarian claim that there are things that one can 'show' but not 'say' (for example, the logical form of a propositional sign shows itself but cannot be said in language), others [here Putnam footnotes Conant and Diamond] have argued (correctly to my mind) that it is a misuse of the 'say/show' distinction to hold that the metaphysical propositions of the *Tractatus* are, at the end of the day, really

supposed to express truths, or even thinkable thoughts. This very difficult and important exegetical question is, fortunately, one that I do not have to get into. In any case, Wittgenstein did at one time accept the picture theory (for example, in the *Notebooks*), and Russell held a very similar view in his logical atomist period; and clearly Wittgenstein still thought the theory worth presenting, even if his purpose may have been primarily dialectical, when he wrote the *Tractatus*" (Putnam, 1994, p. 94).

12. See Wittgenstein, 1953, §107.

13. For ordinary grammatical (opposed once more, perhaps, to *philosophical* grammatical) convenience, Russell would convert this to: "*wisdom* cannot be made into the subject of a sentence."

14. See also Moore, 1899, pp. 79–80: "A concept is not in any intelligible sense an 'adjective,' as if there were something substantive, more ultimate than it."

15. When that claim is interpreted as meaning "that the part is no distinct object of thought [...]," Moore argues: "That this supposition is self-contradictory a very little reflection should be sufficient to shew. We may admit, indeed, that when a particular thing is a part of a whole, it does possess a predicate which it would not otherwise possess—namely that is a part of that whole. But what cannot be admitted is that this predicate alters the nature or enters into the definition of the thing which has it. When we think of the part *itself*, we mean just *that which* we assert, in this case, to *have* the predicate that it is a part of the whole; and the mere assertion that *it* is a part of the whole involves that it should itself be distinct from that which we assert of it. Otherwise we contradict ourselves since we assert that, not *it*, but something else—namely it together with that which we assert of it—has the predicate which we assert of it" (Moore, 1959, §22, p. 33).

See also Russell, 1903, p. 448.

16. I am submerging the complications of 1.1: "The world is the totality of facts, not of things." Some of these complications can be explained via the differences between Moore's *concepts* and Russell's *terms*. For more on those notions, see Gerrard, 1997.

17. Moore writes that the Idealists' doctrine "that a part can have 'no meaning or significance apart from its whole' must be utterly rejected. It implies itself that the statement 'This is a part of that whole' has a meaning; and in order that this may have one, both subject and predicate must have a distinct meaning" (1959, p. 34).

18. See Hylton, 1990, p. 223. The 1899 G. E. Moore clearly held an object-based metaphysics; see the closing page of "The Nature of Judgment": "From our description of a judgment, there must, then, disappear all reference either to our mind or to the world. Neither of these can furnish 'ground' for anything, save in so far they are complex judgments. The nature of the judgment is more ultimate than either, and less ultimate only than the nature of its constituents—the nature of the concept or logical idea" (Moore, 1899, pp. 79–80). Translating Moore's "concept" to "object," Moore is here explicitly claiming that objects are prior to judgments.

This difference in ontology between Russell and Moore, on the one hand, and Frege, on the other, is partly due to a difference in enemies. Russell and Moore originally opposed Idealism with a naive realism, but Frege never saw Idealism as the serious enemy: naturalism was. (Russell and Moore thought naturalism in the guise of John Stuart Mill had already been conquered by the Idealists, so they did not need to engage in that battle. See Gerrard, 1997.)

19. See Thomas Ricketts's work, especially, in this context, "Frege, the *Tractatus*, and the Logocentric Predicament" (Ricketts, 1985). See, e.g., page 4: "[O]ur grasp on the notion of an object is exhausted in our mastery of the logical principles in which first-level variables and the associated proper names figure. Similar remarks hold for concepts. In this way, ontological notions are supervenient on logical ones."

20. See Ricketts, 1985, p. 8: "There is no standpoint from which to ask whether the thoughts expressed by the statements of language really represent reality, whether they are really true or false. Similarly, there is no standpoint from which to ask

whether the statements of language really do express thoughts. Here there is only the work of disambiguation, amplification, and clarification. This task requires no metaperspective.”

21. There is still a further stage, thoroughly investigated by Thomas Ricketts, where Wittgenstein in the *Tractatus* works through tensions in Frege’s thought. See Ricketts, 1985, 1996.

22. See Putnam, 1993, p. 353; 1990, p. 26. For a dissenting view, see the entry “metaphysical realism” in *The Cambridge Dictionary of Philosophy*. Metaphysical realism, according to the dictionary, is “in the widest sense, the view that (a) there are real objects (usually the view is concerned with spatiotemporal objects), (b) they exist independently of our experience or our knowledge of them, and (c) they have properties and enter into relations independently of the concepts with which we understand them or of the language with which we describe them. [...] Metaphysical realism, in all of its three parts, is shared by common sense, the sciences, and most philosophers” (Audi, 1995, p. 488).

Under the No-Nonsense Interpretation, metaphysical realism (as opposed to realism) is not shared by common sense and the sciences.

23. Indeed, throughout his career Wittgenstein expressed strikingly similar thoughts on the dispute between idealists and realists: the very dispute that was raging in England when Wittgenstein arrived. See, in this context, Wittgenstein, 1979a, p. 85; Wittgenstein’s *Tractatus*, 5.64 and 5.62; Wittgenstein, 1979b, p. 23; 1980, pp. 338–39; 1969, §37.

24. Cf. *Tractatus*, 6.521: “The solution of the problem of life is seen in the vanishing of the problem.”

25. Here I am following a suggestion of John McDowell.

26. See McDowell, 1981, p. 150, where McDowell is analyzing Wittgenstein’s criticism of a kind of platonism in mathematics: “The idea is that the relation of our arithmetical thought and language to the reality it characterizes can be contemplated, not only from the midst of our mathematical practices, but also, so to speak, from sideways on—from a standpoint independent of all the human activities and reactions that locate those practices in our ‘whirl of organism’; and that it would be recognizable from the sideways perspective that a given move is the correct move at a given point in the practice But the picture has no real content.” The kind of platonism here is, of course, a kind of mathematical metaphysical realism; a kind that W. W. Tait has called “The Myth of the Model in the Sky.” See also Diamond, 1991, pp. 184–85; Reck, 1997.

27. In 1931 Wittgenstein wrote: “In philosophy we are deceived by an illusion. But this /an/ illusion is also something, and I must at some time place it completely clearly before my eyes, before I can say that it is only an illusion.” Quoted in Stern, 1995, p. 25. See also Stern’s surrounding discussion.

28. See Wittgenstein, 1980, §558: “Those cases in which the inner seems hidden from me are very *peculiar*. And the uncertainty which is expressed in this is not a philosophical one; no, it is practical and primitive.” For an excellent discussion, see Tumulty, 1995, chap. 2, “Wittgenstein and the Inner/Outer.”

29. The central texts of Wittgenstein’s alleged quietism are *Investigations* 124–26, where Wittgenstein writes: “Philosophy may in no way interfere with the actual use of language. . . . It leaves everything as it is. . . . Philosophy simply put everything before us, and neither explains nor deduces anything.” My approach to quietism is meant to be complementary to John McDowell’s excellent discussions in McDowell, 1992; 1994, pp. 92–93, 175–80.

30. This is mirrored by Russell’s increasing suspicion of the philosophical value of ordinary language. In 1903 Russell wrote that “on the whole, grammar seems to me to bring us much nearer to a correct logic than the current opinions of philosophers: and in what follows, grammar, though not our master, will yet be taken as our guide” (Russell, 1903, p. 23). By 1905 and “On Denoting,” all this had changed.

31. Kripke is a partial example here.

32. See Putnam, 1990, p. 20: "Just saying 'That's a pseudo-issue' is not of itself therapeutic; it is an aggressive form of the metaphysical disease itself."

33. See Wittgenstein, 1975, p. 86: "From the very outset 'Realism,' 'Idealism,' etc., are names which belong to metaphysics. That is, they indicate that their adherents believe they can say something specific about the essence of the world."

34. Wittgenstein shortly goes on to say: "Our ordinary language, which of all possible notations is the one which pervades all our life, holds our mind rigidly in one position, as it were, and in this position sometimes it feels cramped, having a desire for other positions as well. Thus we sometimes wish for a notation which stresses a difference more strongly, makes it more obvious, than ordinary language does, or one which in a particular case uses more closely similar forms of expression than our ordinary language. Our mental cramp is loosened when we are shown the notations which fulfill these needs. These needs can be of the greatest variety" (Wittgenstein, 1958, p. 59).

Here we see Wittgenstein genuinely in transition. The *Tractatus* Wittgenstein thought of the *Begriffsschrift* as the route to exposing nonsense. The later Wittgenstein thought there was no privileged route to nonsense. Here we see Wittgenstein in between.

35. "[W]e must focus on the details of what goes on; must look at them from close to" (Wittgenstein, 1953, §51); and "Don't think, but look!" (Wittgenstein, 1953, §66).

36. See, e.g., Wittgenstein, 1953, §133: "There is not a philosophical method, though there are indeed methods, like different therapies"; and §255: "The philosopher's treatment of a question is like the treatment of an illness."

37. This is from the manuscript commonly known as "Big Typescript."

38. The presence of a kind of voice of temptation in the *Tractatus* has the added effect of making the *Tractatus* and the *Investigations* closer not only philosophically, but also stylistically: both works can be heard as dialogues.

39. If, contrary to the standard view, the Early and Later Wittgensteins are so much alike, the question naturally arises: How, then, do they differ? Here is a possible answer, and a program for future research: Early, Middle, and Late, Wittgenstein's method was therapy. The neurosis that we need to be cured of is *not* philosophy. Rather, the disease is unclarity (especially including the subspecies of pseudo-explanations). Philosophy is the cure. What changes is the notion of clarity: from the *Begriffsschrift* to *Übersichtlichkeit* (*perspicuity*: see Wittgenstein, 1953, §122.) (I believe I owe to Warren Goldfarb the suggestion that it was Wittgenstein's view of clarity that changed.) I discuss some of these issues in Gerrard, 1999. After I wrote this essay, Crary and Read, 2000, was published. It is now the best volume on this subject.

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PART II

FREGE

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Frege on the Indefinability of Truth

Hans Sluga

“**T**he content of the word ‘true’ is altogether unique and indefinable”—so Frege in his 1918 essay “The Thought.”¹ The provocative remark has caused his interpreters some embarrassment, for shortly afterward Alfred Tarski was to show how truth is definable in certain formalized languages, and this result is generally now considered one of the major achievements of modern logic. As a result, contemporary critics consider Frege’s thesis commonly as out of date or as ill conceived from the start. They also note that he proposes it only once in his published work, at the beginning of “The Thought,” and that its bearing on the rest of this essay, not to speak of the rest of his work, is far from evident. They express bewilderment, moreover, at the sketchy argument “The Thought” advances in its support.

The easy way out would therefore be to ignore Frege’s claim as an unfortunate blunder. But further scrutiny reveals that he himself considered it crucially important and that it lies behind much else he wrote about truth. What is more, Frege’s thesis contains *philosophical* insights into the notion of truth that are not captured in Tarski’s work and that have generally been ignored by those intrigued with Tarski’s technical brilliance. A popular misreading of Tarski has it that his semantics provides the key to an understanding of the ontological structure of the world. The unfortunate result is a revival of an a priori form of metaphysics that we had thought ourselves well rid off. Proper reflection on Frege’s thesis can serve as a timely antidote to such misconceptions and is, therefore, of more than historical interest.

I. The Place of “The Thought” in Frege’s Work

In order to see what significance attaches to the indefinability thesis, we must first be clear about the special place it occupies in “The Thought” and the equally

special place of the essay itself in Frege's oeuvre. For "The Thought" was by no means an incidental piece of writing. Frege published it in 1918, at the age of seventy, as the first in a series of articles under the general title called "Logical Investigations." It was followed in the same year by a second piece called "Negation" and in 1923, two years before his death, by a third entitled "Thought Connections." His posthumous writings contain, in addition, the beginning pages of an article on "Logical Generality" that was clearly meant to form a fourth installment of the "Logical Investigations." And there are indications that he intended to continue the series even further. Notes written in 1919 for the historian of science Ludwig Darmstaedter outline a course of thought that leads from the content of the four pieces just mentioned to an account of the sense-reference distinction and on to reflections about the nature of arithmetic and the numbers. It seems plausible to conclude that Frege planned to extend his investigations in this direction. This would also make sense of a number of drafts from the last two years of his life in which he sketches an altogether new approach to the analysis of the numbers.

In composing "The Thought" and the essays that followed, Frege drew on material of an earlier date. Since the 1880s he had been at work on an informal account of his logical and philosophical doctrines. In a series of drafts, with such titles as "Logic," "Introduction to Logic," "What Can I Regard as the Result of My Work?," "Short Survey of My Logical Doctrines," and "My Basic Logical Insights," he had tried to expound his ideas in a number of ways without, however, managing to bring them into a publishable form. The 1918 essay and its sequels were, in fact, based on the most extensive of these pieces, a text called "Logic" from 1897. Frege was, therefore, right when he suggested to Wittgenstein in 1918 that there was perhaps "little that is new" in "The Thought." But he was also right when he added that "perhaps it is still said in a new way and thus more comprehensible for some."² For while "The Thought" follows the earlier draft in both conception and language, there remain significant differences between the two pieces, and these are often as interesting as their agreements.

There is no doubt, in any case, that the composition and publication of "The Thought" constituted a surprising new departure for Frege. Disheartened by the inconsistency in his formal system, he had published nothing in the preceding decade. He was retiring from his university that year after forty-four years of teaching and was leaving the city of Jena, his home for half a century, to move to the tiny and isolated town of Bad Kleinen. He had not been in good health for some while, and Germany's military, political, and economic situation clearly depressed him. Given that the moment was in every respect inauspicious, one is left to wonder what may have helped to renew Frege's intellectual energies at this time.

The explanation that suggests itself most strongly is his relation with Wittgenstein. In 1911 he had come into contact with Wittgenstein, who had treated him and his work with unusual consideration. Wittgenstein visited him in 1911, 1912, and 1913 to talk extensively about logic and philosophy, with each of these visits lasting several days. During the war the two maintained contact, though little of their correspondence appears to have been devoted to philosophical issues. Still, Frege knew at least that Wittgenstein was continuing his work even under the most daunting circumstances, and his letters intimate that this reinvigorated his own intellectual energies. Hearing in May 1918 that Wittgenstein was finally putting

his thought into writing, he responded: "Perhaps, I too will be helped by this in the difficult territory in which I am laboring. I always expect some profit from getting to know the ways you have traveled, even when I should not be able to follow you in the essentials" (B, p. 17). One may plausibly conjecture, then, that it was the prospect of Wittgenstein's treatise that spurred Frege on to resume his labors. The supposition is further confirmed by his reaction to the news, four months later, that Wittgenstein had actually completed his work. His response combines congratulations with the announcement that he would soon be sending a copy of an article of his own, that is, "The Thought." His letter also predicts (correctly, as it turned out) that Wittgenstein would "probably not completely agree" with it but voices the hope that it would lead to lively exchanges between them.

Frege's new venture had been made possible by the philosopher Bruno Bauch, who at that time offered to publish something by Frege in his *Beiträge zur Philosophie des deutschen Idealismus*. This was a novel experience for Frege, who had always encountered difficulties in getting his articles printed. Bauch had founded his journal at the end of World War I as a nationalistic alternative to the liberal *Kantstudien*. To assure its success, he had sought to attract distinguished older academics sympathetic to its political line—among them the psychologist Wilhelm Wundt and his former teacher Heinrich Rickert, who had both veered to the right in World War I. As one of his colleagues at Jena, Bauch was presumably also familiar with Frege's increasingly reactionary views and was evidently keen to draw him as well to the journal and its associated organization, the Deutsche Philosophische Gesellschaft. He had, moreover, a genuine interest in Frege's work and had endeavored to use it in his own philosophy—a combination of Kantian idealism with a theory of objective value due to Lotze. Frege had his own obligations to Lotze and had borrowed from him, in particular, the distinction between subjective mental representations and objective thought. His essay "The Thought" concerned, among other things, precisely this Lotzean theme. When it appeared in the first volume of the *Beiträge*, Bauch preceded it appropriately with a piece of his own on Lotze's logic—clearly intended to legitimate the connection between Frege, Lotze, and himself. But Frege's intentions in publishing that essay went in a different direction. For Bauch's generous offer provided him with the unexpected chance to give his views a final and definitive airing before Wittgenstein could lay out his related though distinct ideas.³

Only if we are clear about this prehistory are we likely to appreciate the weight carried by the initial pages of "The Thought." Frege begins that essay with the declaration that logic is fundamentally concerned with truth: "As the word 'beautiful' points the direction for aesthetics and 'good' that for ethics, so 'true' points the direction for logic. . . . To discover truths is the task of all sciences; logic is concerned with recognizing the laws of truth" (T, p. 17). He goes on to denounce the idea that logic might be speaking of the mental process of thinking; the domain of logic is rather that of objective thought and its attendant truth or falsity. He insists on a boundary between psychology and logic that must not be breached, and he proceeds from this to drawing "the rough outlines of what I want to call true in this context."

Truth, he points out, is commonly asserted of "pictures, representations, propositions, and thoughts," and he considers the possibility that in proposi-

tions as in pictures we speak of truth “insofar as there exists a correspondence [*Übereinstimmung*] between the picture and what it depicts.” But a correspondence, he insists, “can only be perfect if the corresponding things coincide and are, therefore, not distinct things at all.” But this is not what we expect in the case of propositions. Here we assume, at most, that idea and reality correspond in certain respects. However, in this case, “we should have to inquire whether it were true that an idea and reality, perhaps, corresponded in the laid-down respect. And then we should be confronted by a question of the same kind and the game would begin again.” He proceeds from this to a generalized attack on all attempts to define the notion of truth, arguing that for the same reasons as the correspondence theory of truth “every other attempt to define truth collapses too.” Any definition whatever of the concept of truth would have to specify that a proposition is true, if and only if it has certain characteristics. But in this case “the question would always arise whether it were true that the characteristics were present. So one goes round in a circle.” The ultimate conclusion is, then, that “the content of the word ‘true’ is unique and indefinable” (T, pp. 18–19).

II. Six Phases in Frege’s Thinking about Truth

This statement of the indefinability thesis marks the terminal point in the evolution of Frege’s thinking about the notion of truth. The process leading to it had extended over forty years and had gone through at least seven distinctively different phases. Mapping these phases is helpful for understanding the important function that Frege assigned to his thesis. It will also help to destroy the common belief that Frege was an essentially static thinker whose system of ideas had come to him in one piece. Nothing reveals the dynamics of Frege’s thought more clearly than following the track of his thinking about truth from 1878 to 1918.

Phase I: A Logic of Judgment Rather Than Truth (1878)

The first phase of Frege’s thinking about truth is marked by the *Begriffsschrift*, which he finished in late 1878. In this phase the notion of truth plays a peculiarly subsidiary role. Frege’s explicit references to it are almost all confined to the first paragraph of the preface of his monograph. After that point the notion is set aside in favor of other concepts. What is distinctive in this manifests itself already in the very first words of the *Begriffsschrift* when he writes: “In apprehending a scientific truth we pass, as a rule, through various degrees of certainty.”⁴ Though there is talk here of truth, one is struck by the fact that Frege’s attention is directed toward its apprehension rather than truth itself. And this focus is maintained in the next few sentences, where he speaks of propositions coming to be more securely established “by being connected with other truths through chains of inferences” and of the division of “all truths that require justification into two kinds,” those that can be proved “purely by means of logic” and those whose proof requires appeal to “facts of experience.” In each of these claims Frege speaks *cognitively* of “recognition,” “certitude,” “justification,” and “secure foundations,” of propositions more or less “securely established,” “derived,” or “confirmed,” “in need of justification” or not, “supported by facts of experience” or depending “solely

on those laws on which all knowledge rests." He is evidently preoccupied throughout this passage with questions of knowledge rather than with truth per se (BS, pp. 5–6).

This is immediately reflected in the logic he proceeds to construct. For it is not a logic of truth and falsity but one of judgment. What he will later call a "thought," that is, the sense of a declarative sentence, he calls here a "judgeable content." And the primary symbol of the new logic is the "judgment sign," \vdash , which has the double function of combining the signs that follow it into a whole and of "affirming" this whole (BS, p. 12). This sign is so fundamental that Frege allows that one might consider it the single and common predicate of all judgments expressible in his symbolism: "We can imagine a language in which the proposition 'Archimedes perished at the capture of Syracuse' would be expressed thus: 'The violent death of Archimedes at the capture of Syracuse is a fact.' . . . Such a language would have only a single predicate for all judgments, namely, 'is a fact.' . . . Our *Begriffsschrift* is a language of this sort, and in it the sign \vdash is the common predicate for all judgments" (BS, p. 12f.).

The second logical symbol of the *Begriffsschrift*, that for the conditional, is also introduced without reference to the notion of truth. In characterizing the conditional judgment, Frege makes use instead of the notions of affirmation and denial. He writes:

If A and B stand for contents that can be judged, there are the following four possibilities:

- (1) A is affirmed and B is affirmed;
- (2) A is affirmed and B is denied;
- (3) A is denied and B is affirmed;
- (4) A is denied and B is denied.

Now  A
B

stands for the judgment that the third of these possibilities does not take place, but one of the others does. (BS, p. 13f.)

The first point to note is that he does not here explain the meaning of a connective but that of a conditional judgment. Equally important is that he characterizes the conditional by the analogue of a truth-table but that his operative terms are those of affirmation and denial, not those of truth and falsity. Instead of saying that the conditional sentence is true or false depending on the truth-values of its constituents, Frege says that the conditional judgment stands for some possibilities of affirmation and denial taking place and others not.

This explains also the peculiarities of the notation he adopts. We tend to think of a sentence of the form "If B, then A" as expressing a relation between the "antecedent" B and the "consequent" A. Frege conceives of it, instead, as an affirmation of A that differs from the straightforward, unconditioned affirmation

 A

only in that it is made on the basis of another affirmation. That the affirmation of A rests conditionally on B, Frege's notation indicates by letting the symbolic expression of the affirmation of A rest literally on B. Hence,



Though Frege speaks of logic throughout the *Begriffsschrift* in terms of affirmation and denial, this should not be taken to mean that truth plays no role whatsoever in his account. His intentions can be made clearer if we distinguish between implicit and explicit notions within his logic. The introductory sentences of the *Begriffsschrift* show him to be aware of the notion of truth as essential to logic, but he reserves for it an implicit role. The basic *explicit* term is that of judgment. But to judge that A is always to judge that A is true and there is, hence, for Frege an internal link between the notions of judgment and truth.

Phase II: The Logic of Right and Wrong (1881)

In three closely related papers written in 1881–82 Frege sets out to reply to criticisms Ernst Schroeder had leveled against the *Begriffsschrift*.⁵ In defending himself in these papers, entitled respectively “Boole’s Calculating Logic and the *Begriffsschrift*,” “Boole’s Logical Formula Language and My *Begriffsschrift*,” and “On the Aim of a *Begriffsschrift*,” he takes the opportunity to restate the fundamental assumptions and concepts of his logic. This he does, in part, by using his old language of affirmation and denial. Thus, he explains a formula we would render as

$$(P \& \neg Q) \rightarrow R$$

as saying that “if P is affirmed and Q is denied, then R is affirmed.”⁶ But in the general characterization of the conditional itself he takes an altogether new route. The first indication of this is that he now defines the conditional *connective* and not, as he had previously done, the conditional judgment. And in introducing that connective he no longer employs the language of affirmation and denial but says, instead, that the conditional sign expresses “the negation of the case that the upper content is wrong [*falsch*] and the lower is right [*richtig*]” (NS, p. 12). From this follows, even more important, a wholly new characterization of logic itself. While the *Begriffsschrift* had maintained the traditional characterization of logic as a concern with the laws of thought, Frege now has it that logic deals with the laws of “right inferring” [*richtiges Schliessen*] (NS, p. 13).

The move is certainly a technical improvement over the *Begriffsschrift* account, which had suffered from at least two flaws. The first was that Frege had introduced the conditional there by speaking of “possibilities” of affirmation and denial when he had at the same time dismissed modality as being “without meaning” for his logic (BS, p. 13). The second flaw lay in the *Begriffsschrift*’s confusion between whole judgments and their judgeable parts. Thus, Frege had said of the conditional judgment as a whole that it *affirms* the conditional “If B, then A,” but he had also said that both its constituents “A” and “B” are affirmed or denied. He had, thus, used the term “affirmation” to characterize both the distinctive function of the judgment as such and the contribution that its components make to the judgment. The new terminology of 1881–82 avoided such pitfalls.

The new terminology signals thereby a turn to a decidedly more “objective” specification of logic. For right and wrong are, presumably, judgment-independent characteristics of propositions. But the question remains why Frege introduced a wholly new pair of terms to make that point. Why did he not fall back on the notions of truth and falsity, which had been available to him since the *Begriffsschrift*? The answer may be this. Both the German terms *richtig* and *falsch* and their English equivalents “right” and “wrong” contain a reference to an outside ruler or standard. *Richtig* and “right” mean originally straight and aligned with a ruler and then, more generally, in accordance with some standard or measure. *Falsch* and “wrong” mean not in accordance with such a standard. The new terms have, thus, an inherently relational meaning. Right inferring is, on this understanding, an inferring according to logical laws, and in this lies for Frege the kernel of an important thought he will pursue later on. What turns out to be more problematic for him is the question of how we are to understand talk of right and wrong propositions. We know that Frege later rejected the idea that truth is relational in character. But the terms “right” and “wrong” have an inherently relational meaning. He does not actually tell us in these writings what standard or measure a right proposition is meant to accord to. But it is not absurd to conjecture that at this point a proposition is right for him when it corresponds to reality, that reality is the standard to which propositions need to be aligned.

If so, Frege never gets to spell this out. The second phase in his thinking about truth proves, instead, merely transitional. He soon abandons the terminology of right and wrong and replaces it by talk about true and false. But if our conjecture is correct, this apparently innocuous shift hides an important move in Frege’s thinking—a turning away from the idea that truth is to be understood as a relational notion and with it, a fortiori, a turning away from the idea that truth can be understood as a correspondence to facts. The seeds of Frege’s later explicit rejection of the correspondence of truth may, thus, be found in his abandonment of the terminology of right and wrong that he had entertained in 1881–82.

Phase III: The Objectivity of Truth (1884)

Truth turns into a central notion finally in the *Foundations of Arithmetic* of 1884. Frege now characterizes logical proof directly in terms of “truth.” A proof, he writes, seeks “to place the truth of a proposition beyond all doubt,” and it also “affords us insight into the dependence of truths upon one another.” He worries now over the question of whether “arithmetical truths” are synthetic or analytic, and he speaks broadly of “truths of a general logical nature,” of truths that “belong to the sphere of some special science,” of “truths which cannot be proved,” and of truths a priori and truths a posteriori. Most important, there are for him now “primitive truths” (*Urvahrheiten*) to which our inquiries will eventually reduce everything.⁷

It is legitimate to ask what motivated Frege to adopt that new language at this point. The notion of truth had remained implicit and hidden in the *Begriffsschrift* period and had been replaced by that of rightness in the next phase of his development. But now, in this third period, it seems to proliferate. What, we may ask, brought about this change of mind? We have noted already that the shift to the notions of right and wrong marks an increasing preoccupation in Frege’s mind

with the objectivity of logic. That concern becomes even stronger in the third phase and coupled with the rejection of any relational account of the rightness of propositions leads now to a new emphasis on the notion of truth.

Frege's increasing preoccupation with the objectivity of logic is made evident, in the methodological principles he formulates for himself at the beginning of *The Foundations of Arithmetic*. The first of these is "always to separate sharply the psychological from the logical, the subjective from the objective" (F, p. x). This had not been his main concern in the period of the *Begriffsschrift*, where he had spoken in a broadly psychological language of ideas and their combination. In his essay "On the Scientific Justification of a *Begriffsschrift*" from the same period, he had talked of sense impressions and memory images as fundamental to human thought and had explained the advantage of linguistic symbols by arguing that they create a new focus around which a group of memory images can gather and, thereby, give rise to thinking in concepts. "Thus, in applying the same symbol to different, but similar things, we actually no longer symbolize the individual thing, but rather what they have in common: the concept."⁶ By 1884, however, Frege has turned staunchly antipsychological in his thinking. John Stuart Mill's *Logic* is now at the center of his attention, and in opposition to Mill he insists now on an absolute separation of the logical from the psychological. In *The Foundations of Arithmetic* he protests accordingly against a "preponderance of the psychological point of view in philosophy, which has even made incursions into logic," and explains in this fashion the reluctance of mathematicians to address philosophical questions, and, to counteract these tendencies, he emphasizes the distinction between the being thought of a proposition and its truth. "One must, so it seems, call to mind once again the fact that a sentence ceases as little to be true when I am not thinking of it, as the sun is destroyed when I close my eyes." Frege's turn to the concept of truth proves thus to be a direct result of his increasingly vociferous antipsychologism (F, pp. 6, 7).

That it is antipsychologism which motivates the recasting of his logic in the language of truth and falsity is confirmed by another text from the same period, a fragmentary piece, called simply "Logic," that contains the earliest of the already mentioned informal accounts of Frege's logic. The editors of his *Posthumous Writings* date the piece somewhere between 1879 and 1891. But we can say with some assurance that it must have been composed immediately before *The Foundations of Arithmetic*. It was certainly written after 1882, since the claims that "the aim of scientific inquiry is truth," that the logical laws are "nothing but an explication of the content of the word 'true,'" and that logic itself is concerned with the clarification of the notion of truth (NS, pp. 2–3), with their emphasis on the notion of truth, belong neither to the first nor to the second phase we have distinguished. But the text was just as certainly written shortly after the end of the second phase. For Frege still characterizes the laws of logic initially in terms of the language of the second phase as "laws of right inferring" but then corrects himself and calls them instead explications of the content of the word "true." This dating is confirmed by another piece of evidence, for the text is preceded by a table of its projected contents, which hints at the possibility of defining objects by means of "judgments of recognition." This idea is actually put forward in section 62 of *The Foundations of Arithmetic* but then rejected, and we are thus justified in concluding that the text must have been written shortly before 1884.

All this is of interest only because the fragment contains a number of ideas and formulations that will eventually recur in “The Thought.” Both share with *The Foundations of Arithmetic* the belief that truth is to be distinguished from its recognition and that in being concerned with truth we are not dealing with something psychological. The business of the logician is, rather, “a continuous fight against the psychological.” Frege also introduces here for the first time the idea that the notion of truth is to be associated with that of good in ethics. The logical laws are in some ways like moral principles, not like natural laws: they are laws “according to which right thinking proceeds,” not laws of actual, psychological happenings. These formulations take up and expand on the idea voiced in the second phase that logic is concerned with the laws of right inferring. By this Frege had, presumably, meant that such laws are normative in character in that they adjudicate between right and wrong acts of inferring. After the second phase the idea that logic is concerned with inferring will drop away, but the belief in the normative character of the logical laws will remain. While Frege understands them no longer as (normative) laws of inferring, he now treats them as (normative) “laws of truth.”

In accordance with this shift he writes now that “anyone who fails to recognize the unique meaning of the word [true] will be unable to get clear about the task of logic.” While this comes close to his later views on truth, we must be careful not to read too much into these early formulations. For two important elements of Frege’s fully matured view of truth are as yet absent. The first is the doctrine of truth-values, the second is an explicit statement of the thesis that truth is indefinable. For the first we will have to wait another six years, for the second another thirteen (NS, pp. 3–7).

Yet another thing is absent from both *The Foundations of Arithmetic* and the early “Logic” fragment. While both assign a central and explicit role to the notion of truth, neither of them makes the slightest attempt to define it. This is particularly striking in *The Foundations of Arithmetic*, where we are offered definitions of the notions of analytic and synthetic, a priori and a posteriori truth but not of truth itself. Neither text talks of truth in terms of the notions of correspondence, facts, or reality, and neither text entertains the possibility of any other kind of definition of truth. The reason for this may be found in the context principle announced in *The Foundations of Arithmetic* (F, p. x). For the claim that words have meaning only in the context of a sentence can be taken to imply that the meanings of sentence components must be explained by reference to the meanings of the sentences in which they occur. On a strong reading, this would seem to exclude any characterization of the truth of a sentence in terms of the meanings of its components, and with the rejection of any compositional account of truth, any definition of truth as correspondence would also be ruled out.

Phase IV: The Idea of Truth-Values

In the next phase Frege inaugurates an idea that is widely considered puzzling or even counterintuitive. It is the assumption of two truth-values, the True and the False—two objects denoted by declarative sentences. This unusual doctrine is clearly intended to serve two ends. It distances Frege from any account of truth as a property or relational characteristic of propositions. It also aims to account for

the normative character of the logical laws of which he had spoken previously. But his talk of sentences denoting objects has been almost universally dismissed as a conflation between the semantics of names and that of sentences and as explicable only by a false economizing with theoretical distinctions. Insofar as logicians still employ the language of truth-values, they do so today as a shorthand for what they take truth to be in reality, namely, a certain kind of correspondence and, hence, a relational property. It would be easy to ascribe to Frege such an attenuated doctrine, but his words point clearly in another direction.

Having come to think of the logical laws as specifically concerned with the notion of truth, the question becomes for him in this fourth phase one concerning the nature of their validity. While these laws are related to human thinking, they do not describe how thought actually proceeds but prescribe how it must proceed, if it is to attain truth. But why should truth and its pursuit matter to us? No account of truth as a natural quality of propositions can give us a satisfactory answer. For why should we be interested only in propositions that have that quality and not some other one? The objection holds obviously also for the correspondence conception of truth. Assume that some propositions correspond to reality and others not. Why should only the corresponding ones matter to us? It is as if propositions were divided into those written in black ink and those written in red, and we then expressed a preference for the former. This would seem to be at best an arbitrary choice. If human thought ought to aim at truth, then truth cannot be a natural property. There is hidden here in Frege's thought an argument equivalent to Moore's argumentation against naturalism in ethics.

Decisive for him is that truth must have a normative force and that its normativity must be objective. Frege finds himself here at a place which the Neo-Kantian philosopher Wilhelm Windelband had also reached at about the same time. Frege and Windelband had both studied with Kuno Fischer in Jena and with Hermann Lotze in Göttingen, and this may explain their convergent views concerning the nature of truth. It was Lotze who had introduced the language of value into philosophy and who had spoken of the Good and the Beautiful as objective values. In the early eighties Windelband had extended this doctrine to include the True as a value. He had argued that it was necessary to distinguish between the content of a judgment and its truth or falsity, and that the latter constituted "truth-values" (*Wahrheitswerte*) at which we aim in making a judgment.⁹ This still left the ontological status of such values uncertain. For Lotze they had been unreal (*unwirklich*) but objectively "valid" (*gültig*). Windelband and Heinrich Rickert, his student, eventually ascribed to them being and objecthood as a guarantee of their objectivity, and in this Frege seems to have followed them.

What separates him from the Neo-Kantians is, however, that he manages to integrate this speculative doctrine into the formal structure of his logic. The period in which he comes to speak of truth-values is also the one in which he works out his distinction between functions and objects and his classical theory of sense and reference. Ever since 1878 he had, in fact, operated with a distinction between functions and arguments meant to subsume the traditional philosophical divisions between subject and predicate, concepts and objects, but generalizing and re-describing them in mathematical terms. None of this had, however, been worked out in the early years. But by 1884 the matter was taking on a new urgency for him. He now thought of numbers as individuated objects rather than predictable

concepts and found it, thus, necessary to insist on the principle “never to lose sight of the distinction between concepts and objects” (F, p. x).

In further development of this distinction, he emphasizes now in the fourth phase that functions must be thought of as “unsaturated” or “incomplete,” and that they are as such *categorially distinct* from objects that are to be conceived as saturated or complete. Into this doctrine Frege inserts the idea of truth-values by identifying concepts with functions whose “value” for any argument is the True or the False. It follows from this that concepts are categorially distinct from objects and that a concept that can be predicated of an object cannot also be predicated meaningfully of a concept and vice versa. This creates a quandary for theoretical notions such as that of “being an object” or that of “being a concept.” We assume that they are legitimate and use them to classify and describe objects and concepts. But the doctrine of categorial distinctions proves this to be logically impossible. While we may have an intuitive grasp of the difference between objects and concepts, there are in Frege’s understanding no legitimate concepts at hand for a theoretical characterization of that difference. Anything that looks like such must be considered a practical device to be discarded once we have caught on to the distinction it seeks to describe.

From this follows that no theoretical account of the semantics of names and functional expressions is possible. For it would have to specify that names refer to objects but not to functions and that functional expressions refer to functions and not to objects. But this proves impossible if the predicates “being an object,” “being a concept,” “being a function” are categorially illicit. It follows, moreover, that even the notions of meaning and reference must be categorially illicit, if they are taken to apply to both names and functional expressions. And this leads almost immediately to the conclusion that truth must be indefinable—one which Frege will not actually state until later. But even in the fourth phase he is committed to the idea that the entities referred to by the components of a sentence are categorially distinct. In the simplest case, a sentence will have to contain a name standing for an object and a predicate standing for a concept. A truth definition would have to say in this case that the sentence is true if and only if the object named falls under the concept referred to by the predicate. It would require us to speak of “the meaning” of the sentence, the name, and the predicate, but on Frege’s account there is no single notion of meaning that could perform such a service. It follows that any definition of the concept of the truth of a sentence in terms of the meanings of its components is categorially defective.

Frege’s second important advance in the late 1880s is his doctrine of sense and reference. He comes to it through a rethinking of the account of identity he had given in the *Begriffsschrift*. In order to resolve the inadequacies of his earlier account, he insists now on a distinction between the sense and the reference of an expression. When he applies this distinction to a whole declarative sentence, he is moved to divide what he had previously called the judgeable content into that which the sentence expresses and the object to which it refers. Following Lotze he calls the former the “thought,” and following Windelband he calls the latter a truth-value. That this is formally satisfactory depends, of course, on the particular way in which he draws the distinction between sense and reference, and this is, in turn, interwoven with his account of concepts as truth-functions. It cannot be our goal here to describe the intricacies of this structure. It is enough to have

indicated that Frege's doctrine of the True and the False as values and objects referred to by declarative sentences is worked thoroughly into the formal machinery of his logic. Together, they lead him inevitably in the next phase to the explicit formulation of the thesis that truth is simple, unique, and indefinable.

Phase V: The Indefinability Thesis (1897)

The claim of the indefinability of truth is made explicitly for the first time in a reworking of the “Logic” fragment of 1884, which we can confidently date as having been written in 1897. Frege begins this text by drawing once more a comparison between good, beautiful, and true. While the first characterizes the “goal” of ethics and the second that of aesthetics, “true” characterizes the goal of science. Even though all science is concerned with truth, logic is concerned with it in a very specific manner. Like ethics, it can be called a normative science. Logic is “the science of the most general laws of being true” (NS, p. 139). Frege continues:

It would now be in vain to try to make clearer by a definition what is to be understood by “true.” If one were to say: “An idea is true if it corresponds to reality,” nothing would be gained, for in order to apply it we would have to ask in each case whether an idea corresponds to reality, in other words, whether it is true that the idea corresponds to reality. One would have to presuppose what is to be defined. The same would hold of any explanation of this form: “A is true if it has such and such a property or if it stands in this or that relation to this or that.” . . . Truth is obviously something so primordial and simple that a reduction to something even simpler is impossible. We are therefore forced to illuminate what is unique in our predicate through a comparison with others. (NS, p. 140)

The first pages of “The Thought” will repeat this kind of reasoning, though in a somewhat more condensed fashion.¹⁰

Both pieces were, of course, written after Frege had introduced the sense-reference distinction, but they seem to pass over it with hardly a nod to the famous conception. There is, in particular, no mention here of the doctrine of truth-values. Frege speaks, instead, in both texts of “true” as a predicate. In the “Logic” of 1897 he writes of it as a predicate which “can be applied to thoughts” (NS, p. 142). And in “The Thought” he writes that “grammatically the word ‘true’ appears as an adjective” (T, p. 18). Still, the sense-reference account is not altogether forgotten and manifests itself in the characterization of “true” as a predicate applying to “thoughts.” Truth, he writes in “The Thought,” arises “for the sense of a sentence” or something “I call a thought” (T, pp. 19ff.). The fact that Frege does not call on the doctrine of truth-values at this point is not evidence that he has abandoned it, but rather of a didactic decision to begin with an intuitive account of truth as a property and then to proceed to an exposition of the doctrine of sense and reference. It is this order that he follows also in his notes for Darmstaedter.

Frege's 1897 statement of the indefinability thesis goes together for him with reflection on the notion of fact. He had not employed that notion since the *Begriffsschrift*, where he had suggested that the judgment sign could be read as a predicate, meaning that the judged content “is a fact.” The 1897 text reintroduces the notion of fact and links it in a surprising turn to the notion of thought. In Frege's

own words: "Thoughts are, for instance, natural laws, mathematical laws, historical facts" (NS, p. 140). What we think of as facts is thus subsumed under the notion of thought. The same idea recurs in "The Thought," where he declares apodictically that "a fact is a thought that is true" (T, p. 35). Facts are, on this account, not what thoughts are about—they are themselves thoughts. We tend to speak of thoughts as mental correlates of possible facts; but these would, in Frege's terminology, be subjective "representations" (*Vorstellungen*) and as such outside the purview of logic. Fregean thoughts are not representations of the world; they rather constitute the world. This may seem odd, at first sight, but is not without its attractions.¹¹ One may plausibly argue that the identity criteria of facts must be intensional, since the fact that Venus is the morning star is surely different from the fact that Venus is the evening star. Facts can therefore, on Frege's scheme, not be located at the level of reference where identity criteria are unfailingly extensional. It follows from this that truth cannot be conceived as a correspondence between thoughts and facts.

Phase VI: The Primacy of Judgment Reaffirmed (1915)

Frege's reflections on truth take yet another striking turn in a note from 1915, published posthumously under the title "My Fundamental Logical Insights." We know nothing about the circumstances under which he wrote it, but its content provides some hints as to Frege's motivations, for it reverts in a surprising twist to the *Begriffsschrift* conception of logic as primarily concerned with judgment and seeks to reconcile it with the later view of logic as primarily concerned with truth. We may take this to mean that the note was part of an overall stocktaking that Frege undertook as a result of Russell's discovery of the antinomy in his logical system.

Russell's communication in 1903 of this discovery had profoundly shaken his earlier assurance that he could ground arithmetic in pure logic. Initially, he seems to have worked on ways to circumvent the antinomy while salvaging the logicist reduction of arithmetic. But as these efforts got nowhere, he appears to have become increasingly despondent. In the text "Logic in Mathematics" from the spring of 1914, the logicist program remains unmentioned. Frege can get himself only to say that "mathematics is more closely related to logic than any other science" (NS, p. 218). And by 1919 he is ready to admit to Darmstaedter the failure of the program. "My Fundamental Logical Insights" was written between these two dates and, thus, at a time of increasing doubt about his achievements. It was natural, then, that Frege should seek to reflect at this moment on his whole development since the *Begriffsschrift*. He characterizes his 1915 note, accordingly, as "a key to the understanding of my results" (NS, p. 271). As such it can be read as the first of his writings from the final period of his life. "The Thought" and the rest of his "Logical Investigations," his notes for Darmstaedter and the various uncompleted drafts, everything he wrote from now on was an attempt to recuperate what was right in the earlier work, to pull the strands of his thought together, and to show the directions in which they might be extended.

The most striking feature of "My Fundamental Logical Insights" is its already noted reversion to the *Begriffsschrift* conception of logic. When we look at Frege's writings between 1879 and 1915, we may easily come to think that he had simply

set the idea of logic as concerned with judgment aside in favor of the unqualified belief that its fundamental concern is the notion of truth. But the apparently forgotten thought is unexpectedly reinstated and argued for in 1915, and one is left with the possibility that Frege had never abandoned it completely.

In "My Fundamental Logical Insights" he takes his departure from the observation that a thought is true even before it is grasped by any human being—something he had said many times since 1884. But now he adds to this an attack on the assumption that the word "true" is an adjective in the ordinary sense. He concludes rather that "the word 'true' has a sense that contributes nothing to the sense of the whole sentence in which it occurs as a predicate" (NS, p. 272). We know this claim as the "redundancy view of truth"—the doctrine that the sentences "A" and "A is true" mean the same. Interpreters often see in it the logical ground for the argument in "The Thought" that truth is indefinable. But "My Fundamental Logical Insights" tells a somewhat different story. For it sets out to justify the redundancy view of truth in terms of another and obviously more basic assumption according to which to assert some proposition A means to assert that A is true. Frege writes: "When I assert 'it is true that sea water is salty' I assert the same thing as when I assert 'sea water is salty.' Through this one recognizes that the assertion does not lie in the word 'true,' but in the assertive force with which the sentence is uttered" (NS, p. 271). He comes back to exactly this point three years later when he argues in "The Thought" that any definition of truth must presuppose a practice of assertion. Given any definition of "P is true" as "Q," we cannot say that P is actually true unless we are already in a position to assert Q; but since the practice of assertion incorporates a grasp of the notion of truth, the attempted definition will turn out to be circular. It presupposes an implicit understanding of that which it tries to define explicitly. When we use the word "true," we are, according to Frege's statement in 1915, merely trying "to make what corresponds to the assertive force appear to be a contribution to the thought." But that is impossible. "And although this attempt fails, or rather because of its failure, it points to the uniqueness of logic" (NS, p. 272). We are thus clearly back in the framework of the *Begriffsschrift*, since the uniqueness of logic of which Frege is speaking here is its fundamental concern with assertion and, hence, with judgment.

But the 1915 text is not simply a reversion to the earlier conception of logic. Frege is now engaged in the challenging project of reconciling that conception with the later idea that truth is primary for logic. He does so in a radically unexpected way by asserting that objectively speaking assertion is, indeed, the primary notion of logic, but that we cannot avoid talking about truth because of the imperfection of our language. We need a notion of truth, in particular, to make the transition from the imperfect language of everyday life to the more perfect language of the *Begriffsschrift*. He writes therefore: "How is it then that this word 'true,' though it seems devoid of content, cannot be dispensed with? Would it not be possible, at least in laying the foundations of logic, to avoid this word altogether? That we cannot do so is due to the imperfection of language. If our language were logically more perfect we would perhaps have no further need of logic, or we might read it off from the language" (NS, p. 272).

These are, indeed, radical ideas, but their bearing on Frege's subsequent work is not immediately obvious. For the first pages of "The Thought" put once again the notion of truth at the center of logic. And the centrality of that notion is em-

phasized even more strongly in the notes to Darmstaedter, where Frege writes: "What is distinctive about my conception of logic is that I give primacy to the content of the word 'true,' and then immediately go on to introduce a thought as that to which the question 'Is it true?' is in principle applicable" (NS, p. 273). Neither in "The Thought" nor in these notes does Frege refer to the possibility that judgment (or assertion) rather than truth is the primary concern of logic. One might therefore conclude that "My Fundamental Logical Insights" was merely an aside, a dead end, a road not taken. But when one attends more closely to Frege's insistence in "The Thought" that truth is not a property of propositions, that it is indefinable and unique, one begins to understand that he might still be gesturing in this essay toward the views contained in the note from 1915. According to it, talk about truth is always of a preliminary and didactic nature; any discussion of logic needs to start from that notion but will ultimately have to set it aside. Talk about truth cannot amount to a systematic theorizing; it can only prepare us for the use of a fully worked out logical language, but in such a language the concept of truth has no place. It will manifest itself there only in our ability to make reasoned and justified assertions. On this understanding "My Fundamental Logical Insight" is not the preface to "The Thought" but the coda to which the whole of the "Logical Investigations" lead up.

Phase VII: The Critique of the Picture Theory of Truth (1918)

We can be sure that Frege wrote "The Thought" with Wittgenstein in mind. For (like Russell) he believed him to be the one most likely to continue the work he had begun. While at work on his essay, he wrote to Wittgenstein in April 1918: "What you have gained from our interchanges will hopefully advance humanity a bit on its assigned road. It will be a comforting prospect for me, if the words I have exchanged with you will live on in their effects" (B, p. 16). And he repeated that thought once more in the following year when he foresaw Wittgenstein standing up one day for "what I believe to have discovered in the area of logic." Their exchange of ideas was important to him, since "in long conversations with you I have got to know a man who like me has searched for truth, though partly on different paths" (B, p. 21).

"The Thought" may be read as Frege's attempt to show Wittgenstein on what paths he himself has searched for truth. The essay was written and published months before Frege had a chance to set eyes on Wittgenstein's *Tractatus*, but their correspondence reveals that he expected to find both agreements and disagreements with his own ideas in that text. He knew, for instance, of Wittgenstein's objections to his doctrine of truth-values from letters they had exchanged in 1913.¹² In conversations Wittgenstein had, moreover, defended Russell's theory of signs, and that theory included a commitment to a correspondence notion of truth. After having taken truth to be simple and undefinable in the early years of the century, Russell had felt "driven back to *correspondence with fact* as constituting the nature of truth," as he wrote in 1912 when he added the proviso, however, that "it remains to define what we mean by 'fact,' and what is the nature of the correspondence which must subsist between belief and fact, in order that belief may be true."¹³ This was the project to which Wittgenstein had devoted himself by developing his own "picture-theory of meaning." The world, he had argued, consists of facts, and a fact

is the existence of a state of affairs which in turn are combinations of objects. The combination of names that makes up a sentence is also a fact, and a sentence is true if it stands in a strict mapping or picturing relation to a fact in the world. The sentence “does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.”¹⁴

We cannot say how much of this doctrine Frege actually knew when he wrote “The Thought,” but the essay contains a number of pointedly anti-Tractarian statements. It begins with a critique of the correspondence conception of truth in the form of a picture theory of the sort we find in Wittgenstein’s treatise. As already quoted, Frege argues that the truth of a proposition cannot consist in its pictorial correspondence to reality, since such a correspondence “can only be perfect if the corresponding things coincide” (T, p. 18). But this is neither expected nor desired in the case of propositions. If we say, on the other hand, that the proposition is true when it corresponds to a fact in a certain respect, we will be forced to ask whether it is true that it corresponds in the laid-down respect, and then we presuppose the notion of truth we are seeking to define. These are, indeed, issues that Wittgenstein is forced to address in the *Tractatus*. He writes accordingly that “in the picture and the pictured there must be something identical in order that the one can be a picture of the other at all” (TLP, 2.161). And he postulates a notion of logical form that can be attributed to both propositions and facts, to language and the world. “What every picture, of whatever form, must have in common with reality in order to be able to represent it at all—rightly or falsely—is the logical form, that is, the form of reality” (TLP, 2.18). This is not, however, a notion Frege would have been comfortable with, and his refusal to define truth may be considered an expression of his resistance to it.

Though Frege’s attack on a picture conception of truth may be taken to be directed against the *Tractatus* view, one must not overlook that he had engaged in similar criticisms already in his “Logik” of 1897. He had insisted then that the sentence is a vehicle for the expression of thoughts, not of ideas (*Vorstellungen*), since it is “not well suited to reproduce ideas. Pictures and musical pieces are, by contrast, unsuited to express thoughts” (NS, p. 137). And he had gone on to characterize talk of the truth of ideas and pictures as merely derivative: “An idea, like any other picture, is not true in itself but only in respect to something to which it is supposed to conform. . . . Without reference to an intention to depict something, one cannot talk of the truth of a picture. From this one can derive that it is not properly speaking the idea to which the predicate ‘true’ is attributed, but the thought that it depicts a certain object” (NS, p. 142). Though the conclusion is the same as the one he comes to in 1918, he reaches it here by a different route. And this difference may be explained by the fact in “The Thought” Frege is concerned with the kind of picture conception of truth we find in the *Tractatus*.

When Frege received the actual text of the *Tractatus* at the end of 1918, his reaction to it was decidedly cool. Despite repeated entreaties from Wittgenstein (WBW, p. 267; B, p. 19), he did not respond to it till the following June when he wrote apologetically: “You must certainly have been awaiting an answer from me for some time now and must have wished for a comment from me on your treatise.” To excuse himself he mentioned “complex business matters” and the fact that “I often feel tired now” (B, pp. 19, 20). While he tried to explain thus his inability to give a reasoned judgment on the *Tractatus*, he could not avoid the even-

tual admission: “I find it difficult to understand” (B, p. 19). This was a complaint he would repeat again somewhat later, in August, when he wrote: “I cannot give an assessment of the treatise itself, not because I disagree with its content, but because that content is insufficiently clear to me” (B, p. 23).

Frege was puzzled by the “dogmatic” style of the *Tractatus*, by the omission of arguments and explanations, by the lack of a clearly stated problem to which the text addresses itself, by Wittgenstein’s insistence that the reader must already have entertained the thought expressed in the work, and by Wittgenstein’s aesthetic intentions. “What you write to me about the purpose of the book, is strange to me,” he remarked in September 1919. “I proceeded in my comments from the assumption that you wanted to communicate a new content. And in that case the greatest clarity would be the greatest beauty” (B, p. 21). It was clear from all this that they would never come to agree with each other. While Frege’s last letter to Wittgenstein, from April 1920, appealed once more to their “old friendship,” they were, in fact, going their separate ways.

At the center of Frege’s critique of both Russell’s correspondence theory and Wittgenstein’s picture theory of truth stands an attack on the notion of fact the two had employed. The observation in “The Thought” that a fact is a thought that is true, far from being a casual aside, is for that reason of critical importance for Frege. He makes this evident in the criticisms of the *Tractatus* he eventually offers Wittgenstein in his letters. Interpreters have occasionally downplayed these comments because Frege prefaces them with an admission that he finds Wittgenstein’s treatise difficult to understand. But this is surely meant to be taken in a philosophical rather than a pedestrian sense. Frege concentrates in his critique on the first pages of the *Tractatus*, writing to Wittgenstein: “You use right at the beginning a fairly large number of words on whose precise sense obviously much depends” (B, p. 19), and he singles out terms like “being the case,” “state of affairs,” “situation,” and above all “fact” as being problematic. He expresses puzzlement, in particular, at Wittgenstein’s statement that a fact is the existence of states of affairs. “I understand this to mean that every fact is the existence of a state of affairs, such that another fact is the existence of another state of affairs. Would it be possible to strike out the word ‘existence’ and say: ‘Every fact is a state of affairs, every other fact is another state of affairs?’” (B, p. 20). These are concerns that later readers of the *Tractatus* have also felt.

Frege continues that these initial doubts have made it difficult for him to advance in his reading of the treatise. His predicament is, however, not merely terminological in nature, as the further course of his argumentation reveals. Since Wittgenstein characterizes states of affairs as combinations of objects, Frege asks whether that means that facts, too, are combinations of things. This is important to him because it implies that facts have extensional identity criteria. He refers in this connection to the principle that the part of a part is a part of the whole, and so, if facts are composed of objects, they must also be composed of the parts of these objects. He writes: “I would like to have an example for the Vesuvius being a component of a state of affairs. That would mean, it seems to me, that the components of the Vesuvius are components of this fact; the fact will thus consist of hardened lava. That does not seem to me right” (B, p. 20). His argument comes to this: the state of affairs that the Vesuvius is near Naples is clearly different from the state of affairs that a heap of hardened lava is near Naples, for the first may be

the case even if the Vesuvius does not consist of hardened lava. The identity criteria for states of affairs and facts must therefore be intensional. But this gets us back to the conclusion that facts are simply true thoughts, and that truth cannot therefore be defined as a relation between thoughts (or the sentences that express thoughts) and facts.¹⁵

Wittgenstein failed to appreciate these critical comments, just as he failed to appreciate the alternative view Frege had laid out in “The Thought.” Wittgenstein had received a copy of that essay after his release from the prisoner-of-war camp on August 21, 1919. But he found it just as difficult to respond to it as Frege had found it to assess the *Tractatus*. His life, too, was in turmoil, and so in September 1919 Frege, in turn, found it necessary to nudge him gently for a response by pointing out convergences between Wittgenstein’s views and his own. Only in March 1920 did Wittgenstein finally get around to addressing Frege’s concerns, and then in such a highly critical manner that he felt it necessary to apologize in advance. “Naturally, I do not resent your outspokenness,” Frege replied. But he was utterly puzzled by Wittgenstein’s comments and, obviously suspicious that Wittgenstein had not seriously studied his essay, urged him “to go through the article on ‘The Thought’ to the first sentence with which you disagree and to write it down together with the reasons for your deviation.” (B, p. 24). It looks as if Wittgenstein’s exchanges with Frege had come to an end at this point. While he retained a general admiration for Frege, he never seems to have come around to appreciating “The Thought” because in later years he dissuaded Max Black and Peter Geach from including it in the collection of Frege’s writings that he encouraged them to publish.

Despite their patent disagreements, there was, however, one fundamental point on which Frege and Wittgenstein might have come to a mutual understanding at this point, for they both entertained similar doubts about the possibility of a *theory* of truth. Frege had expressed his doubt most directly in “My Fundamental Logical Insights,” which Wittgenstein did, of course, not know, since it remained unpublished until 1969. If he had been acquainted with that text, he might have been surprised by the almost Tractarian tone of Frege’s conclusion that in a fully functioning language talk about truth would eventually drop away. If Frege had not been put off by the first pages of the *Tractatus*, he, in turn, might have come to appreciate how close the final pages of that work came to his 1915 arguments. While the early parts of the *Tractatus* spell out an account of truth, Wittgenstein proceeds there to deconstruct all semantic theorizing and concludes that all attempts to speak *about* logic are bound to fail. The result is the concluding declaration in the *Tractatus* that “my propositions serve as elucidations in the following way: anyone who understands me eventually recognizes them as nonsensical. . . . He must transcend these propositions, and then he will see the world aright” (TLP, 6.54). For on Wittgenstein’s conception, “logic is not a theory but a reflexion of the world,” and as such “transcendental” (TLP, 6.13, and 6.2). Similarly to the Frege of 1915, he concludes in a memorable phrase that “logic must take care of itself” (TLP, 5.4731).

III. Frege against Tarski

Thirty years ago, Jean van Heijenoort argued in a seminal article that none of the original authorities in the rise of mathematical logic, neither Frege nor Russell

nor Wittgenstein, had been concerned with questions of semantics, whose formal development had been left to another generation. The “logicians” had taken logic instead as a universal language such that “nothing can be, or has to be, said outside the system.” In consequence, Frege had never raised “any metasystematic question,” and “questions about the system are as absent from *Principia Mathematica* as they are from Frege’s work. Semantic notions are unknown.”¹⁶

It is certainly true that none of the logicians constructed a *formalized* metalanguage or a *formalized* theory of meaning or truth. What we have in their writings are incidental arguments that are sometimes disowned as soon as they are made. But van Heijenoort overstated his case by ignoring Frege’s, Russell’s, and Wittgenstein’s extensive discussions of the notions of meaning and truth. There is, in fact, no other period in the whole history of logic in which the concept of truth was examined with such passion and in which such different proposals emerged about how it should be understood. And this was no accident. Since the old Aristotelian logic had been tied to the classical correspondence conception of truth, the logicians found it necessary to rethink, revise, or reject the traditional conception before they could make advances in their own logical investigations.¹⁷

It has been said that Tarski’s definition of truth returns us to the prelogician tradition, that it salvaged the Aristotelian notion of truth and showed how it could be made precise, that Tarski achieved what the logicians either neglected or claimed to be impossible. This would imply that he opened the door once again for the kind of metaphysical speculation that surrounded the classical notion of truth. But there is another and more compelling way to read Tarski. It holds that his truth-definition has nothing to do with the traditional conception of truth as correspondence to the world. Tarski offers us, instead, an account of how sentences of one language, the object-language, are correlated with those of another, the metalanguage. On this account Tarski’s criterion of adequacy declares in metalinguistic terms that an object-language sentence can be called true when we are in a position to assert its metalinguistic translation. No metaphysical consequences follow from this.

Understood in this minimalist fashion, Tarski’s theory may be taken to confirm the Fregean intuition that assertion is primary to logic rather than truth. However, if the notion of truth is contained in that of assertion, it also holds for both Tarski and Frege that we must understand the notion implicitly in one language (the metalanguage) before we can explain it explicitly in another. Tarski can offer us therefore at most a partial definition of the concept of truth, and since Frege opposed any kind of partial definition, he would have had to reject also Tarski’s formal construction. There are additional reasons why Frege would not have been happy with Tarski’s theory. His claim that words have meaning only in the context of a sentence and his related assertion of a categorial difference between functions and objects rule out any theoretical account of the truth of sentences in terms of the meanings of their components. That these doctrines are not reconcilable with Tarski’s theory can be seen from Rudolf Carnap’s initial resistance to formal semantics, which stemmed from his early reliance on Fregean assumptions. Carnap changed his mind on the prospects of a Tarski-style semantics only after he had abandoned these inherited commitments.¹⁸

Most logicians today will think that Carnap did right in this. But even if we concede this point and allow for the legitimacy of Tarski’s technical constructions,

we are forced to conclude that there remain insights to be gleaned from Frege's alternative line of reasoning. One of these is the observation that the concept of truth is so fundamental to our understanding that it is impossible to reduce it to other, more primitive notions. A second concerns the normative character of the notion of truth, a topic on which Tarski's theory has nothing to say. While we may want to resist Frege's own value-theoretical explanation of this normativity, we still need to account for it in some way or another. Finally, there is the insight which Frege shared with Wittgenstein, that our practice of assertion is certainly responsible to the world, but that the attempt to explicate the relation between language and world must remain problematic because it is, of necessity, always confined within the bounds of language.

Notes

I have learned much from questions and comments on earlier versions of this essay. I am particularly grateful to Jamie Tappenden for extensive notes on a somewhat earlier draft. I wish that time and space had allowed me to address the serious concerns and views he has expressed in Tappenden, 1997, in the manner they deserve.

1. Frege, 1967b, p. 19 (hereafter referred to as "T"). In this passage Frege characterizes his conclusion as merely "probable," but this is not to be taken as a sign of his uncertainty about it. He is expressing rather his realization that it depends on a long string of considerations that go all the way back to the *Begriffsschrift* and not simply on the sketchy argument he provides in "The Thought." This is, in any case, the assumption on which I proceed in this discussion.

2. Frege, 1989, p. 18 (hereafter referred to as "B").

3. On Bauch, his journal, and the Deutsche Philosophische Gesellschaft, see Sluga, 1993.

4. Frege, 1967a, p. 5 (hereafter referred to as "BS").

5. See Sluga, 1987.

6. Transcribed from Frege, 1969, p. 47 (*Nachgelassene Schriften* hereafter referred to as "NS").

7. Frege, 1959, pp. 2–4 (hereafter referred to as "F").

8. Frege, 1972, p. 84.

9. Windelband, 1882, p. 3; see also Windelband, 1884. The observation that the term "truth-value" is due to Windelband was first made by Gottfried Gabriel in Gabriel, 1986. For a discussion of these issues, see also Sluga, 1996. Given Frege's interest in the nature of negative judgments, it seems to me likely that he discovered the notion in Windelband's 1884 essay rather than in the less accessible earlier piece. Windelband writes in 1884 (in an admittedly psychologistic language) of the need to distinguish in every judgment between the mere combination of ideas and an act of adjudication (*Beurtheilung*) that concerns "the truth-value of the judgment" (p. 170), adding that "the truth-value must be correlated with the other values" (p. 174).

10. Rudolf Carnap recorded an even more abbreviated version of the argument when he attended Frege's lectures at Jena in the winter of 1910. Frege concluded then that "truth cannot be defined, analyzed, or reduced. It is something simple, primordial" (Frege, 1996, p. 15).

11. Frege's language is certainly no more odd than G. E. Moore's doctrine in his seminal 1899 essay "The Nature of Judgment" that the world consists of judgments or Russell's related claim in *The Principles of Mathematics* that it consists of propositions; see Moore, 1899; Russell, 1959a.

12. Frege, 1976, p. 266 (hereafter referred to as "WBW").

13. Russell, 1959, p. 123.
14. Wittgenstein, 1922, 5.542 (hereafter referred to as “TLP”).
15. There are some other notable disagreements between “The Thought” and the *Tractatus* that cannot be discussed here. In contrast to Wittgenstein but in agreement with Russell, Frege maintains in “The Thought” that there are real subjects in the world and that each subject has an incommunicable knowledge of itself. And in further contrast to Wittgenstein, but once again in agreement with Russell, he argues that logical truths have substantive content and are not mere tautologies.
16. van Heijenoort, 1993, p. 74.
17. Sluga, forthcoming.
18. Sluga, 1999.

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On Interpreting Frege on Truth and Logic

Sanford Shieh

No one disputes that Frege is a founder of modern logic. Few have asked whether Frege thought of logic in the same way as do contemporary philosophical and mathematical logicians. Of course, this last claim, as formulated, suggests that there is a single fully articulated contemporary conception of logic, a presupposition that may well be questioned. But what I have in mind in writing of how we now view logic is a quite imprecise idea: that semantic concepts, principles, and theories occupy a legitimate, if not indeed central, place in logic. It is the attribution of *this* idea under some reading to Frege that has rarely been doubted by analytic philosophers. A plausible reason is this. Frege's philosophy of language, in particular the sense/reference distinction considered as an account of linguistic meaning, has been the aspect of his work that had the most substantial philosophical impact on the analytic tradition. Thus, most analytic philosophers would have found nothing surprising or controversial in the following remark of Montgomery Furth, in his introduction to his partial translation of *Grundgesetze der Arithmetik*: "Frege's *explanation* of the primitive basis of his system of logic, and particularly of the primitive symbolism, is undertaken in terms of a . . . *semantical interpretation*."¹ That is, most analytic philosophers would have found it natural to accept that Frege's philosophy of logic is in some way grounded in his philosophy of language.

Recently, however, a number of historians of analytic philosophy have criticized the ascription to Frege of the conception of logic suggested by Furth's remark.² More specifically, they argue that Frege did not think that there is, or can be, any "explanation" of his system of logical principles in "semantic" terms. Thus, they hold that Frege did not accord semantics any, or any significant, role in logic; and so, in this sense, Frege did not share our contemporary view of logic. For convenience, and despite the fact that we will come to see these names as mis-

leading, I will call this revisionary reading of Frege's philosophy of logic the "antisemantical interpretation," and the traditional, somewhat less explicitly articulated view the "semantical interpretation."

The present essay is motivated by the fact that, as I see it, the precise content of the disagreement between these two types of interpretation has never been made altogether clear by those engaged in the controversy. While the textual bases of the disagreement are reasonably evident, there has been no explicit account of the lines of reasoning underwriting the interpretations. Nor, more important, has either side fully spelled out what counts as an explanation of logical principles, and what are the semantic terms that might be involved in such an explanation.

I hope in the following to take some steps toward clarifying what is at stake in the controversy, by discussing a representative instance of each style of interpretation. Specifically, I will focus on the reading of Thomas Ricketts, which constitutes what I take to be the most detailed and sharply formulated instance of the antisemantical interpretation, and that of Michael Dummett, because it is, or at any rate is taken by the antisemantical interpreters to be, the most representative version of the semantical interpretation. In the case of Ricketts, I will try to clarify and assess his central interpretive argument, and thereby make clear what I take to be its philosophical underpinnings. In the case of Dummett, considerations of space alone would preclude one from more than a superficial analysis of more than a fraction of his interpretive arguments. So I will content myself with a presentation of some central strands of his interpretation, in order to spell out the ideas of semantics and semantic explanation of logic that he discerns in Frege.

The central result of this investigation is that the substance of the disagreements between the antisemantical interpreters and their perceived opponents is not what they suggest it to be. As we will see, the arguments Ricketts ascribes to Frege fail to support the strong antisemantical doctrine that he takes Frege to hold; moreover, the general conception of logic that the premises of these arguments do actually support is shared by Dummett. But, the failure of these antisemantical arguments does not deprive this interpretation of philosophical significance. For, the materials upon which this interpretation is based suggest a picture of what could count as a philosophical explanation, in semantic terms, of logic, based on intuitively compelling ideas of the natures of judgment and assertion, and their relation to logic. Thus, the interest of the antisemantical reading is not simply exegetical but lies also in its indicating an approach to a philosophical investigation of the senses and grounds of our contemporary assumption of the importance of semantics in logic. This is my excuse for saying very little in the ensuing about the bearing of this interpretive controversy on Frege's actual views.

I. Dummett's Semantical Interpretation

I begin with an initial impression of Dummett's interpretation. According to Dummett, Frege's claim to being the founder of modern logic rests on the following grounds:

Logic began with Aristotle's discovery that the validity of an argument could be characterised by its being an instance of a valid argument-

schema, where an argument-schema is like an argument save for containing schematic letters at certain places instead of actual words or expressions, and is valid if every instance with true premises has a true conclusion, an instance being obtained by replacing each schematic letter with an actual expression of the appropriate logical category. This pre-semantic notion of an interpretation of a schema by *replacement* was the only one that logic had to operate with until Frege. Frege supplied us for the first time with a semantics, that is to say, an analysis of the way in which a sentence is determined as true or otherwise in accordance with its composition out of its constituent words. (Dummett, 1975, p. 118)

Once we have such a semantics, we can substitute for our notion of an interpretation by replacement that of a semantic interpretation, under which make a direct assignment to the schematic letters of the semantic values of expressions of the appropriate categories, bypassing the expressions themselves. (Dummett, 1975, p. 121)

Given this notion of a semantic interpretation of argument schemata, “there was for the first time offered an account of the determination of the sentences of a considerable fragment of language as true or as false, and therefore, also for the first time, the possibility, not merely of specifying certain rules of inference as valid, but of demonstrating their validity in the sense of yielding true conclusions from true premises” (Dummett, 1981a, p. 82).

It seems plausible that Dummett conceives of such demonstrations as soundness arguments: proofs that any interpretation under which all the premise-schemata of a form of inference are true is also an interpretation under which the conclusion-schema is true. Up to this point there is nothing in the conception of logic Dummett ascribes to Frege that a contemporary logician would find controversial.³

It is clear that, according to Dummett, the importance of Frege’s semantic theory for logic lies in its making soundness proofs possible. But what is the importance of soundness? What exactly do soundness proofs establish? Dummett does not directly address these questions with respect to Frege, but, in another context he writes,

The semantic notion [of logical consequence] always has a certain priority: the definition of the syntactic [consequence] relation is required to be responsible to the semantic relation, rather than the other way about. The syntactic relation is defined by devising a set of primitive rules of inference, and a corresponding notion of a formal deduction. *If a semantic notion can be defined with respect to which a soundness proof can be given, we then have a reason for regarding the primitive rules of inference as valid.* (Dummett, 1973, p. 290; my emphases)

So we may summarize Dummett’s view of the relationship between Frege’s semantics and his logic as follows:

1. For Frege, logical laws are rules of inference, and these are schemata standing in certain relations to one another. That is, logical laws are metalinguistic generalizations about statements of an object language.
2. The metalinguistic claims are formulated using a truth predicate: to claim that a rule of inference is valid is to claim that if any instances of

its premise-schemata are *true*, then the corresponding instance of its conclusion-schema is also *true*.

3. Frege's semantics is a compositional account of the truth conditions of sentences, given in terms of certain semantic properties of their constituent expressions. Statements of the truth conditions of sentences of a given schematic logical form are also generalizations formulated using a truth predicate.⁴
4. This account forms the basis of a notion of the interpretation of schemata, which in turn is the basis of a semantic notion of logical consequence.
5. Given the semantic notion of consequence, it is possible to prove, and thereby justify, the validity, that is, the soundness of rules of inference.

On the basis of these features, it *seems* reasonable to conclude that Dummett's Frege is committed to according semantics conceptual priority over syntactically specified inference rules: a form of inference can genuinely count as a logical law only if it is justified with respect to a semantics. It is in this sense, then, that semantics provides an explanation of logical laws: it provides an analysis of the basis of deductive validity in terms of a more fundamental notion of truth conditions.⁵

II. Puzzles for the Semantical Interpretation

The antisemantical interpretation is motivated by two interpretive puzzles faced by Dummett's account, which pinpoint differences between Frege's logic and ours. First of all, for Dummett's Frege, as we have just seen, logical laws are schematic statements or forms of inference. But, in neither *Begriffsschrift* (1879) nor *Grundgesetze der Arithmetik* (1893 and 1903) did Frege in fact formulate his fundamental logical laws in terms of schemata. They are *not stated* as generalizations about expressions of an object language; *nor* are they *used* as such generalizations in derivations. Rather, Frege's logical laws are simply generalizations, stated using first- and second-order unrestricted quantifiers, over *everything*, all objects and functions.

The second interpretive puzzle is that there is no evidence that Frege ever even formulated the idea of a completeness proof for his logical system. If Frege thought of his semantic notion of logical consequence as the most fundamental account of deductive validity, it seems strange that he never considered the possibility that the syntactically specified basic laws of his logical system might fail to capture all genuinely valid deductive inferences.⁶

Dummett is not unaware of these facts. He writes:

It would be misleading to say that Frege anticipated model theory. On the contrary, he cannot concern himself with a variety of interpretations of a formula or set of formulas, since he does not officially recognize any such thing as a schematic letter. In *Begriffsschrift*, the italic letters do, for all practical purposes, function like schematic letters, but officially they are bound variables governed by an initial universal quantifier. This way of looking at things impeded Frege from framing any semantic notion of the validity of a formula, or, therefore, of the completeness of a formal system for logic. But his theory of reference anticipates later formal semantics in

the sense that it provides precisely the framework within which the notion of a semantic interpretation can be defined; and, within that framework, his specification of the references of the primitive expressions of his system constitutes, in precisely the sense of classical two-valued semantics, a stipulation of the one intended interpretation of the system. (Dummett, 1981b, p. 151)

Hence Dummett also claims, “Frege would . . . have had within his grasp the concepts necessary to frame the notion of the completeness of a formalization of logic as well as its soundness . . . but he did not do so” (Dummett, 1981a, p. 82). Perhaps Dummett would similarly claim that Frege could have chosen to adopt the schematic conception of logical laws, but he just did not.⁷

These responses, in the end, amount to an assumption that the differences between Frege’s and the contemporary conceptions of logical laws are philosophically unmotivated. It is this assumption that Ricketts’s antisemantical interpretation questions.⁸ This interpretation comprises two principal theses:

1. Frege’s regress argument against the definability of truth in fact shows that truth is not a property of thoughts, and, therefore, no purported property in terms of which truth can be analyzed is in fact a property.
2. As a consequence, no genuine claim can be made by the use of a truth predicate, or any predicate purporting to ascribe properties in terms of which truth can be analyzed; hence Frege was in principle debarred from adopting a schematic conception of logic.

Thus, a central strand of Frege’s thought actually precludes him from accepting any “genuine” semantic theorizing at all, much less one through which he might provide a “justification” for logic.⁹

III. Ricketts’s Argument, First and Third Parts

Ricketts’s interpretive argument has three main parts:

1. A reconstruction of Frege’s argument against psychologism, from which Ricketts extracts a conception of judgment and of assertion that he takes to be fundamental to Frege.
2. A reconstruction of Frege’s argument against the definability of truth, using the conception of judgment and assertion established in the first part. Ricketts takes this argument to demonstrate the conclusion that truth is not a property.
3. From the claim that truth is not a property, Ricketts infers the conclusion that Frege could have no genuine semantic theory.

The third part of this argument appears the least problematic.¹⁰ If truth is not a property, then there is no such thing as ascribing the property of truth to thoughts. It follows that apparent assertions or statements that *appear* to be making such ascriptions cannot really be doing so; they cannot be genuine assertions or statements unless they can be successfully reinterpreted.¹¹ Simple truth predication of the form ‘*p* is true’ may be reinterpreted as assertions made using *p*; that is to say, on this interpretation, Frege is committed by the regress argument to the same

treatment of the truth predicate as that which results from the redundancy theory of truth.¹² Now, as noted above, statements of the truth conditions of sentences of a given schematic logical form are complex generalizations formulated using a truth predicate, which quantify over both constituents of schemata and over infinite domains. It is unclear that the reinterpretation strategy for simple truth predications works for these generalizations.¹³ In the absence of a clearly viable reinterpretation, the statements comprising most semantic theories, certainly semantic theories that Dummett appears to attribute to Frege, cannot be taken to express anything.

The second part of the argument is the most important for assessing the differences between the semantical and the antisemantical readings, and will be analyzed at length here, in sections IV–VI.

In this section I will give a brief summary of the first part of Ricketts's interpretive argument, primarily in order to set out the necessary background for our discussion of the second part.

One final note before beginning. While the conclusion of Ricketts's argument yields a philosophically motivated explanation of Frege's universalist conception of logic, it faces interpretive difficulties of its own. As a number of commentators have pointed out, Frege certainly appears to present arguments whose premises use a truth predicate in ineliminable ways.¹⁴ Ricketts has argued for ways of understanding these Fregean texts consistent with his overall interpretation. But I mention this only to note that I will not, in the ensuing, discuss *these* interpretive issues. They are instructive both in themselves and for the light they shed on Fregean logicism. But it seems to me that their conceptual place lies after the issue of the cogency of Ricketts's interpretation of Frege on truth.

On Ricketts's view, the most fundamental resting point of Frege's philosophy consists in his conception of judgment, a conception revealed in his opposition to psychologistic accounts of logic. What is essential to such views is the claim that the laws of logic are, and can only be, empirical generalizations about human psychology, that is, generalizations that describe how human beings actually reason. This view of the laws of logic has the following consequence. Since it is clearly a coherent possibility that there exist beings whose inferential processes are different from ours, it is conceivable that there exist laws of logic different from ours, describing how those beings think. Ricketts takes Frege's criticism of psychologism to develop from consideration of such a possibility, a case involving beings who do not accept "der Grundsatz der Identität," that is, "Jeder Gegenstand ist mit sich selbst identisch" (Frege, 1964, p. 14). I follow Ricketts in calling such beings "logical aliens."

Two features of the possibility of such beings are important for the argument. First, it certainly seems as if we and the aliens *disagree* over the law of identity: we affirm it, they deny it, thereby contradicting us. Second, on a psychological construal of logic, the notion of truth or of correctness, as applying to the affirmation and denial of logical laws, must be relativized to the population that these laws describe. Hence it makes no sense to describe the aliens' rejection of the law of identity as either correct or incorrect *simpliciter*; their rejection can only be correct for them, or incorrect for us. So the question, as Frege puts it, of who is right about the self-identity of all objects can have no content.

But, Frege argues, once correctness is relativized, the notions of agreement and disagreement could no longer be properly applied to the apparent disagreement

between us and the aliens. I will develop this argument somewhat differently from Ricketts.

Consider the following question: Can the apparent disagreement between us and the aliens be distinguished from differential responses to stimuli exhibited by two groups of animals that have been trained differently? Suppose a group of parrots has been trained to make the sounds “the moon is made of green cheese” when given a biscuit, while a second group has been trained to make the sounds “it is not made of green cheese” in response to the sounds “the moon is made of green cheese.” If two parrots, one from each group, are placed together, and a biscuit is given to the one from the first group, we would not take the resulting noises to constitute a disagreement with one another. Intuitively, we do not take these birds to be contradicting one another. It makes little apparent sense to think that either *ought* or *ought not* to make the noises that it does, and so it makes no sense to think that either might under certain circumstances have to “withdraw” the noise it makes.

If the psychologistic view of logic is right, it is unclear how the apparent disagreement between us and the logical aliens is, at bottom, any different. We think the way we do. The aliens think the way they do. There is no conflict or contradiction between the conclusions we reach and the conclusions they reach. Moreover, the claim that one *ought* to reason the way the aliens do, or the way we do, makes no sense, since there is no question of who is right, *simpliciter*. Why isn’t this situation a mere difference in reactions to stimuli?

Moreover, it is not clear that, if psychologism is right, we can agree with one another. Consider the example of the parrots again. Suppose the first group of parrots has been trained to make the sounds “it is indeed” in response to the sounds “the moon is made of green cheese.” If now two parrots, both from the first group, are placed together, and a biscuit is given to one from the first group, we would still surely not take the resulting noises to show them to be agreeing with each other. Rather, again, all that is happening is that the parrots are determined to say what they say by their training. What differentiates our apparent agreement among ourselves from these parrots, if psychologistic logic is right? Each of us is simply determined to have sequences of mental states in certain orders, in virtue of our belonging to a population all of whose members have a certain common psychological constitution, and similarly for the aliens.

So Ricketts’s Frege concludes that psychologism implies that what we think of as agreement and disagreement are ultimately not coherently distinguishable from physiological reactions to stimuli. That is to say, there are, in the end, no such things as agreement or disagreement in our assertions and judgments, as we intuitively conceive of these actions.

This consequence does not, of course, result in a conclusive refutation of psychologism, since Frege has not shown that there are indeed such things as agreement and disagreement, as we conceive of them.¹⁵ Ricketts concludes that the argument serves principally to make explicit Frege’s commitment to a particular conception of assertion and judgment, a conception that I will formulate thus:

Judgment and assertion are actions that have certain constitutive norms. That is, no one can count as making a judgment, or an assertion, unless she is appropriately sensitive to the evaluation of her action as correct or

incorrect, on the basis of impersonal standards; she cannot be performing these actions and yet disregard completely the correctness of what she says or thinks. The sensitivity or regard that she must have may be spelled out as follows. She must (at least) represent herself as acting in order to achieve the purpose of having her action assessed as correct by the standards in question, and so as holding herself responsible to these norms, or, alternatively, as having incurred commitments determined by these standards. Specifically, she must take herself to be committed to defending her judgments or assertions and to “withdrawing” them should they be evaluated as incorrect, with respect to the norms that she acknowledges.¹⁶

Call this the normative conception of judgment and assertion.

IV. Ricketts's Argument, Second Part

The main issues about the second part of Ricketts's argument that we will focus on are:

1. What exactly *is* the regress argument, according to Ricketts?
2. Does it indeed establish that truth is not a property of thoughts, on the basis of the conception of judgment just outlined?
3. If not, what premises would be needed to do so?
4. Finally, and most important, what relation, if any, is there between a normative conception of judgment and the possibility of foundational or justificatory theorizing about logic?

Frege's regress argument against the definability of truth is given in a number of places, of which the following passage from “Der Gedanke” may be taken as representative:

Kann man nicht festsetzen, daß Wahrheit bestehe, wenn die Übereinstimmung in einer gewissen Hinsicht stattfinde? Aber in welcher? Was müßten wir dann aber tun, um zu entscheiden, ob etwas wahr wäre? Wir müßten untersuchen, ob es wahr wäre, daß—etwa eine Vorstellung und ein Wirkliches—in der festgesetzten Hinsicht übereinstimmen. Und damit ständen wir wieder vor einer Frage derselben Art, und das Spiel könnte von neuem beginnen. So scheitert dieser Versuch, die Wahrheit als eine Übereinstimmung zu erklären. So scheitert aber auch jeder andere Versuch, das Wahrsein zu definieren. Denn in einer Definition gäbe man gewisse Merkmale an. Und bei der Anwendung auf einen besonderen Fall käme es dann immer darauf an, ob es wahr wäre, daß diese Merkmale zuträfen. So drehte man sich im Kreise. Hiernach ist es wahrscheinlich, daß der Inhalt des Wortes ‘wahr’ ganz einzigartig und undefinierbar ist. (Frege, 1918, p. 344)¹⁷

In the first version of his interpretive argument to appear, Ricketts begins by claiming that Frege's argument, as presented in “Der Gedanke,” is “cogent only in the context of [his] conception of judgement” (Ricketts, 1986b, p. 77). The argument itself goes as follows:

The connection between judgement and truth is not casual. . . . We cannot take someone to be making assertions in complete disregard of the correctness of what he asserts; such a person would be understood to be play-acting or perhaps merely mouthing words. To take truth to be definable forces a particular construal on this talk of standards of correctness. For to have a definition of truth is to have a general description of the conditions that have to be satisfied for a judgement to be correct. So, if truth is definable, then any person who makes a judgement must have ascertained, or taken himself to have ascertained, whether these conditions, applied to the thought under consideration, are satisfied. . . . But to ascertain that some condition holds is to make a judgement. So the regress begins. Given the definition of truth, a person cannot have judged that Socrates is mortal corresponds with Reality unless he has judged that the content of this second judgement corresponds with Reality. On the correspondence theory then, a person is never in a position to make a judgement; for no one is ever in a position to have satisfied, or even think of himself as having satisfied, the standards for judgement that would *ipso facto* be provided by its definition of truth. (Ricketts, 1986b, p. 78, emphases mine)

The argument requires one further claim, which constitutes another aspect of Frege's conception of judgment: "A judgement is not . . . a special recognitional capacity, a species of some genus. Judgement is itself the genus; to recognize anything to have a property is *ipso facto* to make a judgement" (Ricketts, 1986b, p. 77). And Ricketts claims, further, that the regress argument demonstrates how taking truth to be definable is taking judgment to be a species of recognitional capacity, and hence confusing the genus with the species. I will therefore take the term Ricketts uses in the quoted passage, "ascertain," to be interchangeable with the term "recognize." I should note that it is not made clear whether this aspect of judgment follows from the normative conception of judgment.

I will formulate the argument, not in terms of the definability of truth, but directly in terms of the claim that truth is a property. It contains three principal premises:

1. It is not possible for someone to make an assertion if she disregards the correctness of what she says. (From the normative conception of judgment.)
2. If truth is a property of thoughts, then the condition for a judgment to be correct is that the thought judged has that property.
3. To ascertain that a condition holds is to judge that it holds. (From Frege's conception of judgment as the genus of recognition.)

The argument itself proceeds by *reductio*. First assume as premise for the *reductio*:

4. Truth is a property of thoughts.

Then, by, (2) and (4), it follows that

5. The condition for a judgment to be correct is that the thought judged has the property of being true.

Now, from (1) Ricketts infers:

6. If someone makes a judgment, then she must have ascertained that the judgment is correct.

Thus, from (5) and (6) we may infer that

7. If someone makes a judgment, then she must have ascertained that it has the property of being true, that is, that it is true.

From (3) and (7) it follows that

8. If someone judges that p , then she must have judged that p has the property of being true. It is (8) that generates the regress.

From (8) it follows that

9. If someone judges that p , then she must have made an infinite number of judgments, namely, those expressed by p , ' p , is true', ' $'p$ is true' is true', ...

This regress claim leads to the *reductio* conclusion if we assume further that

10. Human beings make judgments; but no human being makes an infinite number of judgments.

Assumption (10) contradicts (9). Hence we may infer that (1) through (3) and (10) imply the conclusion that truth is not a property of thoughts (i.e., the negation of (4)).

V. First Objection to Ricketts

One objection to the regress argument concerns step (6), which seems, on the face of it, implausible, since it appears to rule out the possibility of unsupported assertions and unwarranted judgments.

From an interpretive standpoint, one might reply that the objection is based on evidence about how the words “assertion” and “judgment” are actually used, something in which Frege had little interest. Frege should simply be taken to have had an especially stringent conception of judgment, one that deviates from the ordinary one.¹⁸

But in the overall context of Ricketts’s interpretive argument, it is not clear that this reply is adequate. For, according to Ricketts’s regress argument, (6) is supposed to follow from (1). Hence, the question is whether the implausible pictures of assertion and judgment are really required by Frege’s normative conception of judgment and assertion. The answer seems fairly clearly to be no. This conception holds that judgment and assertion are constitutively subject to assessment as correct or incorrect. It is a further step, not warranted by this conception, to conclude that the assessment has to have been conducted, before one could perform these actions. Thus, it is not clear that Ricketts’s argument as it stands succeeds in establishing that truth is not a property.

But it seems possible to reformulate the argument to meet this objection. According to the normative conception, in order to make an assertion or judgment, one has to hold oneself responsible to certain norms or, alternatively, to have incurred certain commitments. Specifically, one must take oneself to be committed to withdrawing one’s assertion, should it be assessed as incorrect with respect to the norms that one acknowledges. Now the regress argument may be reformulated in the following way. Suppose that the assessment of the correctness of an

assertion or a judgment required determining whether the thought that constitutes its content has the property of being true. And suppose further that determining whether this holds required making a further judgment. Then, we could not determine whether the original assertion or judgment is correct until we have determined whether this second judgment is correct. And *this* gets the regress going.¹⁹ As before, assume that human beings do not make an infinite number of judgments, which now implies that no one can be in a position to assess the correctness of any assertion or judgment that one makes. So the regress implies that the norms governing the practices of assertion and judgment are unattainable, and this conflicts in two ways with the normative conception. First, if the requirements of these practices are in principle impossible to fulfill, it is at least a question how they could have normative force. Second, if no assertion or judgment can ever be warranted, it is unclear why the justification and criticism of assertions and judgments should play so central a role in inquiry as we conduct it.

VI. Second Objection to Ricketts

A second, more substantial, objection derives from one of Dummett's objections to Frege's regress argument: "It is true enough that, in determining that some statement A is true, I thereby also determine the truth of infinitely many other statements . . . But there is no harm in this, as long as we recognise that the truth of every statement in this series is determined simultaneously: the regress would be vicious only if . . . in order to determine the truth of any member of the series, I had first to determine that of the next term in the series" (Dummett, 1981a, p. 443). It is clear how this objection applies to Ricketts's regress argument. If each judgment in the series expressed by p , " p is true", "' p is true' is true", . . . , can be made simultaneously, then (10) is false. So, even if we accept the validity of the regress argument, we need not accept that it is sound, and so we have been given no reason to believe that truth is not a property of thoughts.

The cogency of this objection clearly depends on whether, and if so how, in making a judgment a subject can thereby simultaneously make the infinitely many judgments in the regress. Dummett, unfortunately, does not give any account of this. But the following, I hold, is a plausible account. What I have in mind is modeled on the sorts of examples frequently encountered in the theory of action: raising an arm and hailing a taxi are distinct actions, and yet they are done simultaneously. In the case of judgments, it would work as follows. Let us consider the judgment that it is raining, which we abbreviate as p . According to the regress argument, one has to ascertain that p is true before one can judge that p . But now, how do we usually, in fact, do this? That is, how do we usually recognize that it is true that it is raining? Well, one common way is to look out the window, if you are indoors. So we ascertain that p is true by observation. But, surely, by this same observation, we also recognize simply that it is raining, that is, simply that p . So by this very same observation we simultaneously ascertain the truth of all the thoughts expressed by the judgments in the regress.

But supposing that we grant the cogency of this objection, what is its significance? In order to bring this out, note one further point about our account of how one may make infinitely many judgments simultaneously. Consider the examples of actions on which the account is modeled: raising an arm and hailing a taxi count as dis-

tinct actions, and yet they are done simultaneously. It is plausible that, while these actions are distinct, there is a certain dependence among them. Hailing a taxi depends, in some way, on raising one's arm. In raising one's arm, given the circumstances, one is thereby also performing the distinct action of hailing a taxi; but in those circumstances, hailing a taxi in that way is not possible without raising one's arm. So the act of arm raising is, in some sense, more fundamental than the act of taxi hailing. One way of describing these dependencies is by using the notion of supervenience, a relatively weak reduction relation. A corresponding description of judgment is that the making of each of the infinitely many judgments in the regress is supervenient on the making of the first, the judgment that p . In other words, all the acts of judging that ' p is true', ' $'p'$ is true', ... are, in this weak way, reducible to the act of judging that p . In judging that p , one is thereby also performing all the other distinct judgments in this series; but making these other judgments is not possible without judging that p .²⁰

Let me put the point in a slightly different way. The regress can be blocked if it is assumed that, of the two judgments:

- a. p
- b. p is true

the first is conceptually prior in the following sense. What puts one in a position to make a judgment whose content is a thought expressed by p is not the recognition that this thought falls under the concept expressed by a truth predicate. Rather, what the regress argument shows is that the conceptual dependence is the reverse: What puts one in the position to recognize that a thought has the property of truth is that which puts one in a position to make a judgment whose content is that thought. The regress is *not* generated by the assumption that truth is a property of thoughts, but rather by the assumption that the recognition of the possession of the property of truth by a thought is conceptually prior to the recognition of the correctness of the corresponding judgment.

It should be noted that the relations among the judgments in the regress are not exactly analogous to the case of action described above. For instance, in the latter case there is a counterfactual dependence between the two types of acts: I might have raised my arm without thereby hailing a taxi. In contrast, I could not judge that p without also thereby judging that ' p is true', and so on. But the preceding was not intended to establish that the upshot of the regress argument is a special case of the notion of supervenience. The role of the case of action was to provide a model of how one can perform several actions simultaneously, in which, in addition, one of these actions is in some sense more fundamental than others. The sense at issue was what I have just tried to spell out. With these caveats in mind, I will call the objection just described the "quasi-supervenience" objection.

The quasi-supervenience objection shows that Ricketts overstates his case. The regress generated by the premises of his interpretive argument does not in fact have any implications for whether truth is a property of thoughts. Central to those premises is the normative conception of judgment and assertion; it thus follows that that conception does not support the thesis that truth is not a property, or that truth predicates have no *Bedeutungen*. Hence it does not imply that uses of truth predicates must be reinterpreted; nor does it show that semantic theory, in the sense of a theory of truth conditions, has to be reinterpreted as anything other

than generalizations about thoughts with respect to the property of truth. *A fortiori*, Ricketts's regress argument fails to show that semantics lacks content.

The quasi-supervenience objection also points to what Ricketts's premises do in fact support: a conceptual connection between recognition of truth of thoughts and recognition of correctness of judgments. I take this to be the right conclusion to draw from the regress argument. I will call this account of what is shown by the regress argument the "quasi-supervenience" interpretation of that argument, and I will, from now on, sometimes just call it "the regress argument," as opposed to "Ricketts's regress argument."

One point not entirely clear in the version of Ricketts's regress argument just discussed is the precise meaning of step (8) (which depends on (6)): In order to judge that *p*, one must have judged that *p* has the property of being true. The quasi-supervenience objection clearly must insist that the two judgments mentioned here are made simultaneously. In a more recent presentation of his regress argument, Ricketts appears to reformulate his argument to make explicit that (8) is not to be understood in this way: "the regress argument assumes that if truth is characterized as correspondence, then the judgment that *p* requires the *prior* judgment that *p* corresponds to Reality, i.e., that it is true that *p*" (Ricketts, 1996, p. 130, my emphasis). Let us call this assumption the priority thesis. If the priority thesis holds, then clearly the quasi-supervenience objection cannot get off the ground.

But now the question is, what justifies the priority thesis? In the essay in which he formulates that thesis, Ricketts acknowledges that "to many contemporary ears, [it] will sound gratuitous" (1996, p. 130), and then appears to claim that it follows from yet another characterization of Frege's conception of judgment: "For Frege, the conceptions of judgment and truth are intertwined, as enunciated in his dictum that judgment is the recognition [*anerkennen*] of the truth of a thought. . . . Truth is thus the goal of judging and judging is the recognition of truth. We have no grasp on the one independent from grasp of the other" (pp. 130–31).

Ricketts does not explicitly signal where he spells out the argument from this conception of judgment to the priority thesis (nor, indeed, it should be noted, does he state whether he gives such an argument at all). So I indulge here a mere conjecture. It is not altogether implausible that the following passage contains his argument:

In the context of this view of judgment, Frege considers the proposal to identify truth with some particular property of mental states. This proposal, in giving us a characterization of truth independent from our purchase on judging, foists a particular construal on Frege's elucidatory dictum: to recognize a thought to be true must be to recognize a mental representation to have a particular property. The genus judgment is thus assimilated to one of its species, the discrimination whether things of some restricted range have a particular property. (p. 131)

I reconstruct the argument as follows:

1. Judgment is the recognition of the truth of a thought.
2. Truth is a property of thoughts, that is, of contents of judgments.
3. From (1) and (2) it follows that judgment is the recognition that a thought has the property of being true.
4. (3) implies that a prior recognition that a thought has the property of being true is what puts one in a position to make the corresponding judgment.

5. To recognize that a state of affairs obtains is to make a judgment.
6. To make a judgment that p requires making a prior judgment that p has the property of truth.

If, indeed, this is the argument, then the question is, what licenses the transition from (3) to (4)? It seems that Ricketts reads (3) as an analysis of what (correct) judging consists in, in terms of a prior and independent notion of recognition of a property of thoughts. The source of this reading, I conjecture, is Ricketts's account of Frege's "elucidatory dictum" (premise (1)) as claiming that neither truth nor judgment can be understood independently of one another. Given this account, (2) appears as an attempt to explain truth in terms prior to and independent of judgment.

But this reading of (2) is not compulsory. One can take (2) as a claim whose precise content has to be explicated in terms of the fundamental conceptual connection embodied in Frege's dictum. In which case, we arrive at an analysis of what it is to recognize that a thought has the property of truth in terms of the making of a judgment expressing that thought. But this analysis does not imply (4); indeed, it is nothing other than the conclusion of the quasi-supervenience interpretation of the regress argument.

Another consequence of the quasi-supervenience objection is that it is not possible to judge any member of the infinite series expressed by p , " p is true", " p is true' is true", ... independently of any other, and, more important, that it is not possible to judge any member to have an opposite truth value from any other. But this, together with Frege's test of sameness of sense, implies that each of the infinite series of judgments have the same sense.²¹ This implication has been singled out by Ricketts, in recent work,²² as the basis of what is apparently a different argument for the conclusion that truth is not a property. Ricketts claims that, in the context of Frege's philosophy, a property can only be the *Bedeutung* of a one-place first-level concept expression. He then extends his former argument as follows: the only way to escape the regress is to take p and " p is true" to have the same sense. This, however, is possible only if the concept expression ' ξ is true' has no *Bedeutung*, that is, does not in fact mean a concept. So it is in this sense that truth is not a property for Frege.

It is, however, unclear that this possibly distinct argument is consistent with other views of Frege. For one thing, Frege claims that p and " p " have the same sense, and yet Frege clearly does not think that the horizontal is *bedeutunglos*.²³ More generally, as Cora Diamond has shown, it is consistent with Fregean principles to hold that there exist function expressions whose senses "collapse" when applied to certain complete expressions, and which are nevertheless well-defined and so have *Bedeutungen*. Her example is a function expression "W()" defined by specifying its value to be "Wismar whenever the argument is Wismar and . . . otherwise Frankfurt" (Diamond, 1991, p. 123).

Now, one might think that Ricketts does not need to extend his argument to the conclusion that truth predicates do not have *Bedeutungen*. Instead, all he needs is the conclusion that p and " p is true" have the same sense, in order to establish that semantical claims have no genuine content. But this does not follow because that conclusion does not support the demand for a reformulation of all claims stated using a truth predicate.

I end this section with some remarks about the quasi-supervenience view as an interpretation of Frege.

First of all, is the quasi-supervenience view consistent with the conclusion that Frege actually draws from his regress argument? In the passage in “Der Gedanke” where Frege presents the argument, he concludes, “der Inhalt des Wortes ‘wahr’ ganz einzigartig und undefinierbar ist” (Frege, 1918, p. 344).²⁴ On a natural interpretation, this states that truth is an indefinable (and primitive) property of thoughts, which, I claim, conflicts with the view that our recognition of the instantiation of this property is conceptually dependent on our capacity to make correct judgments.²⁵ For, as I will argue in the next section, the quasi-supervenience objection supports a conception of truth as an analyzable, nonprimitive property. I leave it as an open question whether there are reasonable alternatives to the natural interpretation.

Second, the claim that truth is quasi-supervenient on assertion or judgment accords reasonably well with some of Frege’s very late remarks about truth and assertoric force:

[D]ie Behauptung nicht in dem Worte ‘wahr’ liegt, sondern in der behauptenden Kraft, mit der Satz ausgesprochen wird. (Frege, 1915, p. 271)²⁶

So scheint das Wort ‘wahr’ das Unmögliche möglich zu machen, nämlich das, was der behauptenden Kraft entspricht, als Beitrag zum Gedanken erscheinen zu lassen. (p. 272)²⁷

In these passages Frege seems to reject the claim that the truth predicate, or the property of truth, can explain what qualifies a speech act as an assertion. Since he also nowhere explicitly denies truth the status of a property, this suggests that he is here insisting on the conceptual priority of assertoric force over the property of truth. Now, we can read “assertoric force” as whatever it is that constitutes a practice as one of assertion. Further, if we ascribe the normative conception of assertion to Frege, we can surmise that assertoric force includes being subject to standards of correctness. These interpretive conjectures lead to the view that the property of truth is analyzable in terms of the norms of correct assertion (and thus also judgment). This is not the conclusion of the regress argument; but, in the next section, I will present an argument connecting the two.

VII. Quasi-Supervenience and Semantic Justification of Logic

In this section I consider the implications of the preceding discussion for our principal question: Did Frege provide an explanatory justification of logical laws in semantic terms? We have seen that Ricketts’s attempt to return a negative answer fails. However, this fact by itself does not imply anything about what a Fregean semantics could be, or whether it could explain or justify the laws of logic.

It is possible to go from the conclusion of the quasi-supervenience interpretation of the regress argument to a view of the justificatory or explanatory status of Fregean semantics. In order to do so, we must first see what might be said about truth, if it is a property, on the basis of this argument. I claim that it supports the view that *if* truth is a property of thoughts, then it is *not* a primitive and unanalyzable

property of thoughts, but is to be analyzed in terms of the correctness of judgments expressing those thoughts. I say “support” because the argument to be given cannot rule out the viability of the view that truth is an unanalyzable property. The argument is this. To say that truth is an unanalyzable property of thoughts is to say that if a thought is true, it is true *not* in virtue of any other facts about it. In particular, it is not true in virtue of its being expressed by a judgment that is correct. But now if, nevertheless, according to the regress argument, we recognize the truth of a thought only in virtue of recognizing the correctness of the corresponding judgment, a question immediately arises about why this should be the case. Since the correctness of the corresponding judgment has no constitutive relation to the truth of the thought, why should our epistemic route, as it were, necessarily run through the former to the latter? In contrast, if a thought is true only in virtue of its corresponding judgment’s being correct, then this by itself is an explanation of that epistemic route. No further explanatory problem arises. The conclusion is that correctness of judging is the more fundamental (perhaps the conceptually prior) property in terms of which truth of thoughts is analyzed.

But now, if we accept this conclusion, how are we to understand the very claim that a rule of inference is valid with respect to a semantic definition of logical consequence? As stated in section I, to claim that a rule of inference is valid is to claim that if any instance of its premises-schemata is *true*, then the corresponding instance of its conclusion-schema is also *true*. Since according to the conclusion just canvassed truth is dependent on correct judgment, the claim that a rule of inference is valid in this sense must be based on the claim that, if each judgment in the set of judgments whose contents are expressed by the instances of the premises is correctly made, then making the judgment whose content is expressed by the corresponding instance of the conclusion is (necessarily) correct. It is not clear, however, that this is any more than a restatement of the claim that the rule of inference is (syntactically) valid. So it appears that, after all, Ricketts’s overall conclusion is vindicated: even if truth is a property, it is not a property conceptually prior to deductive validity, and therefore a theory of the truth conditions of thoughts, while not contentless, cannot be used as part of an analysis of logical validity in terms of more fundamental notions.

VIII. Semantic Antifoundationalism about Logic

In this section I pause to take stock of the preceding discussion. We have considered three readings of Frege’s conception of the relation between semantics and logic:

1. Our initial impression of Dummett’s interpretation is that for him Frege’s semantics is a theory of truth conditions, which comes from the theory of meaning, or is a part of the theory of meaning. Semantics on this view provides an explanation of logical laws, in the sense of furnishing the conceptually fundamental standard for determining whether a form of argument is genuinely valid.
2. Ricketts’s Frege holds that truth is not a property (not the *Bedeutung* of a concept-word), so semantics has no content. Since semantics has no content, it cannot figure in an explanation of logical laws.

3. The quasi-supervenience interpretation holds that if truth is a property of thoughts, it is conceptually dependent on (correct) judgment. So semantics as a theory of the truth conditions of thoughts has content. But it does not provide an independent and prior standard for determining deductive validity.

Our findings about these interpretations may be summarized by the following points:

1. A tacit assumption made by all three interpretations is that semantics is a theory or science consisting of statements of conditions under which thoughts have the property of truth.
2. Another tacit assumption, made at least by the first and third interpretations, is that a semantic explanation of logic is an analysis of logical validity in terms of prior and independent semantic concepts.
3. All three interpretations take Frege to have a single notion of truth: as a property of thoughts and as that which a truth predicate expresses. The disagreement is over whether this notion is the explanatory basis of deductive validity.
4. The second and third interpretations are united by their rejection of the possibility of a semantic explanation of logical laws.
5. Thus the principal division among these interpretations is really between what we have been taking to be Dummett's interpretation, which accepts the possibility of a semantic foundation of logic, and the two mentioned in the last point, which do not. So it is perhaps less misleading to label both Ricketts's and the quasi-supervenience interpretations as instances of an "antifoundationalist" interpretation of Frege:
 - a. Ricketts's view is an antifoundationalism about logic based on the elimination of semantics.
 - b. The quasi-supervenience interpretation is antifoundationalism without the elimination of semantics.

In these points we have, finally, an account of what is at stake in the interpretive controversy that I initially characterized as a disagreement over Furth's remark. Unfortunately, this account is not fully adequate. Perhaps some philosophers who would endorse Furth's remark do also hold some version of a "foundational" interpretation. But Dummett does not. In the next section I will show that Dummett conceives of semantics, and of its explanatory justification of logic, in rather different terms.

IX. Dummett's Conceptions of Semantic and Logic

Let us begin with a recent text in which Dummett explicitly addresses Frege's conception of how truth and logic are related:

[Frege] did make a connection between [truth and assertion], and an extremely strong connection at that. The subject-matter of logic is not truth, at least if truth is what is expressed by the predicate "is true"; for that is merely an identity operation, and an operation at the level of sense rather than of reference, which maps any thought on to that very thought. Rather, the subject-matter of logic is, not indeed assertoric force itself, but that to which assertoric force constitutes a claim. This we may, very

naturally, term “truth”; but *truth in this sense is not given to us in the first instance as the reference of any singular term or concept-word, still less as the sense of either, but precisely through the peculiarity of assertoric force*, which neither modifies nor contributes to the thought expressed, but is *sui generis*, differentiating the expression of a judgement from the mere expression of a thought.

It is, indeed, a peculiarity of Frege’s theory that truth—presumably truth in the sense of the term which we are discussing—is taken to be the reference or *Bedeutung* of all true sentences; but we could not attain a grasp of what truth was by this explanation. One cannot know what truth is unless one knows what judgement is. . . .

It is, then, truth in this sense of “truth”—the sense in which assertoric force embodies a claim to truth, and in which judgement aims at truth—that truth is the subject-matter of logic, or that logic comprises the laws of truth. (Dummett, 1994, pp. 283–84)

The principal theses that Dummett here attributes to Frege are as follows:

1. There are two notions of truth:
 - a. One is that at which judgment and assertion aim.
 - b. The other is what is expressed by a truth predicate, or is the *Bedeutung* of such a concept expression or of a singular term (presumably such as “the True”).
2. The subject matter of logic is not what is expressed by a truth predicate but is rather the aim of judgment and assertion.
3. The notion of truth given as the *Bedeutung* of a truth predicate is not the primary notion. It is, on the contrary, explained in terms of the fundamental notion of truth as the aim of judgment.

These points certainly suggest that Dummett would resist attribution to Frege of an account of logical properties based on what is expressed by a truth predicate. And this conclusion further suggests that Dummett’s interpretation of Frege is not identical to what we have been calling the semantical interpretation. This makes it much harder to see what, if anything, is the substance of the disagreement between Dummett’s interpretation and the antifoundationalist interpretations.

It must be noted that the text just quoted represents a recent aspect of Dummett’s view of Frege; so far as I know, he has not previously ascribed to Frege the two conceptions of truth, and their relations to logic. Thus, one possible account of the apparent tension between this text and those cited in section I is that Dummett has changed or expanded his views of Frege without being aware of or calling attention to inconsistencies with his former view.

I have no definitive grounds for rejecting this account, but it seems to me that a philosophically more fruitful approach is to consider how this recent interpretation might be continuous with Dummett’s earlier reading. To that end, I will spell out a bit more Dummett’s conceptions of semantics and of the semantic explanation or justification of logical laws.

For Dummett’s conception of semantics, consider the following passage:

The question, “What confers on a sentence the sense which it has?,” may be answered, “The speaker’s understanding it as having that sense.” The next question, “In what does their so understanding it consist?,” may be

answered, after the manner of Frege's *Grundgesetze* account, . . . "In their taking the appropriate condition as being required to obtain for the sentence to be true." Frege's account of sense thus rests on the notion of truth. . . . But now a further question arises: "What determines under what conditions a sentence is true?" What Frege fails to stress as heavily as that sense is bound up with truth is that *the concept of truth is in turn bound up with that of assertion*. He does not wholly neglect this connection: in a remarkable unpublished fragment of 1915, he speaks of the word "true" as attempting, in vain, "to make the impossible possible, namely to make what corresponds to the assertoric force appear as a contribution to the thought" . . . It would be impossible to discover from specifications of the conditions under which the sentences of a language had the value *true* or the value *false*, but in which the two truth-values figured only with the labels "A" and "B," which of "A" and "B" represented the value *true* and which the value *false*. To ascertain this, it would be necessary to know, for some sample sentence, with what significance speakers of the language invested *assertoric utterances of it*. . . . *A form of utterance employed to transmit information can only be taken as carrying assertoric force: and a statement is true just in case an assertion of it is correct.* (Dummett, 1993a, pp. 11–12, my emphases)

It thus appears that the notions of truth and of truth condition have to be explained in terms of the linguistic practice of assertion. In particular, as the last sentence of this text makes clear, the truth of a statement is explained in terms of the correctness of an assertion made with it.²⁸ Since the notion of truth condition is the object of study of theories of meaning,²⁹ this shows that theories of meaning turn out to be theories of the conditions of correctness of assertions. Thus, for Dummett semantics is not a science of conditions under which thoughts have the property of being true, but a science of the conditions under which assertions are correct.

For Dummett's conception of the semantic justification of logical laws, consider his response to Nelson Goodman's argument that all justifications of deductive inference are circular:³⁰

The circularity that is alleged against any attempt to justify deduction . . . is not of the usual kind. The validity of a particular form of inference is not a premise for the semantic proof of its soundness; at worst, that form of inference is employed in the course of the proof. Now, clearly, a circularity of this form would be fatal if our task were to convince someone, who hesitates to accept inferences of this form, that it is in order to do so. But to conceive the problem of justification in this way is to misrepresent the position that we are in. *Our problem is not to persuade anyone, not even ourselves, to employ deductive arguments: it is to find a satisfactory explanation of the rôle of such arguments in our use of language.* . . . Characteristically, in an explanation, the conclusion of the argument is given in advance; and *it may well be that our only reason for believing the premises . . . is that they provide the most plausible explanation for the truth of the conclusion.* (Dummett, 1973, pp. 295–96, my emphases)

Here Dummett is clearly willing to contemplate the possibility that our *only* reason for adopting a semantic theory is that it provides an explanation for the sound-

ness of (at least some of) the logical laws we accept. If so, then surely when that semantic theory fails to provide the appropriate explanation, *it*, rather than the logical laws in question, may be rejected. In this sense, it is clear that Dummett in fact does *not* hold that what counts as a logical law is necessarily determined by semantic theory. That is, semantics is not conceived of as conceptually prior to logical laws.

But I think we can go further here. What is “an explanation of the role of deductive arguments in our use of language”? It is a solution to the central problem posed by the existence of deductive inference: “When a statement is established, conclusively but indirectly, by the use of a deductive argument, in just what sense would it be right to say that, in accepting it as so established, we have remained faithful to the meaning we originally gave it? Or: in what sense, if any, can we say that, when it is established indirectly, we had already implicitly established it directly?” (Dummett, 1973, pp. 299–300).

The direct way of establishing a sentence is determined by what Dummett calls its “essential structure.” This latter notion is part of “Frege’s account of what is involved in grasping the thought expressed by a complex sentence” (Dummett, 1981b, p. 49). As we have seen above, to grasp the thought expressed by a sentence is to know the conditions under which it is true or false. In the case of a complex sentence, Frege holds, according to Dummett, that “it is only by reference to [essential structure] that we can explain how the sentence is determined as true or false in accordance with its composition” (p. 49).

Now, on Dummett’s interpretation, Frege’s account of the grasping of thoughts expressed by sentences is his theory of sense, which is one of the central components of his theory of meaning. Since, as we have just seen, the notion of truth used in the theory of meaning is the notion of the correctness of assertions, what this means is that the conditions under which an assertion made with a complex sentence is correct or otherwise is determined by that sentence’s essential structure.

The picture, then, is this. The meaning of a sentence determines, via its essential compositional structure, a set of conditions of correct assertion of that sentence. But there also exist forms of deductive inference applicable to that sentence that determine a distinct set of conditions of correct assertion. So it turns out that an “explanation of the role of deductive arguments in our use of language” requires a demonstration that the standards of correct assertion embodied by logical laws cohere with those determined by the meanings of sentences. Hence, a semantic explanation of logical principles is not an analysis of logical validity in terms of prior and independent semantic concepts; it is, rather, an attempt to demonstrate the mutual coherence of equally fundamental norms of the speech act of assertion.

I note two final points about the conception of semantics that I have just ascribed to Dummett.

First, a semantic theory, on this view, is not really a distinct kind of theory from the syntactic delineation of general logical laws: each is an attempt to characterize standards of correctness of assertions.

Second, we can now give a rather different answer to the questions raised in section I above: what does a soundness proof show, and what is its importance? From our present vantage point, a soundness proof appears to be precisely that which secures the above-mentioned mutual coherence of two equally fundamen-

tal norms of assertion. So, soundness is important because its failure exhibits an incoherence in our language and thought.

X. Dummett and Antifoundationalism

In this section I comment briefly on the relation between Dummett's views and the antifoundational interpretations. The first point to be made is that there is in fact no dispute over whether logic can be given a foundation in semantics in the sense of a theory of truth conditions of thoughts.

Second, as we have just seen, Dummett's conception of semantics, and of semantic explanation of logic are, *prima facie*, completely consistent with antifoundationalism about logic based on the quasi-supervenience view of truth. Thus, Dummett's views show how, even if one accepts in full the normative conception of judgment, there is a viable notion of semantics and a cogent account of its role in logic.

Third, one possible substantive difference between Dummett's view and Ricketts's is over whether correctness is a property of judgments and assertions. But this, to my mind, is rather unlikely. If correctness is not a property of judgments, then, it seems plausible, there cannot be standards of correctness applying to judgments. For what could such standards be, if not conditions under which judgments are correct, that is, have the property of correctness? The problem that this raises for Ricketts, or, indeed, for any interpreter of Frege, is that Frege surely takes logic to constitute the most general standards of correct judgment. If there are no such standards, how can there be such a thing as logic?

I note in passing that the ultimate basis of the thought that correctness is a property of judgments is the fact that there is an objective distinction between correct and incorrect judging. So, regardless of what we make of the idea of correctness as a property of assertions, there must surely be a difference between the conditions of correct judgment and the conditions of incorrect judgment. Moreover, it seems to me that it is this distinction that underlies a number of claims that truth is a "substantial" notion.³¹

Finally, I would speculate that the true divergence between Dummett's and Ricketts's Frege appears at the level of how the notion of standards or conditions of correct assertion or judgment should be characterized. For Ricketts, the sole standard of correctness Frege acknowledges is given by the totality of the basic laws of logic.³² For Dummett, in contrast, as we have seen, Frege thought that there is in addition a distinct notion of standard of correctness for each sentence characterizable in terms of its essential compositional structure.³³

XI. Some Varieties of Antifoundationalism

If Dummett, as I have argued, does not hold a foundationalist conception of logic, who nowadays, one might ask, does? One answer, suggested in a recent essay by Jamie Tappenden, is: just about no one.³⁴ That is, according to Tappenden, just about no contemporary philosopher or logician would think of semantics as providing a "reduction" of the (intuitive) concept of logical consequence to more

primitive concepts. If this is right, then two questions arise. First, what are the grounds for this rejection of semantic foundationalism? Second, why, if semantics does not provide a foundation for logical consequence, is it thought to play an important role in logic? I doubt that there are simple answers to these questions; for one thing, there may well be at present a spectrum of significantly distinct conceptions of semantics and of logic which nevertheless concur in rejecting antifoundationalism. Perhaps for this reason, Tappenden does not in his essay directly address these questions. Nevertheless, one of his criticisms of what is here called the “antisemantical interpretation” suggests a conception of semantics with which one could answer these questions.³⁵ This is what I will do in this concluding section, in order to spell out the differences between this type of anti-foundationalism and the ones discussed above. By doing this I hope to say a bit about the significance of the preceding discussion.

I begin with an objection to the third part of Ricketts’s interpretive argument which I have not brought up. The third part, recall, moves from the claim that truth is not a property to the conclusion that there is no genuine semantics, in the sense of a theory of the conditions in which thoughts expressed by sentences used to make assertions have the property of truth. The objection is based on Tappenden’s claim that semantic theory in the sense of Tarski’s model theory rests on the *stipulative* definition of a notion that can be proven to satisfy Tarski’s adequacy and correctness constraints. Hence, even if we accept that truth is not a property, nothing follows about *Tarski’s* semantics. Thus, the thesis that truth is not a property does not imply that there is no genuine semantics. As Tappenden points out, Tarski himself had already made a similar point, in a well-known passage from “The Semantic Conception of Truth”:

It is undoubtedly the case that in philosophical discussions—and perhaps also in everyday usage—some incipient conceptions of th[e] notion [of truth] can be found that differ essentially from [mine]. . . .

Personally, I should not feel hurt if a future world congress of the ‘theoreticians of truth’ should decide . . . to reserve the word ‘true’ for one of the [other] conceptions, and should suggest another word, say, ‘frue’ for the conception considered here. But I cannot imagine that anybody could present cogent arguments to the effect that the semantic conception is ‘wrong’ and should be entirely abandoned. (Tarski, 1944, p. 58)

Since the property (of sentences) that is the subject matter of Tarskian semantics is defined by stipulation, there is no particular reason to think that it is in any way conceptually more primitive than intuitive notions of logical consequence. So a definition of logical consequence in terms of Tarskian semantics is certainly not a foundational explanation of logical consequence.

What then is the relation between Tarskian semantics and logic? We can see how Tarski might respond to this question by considering why he “cannot imagine that anybody could present cogent arguments to the effect that the semantic conception is ‘wrong’.” The reason, I conjecture, is that Tarski is thinking of his stipulative definitions as the basis of a mathematical theory. The criterion of evaluation of such stipulations is simply *not* faithfulness to an antecedently given concept; in this sense, the semantic conception cannot be shown to be wrong. Rather, the stipulations are to be evaluated by the consistency and fruitfulness of the

mathematics that results; and we, who have seen the development of model theory from Robinson to Hrushovski, know, better than Tarski could, just how mathematically successful his semantic conception has been. On this view, the legitimacy of Tarskian semantics is the legitimacy of any mathematical theory, and is quite independent of whether it has any relation to language or to reasoning. Furthermore, if logic consists of mathematical theories, as contemporary mathematical logic surely does, then the relation of Tarskian semantics to logic is simple: model theory is a branch of mathematical logic.

Now that we have a characterization of Tarskian semantics and its relation to logic, we can come back to our objection to the third part of Ricketts's argument. It is certainly right that the legitimacy of Tarski's semantics is not impugned by the thesis that truth is not a property. But, *by the same token*, nothing follows from the legitimacy of Tarski's semantics about the correctness of the third part of Ricketts's argument. The reason should be plain. Tarskian semantics and logic are mathematical theories with no essential connections with judgment or assertion. In contrast, the semantics and logic that figure in Ricketts's argument *are*: the property that semantics purports to be a theory of is essentially connected to the aim of judgment, logic consists of the most general standards of correct judgment.

A similar contrast holds between the Tarskian conceptions and Dummett's views of semantics and logic. For example, Dummett writes, "semantic notions are framed in terms of concepts which . . . have a direct relation to the use . . . of the sentences of a language; to take the most obvious example, the concepts of truth and falsity" (Dummett, 1973, p. 293). In addition, he suggests that the use of "the words 'true' and 'false'" "in defining the semantic notion of logical consequence" is warranted "in so far as [these words] are taken as . . . connected with the manner in which we effect communication by the use of sentences" (p. 294). For these reasons, Dummett rejects the view that a semantic definition of logical consequence may be based on a definition of a valuation of schemata "as a purely mathematical object—an open set, or a natural number—which has no intrinsic connection with the uses of sentences" (p. 293).

Clearly, then, the semantic antifoundationalisms previously discussed in the present essay are quite different from the semantic antifoundationalism of Tarskian semantics, since they proceed from distinct conceptions of semantics and of logic. What, if anything, should we make of this difference? I would like to urge that we think of it in three ways.

First, let us think of this difference in connection with the idea with which I began this essay: our contemporary assumption of the importance of semantics in logic. The characterization of Tarskian semantics in this section yields, in effect, a way of making this assumption more precise, and a justification of the resulting thesis: semantics plays, and ought to play, an important part in logic because it is one of the most mathematically successful branches of the mathematical science that is logic. The discussion of Dummett's views in sections IX and X, it should now be clear, represents another specification and justification of the place of semantics in logic: semantics is a compositional account of the correctness conditions of assertions, and the importance of the explanation that it might provide of logical laws lies in the fact that a successful explanation shows the coherence of our linguistic and inferential practices.

Second, the discussion in this and the last two sections pinpoints two salient features of Dummettian antifoundationalist semantics that contrasts with Tarskian semantics. First, it is based on what seems an inescapable truism about logic: surely *logic* could not possibly lack essential connections with judgment and assertion. Second, it is consistent with the quasi-supervenience view of truth and therefore with the normative conception of assertion, a conception which is, to my mind at any rate, also intuitively attractive. Thus, to the extent that philosophy should take seriously what we find intuitively compelling, it seems that there is a question to be asked about what significance Tarskian semantics has for a *philosophical* account of logic.³⁶ The reason is that this semantics has no intrinsic connection with correctness in judgment and assertion, so, it seems only incidentally connected with what we intuitively think of as logic. Now, this question of the philosophical significance of Tarskian semantics is, I hold, an open one, not one that can or should be settled by doing no more than either citing our intuitions about what is logic or alluding to the undoubted mathematical interest and importance of model theory. But, for this very reason, as things stand, Tarski's antifoundationalist semantics, considered as an attempt to provide a philosophically viable conception of the nature of semantics and its importance for logic, seems somewhat less satisfying than Dummett's.

Finally, I want to note that these last points do not amount to an endorsement of Dummettian antifoundationalist semantics or a rejection of Tarskian semantics. Rather, I take the foregoing to show that our contemporary assumption of the importance of semantics in logic deserves further philosophical investigation and to highlight some considerations which such an investigation ought not ignore.

Notes

I would like, first and most importantly, to thank Tom Ricketts for helpful discussions about his views. Second, I wish to thank Gary Ebbs for detailed comments that led to far-reaching revisions and reconceptions in my account of the regress argument and its consequences; this essay could not have reached its final form without his help, although, naturally I don't expect him to endorse fully either my conclusions or my ways of meeting his points. Third, I am grateful to Jamie Tappenden for a number of helpful comments and useful discussions, which enabled me not only to avoid some egregious errors, but also to express more clearly my tentative ideas of how to think philosophically about the relation of semantics to logic. I am also pleased to acknowledge my debt to Stephen Menn for forcing me to give two talks to the Philosophy Department of McGill University, even though I had promised only one, since, at the second, the insightful questions and comments of Emily Carson, Michael Hallett, and Stephen himself more or less drove me to write the present essay. Thanks are due, as usual, to Warren Goldfarb and Juliet Floyd for their encouragement and support. I would like to thank the editor, Erich Reck, for his generous help in making some last minute changes possible. Finally, I dedicate this essay to the memory of my teacher, Burton Dreben.

1. Frege, 1964, p. vi. I am indebted to Floyd, 1998, for coming to see that this passage can be used to represent the focus of the interpretive disagreement to be characterized below.

2. See Baker and Hacker, 1984; Diamond, 1991, Essays 2–5; Dreben and van Heijenoort, 1986; Goldfarb, 1979, 2001; Hintikka, 1979; Hintikka and Hintikka, 1986;

Ricketts, 1985, 1986a, 1986b, 1996, 1997, 1998; Sluga, 1980; van Heijenoort, 1967, 1986; Weiner, 1990.

3. See Goldfarb, 2001, for a clear exposition of the contemporary conception of logic often associated with Tarski and Quine.

4. E.g., “For any simple predicate ‘ $F\xi$ ’, ‘ $\forall x Fx$ ’ is true just in case, for every assignment of an object in the domain to ‘ x ’, ‘ Fx ’ is true.”

5. The full reasons for my caution in formulating Dummett’s interpretation will become evident in section IX below. Here, however, I note that, in view of what we will see in the next section, one should take Dummett, in the second and third of the quoted paragraphs, as describing, not Frege’s actual views, but subsequent refinements of them into the schematic approach and soundness proofs. I’m grateful to Jamie Tappenden for pointing this out. Still, this point does not alter the impression that according to Dummett Frege would have endorsed these refinements.

6. Both of these differences between Frege’s and contemporary logic are established in van Heijenoort, 1967, Goldfarb, 1979, and Dreben and van Heijenoort, 1986. As Goldfarb notes in 2001, Burton Dreben, in lectures and seminars as far back as the early 1960s, had been pointing out differences between Frege’s and our views of the subject matter of logic.

7. It should be noted that in part III of Frege, 1906, especially at pp. 321–323, Frege might be read as contemplating a schematic conception of soundness. It is unclear what attitude he holds toward this conception. For discussion see Ricketts, 1998, section III.

8. Ricketts’s interpretation is developed in Ricketts, 1985, 1986a, 1986b, 1996, 1997, 1998.

9. See Ricketts, 1996, p. 136.

10. But see section XI below for further discussion.

11. It might be useful to note here that the view Ricketts ascribes to Frege is best understood as an error theory about uses of the truth predicate. Thus, for instance, on this view Tarski’s elimination of truth in terms of primitive denotation and satisfaction implies, either that there are no such relations as denotation and satisfaction, or that the definition is not of truth.

12. Ricketts (1996) appears to deny that on his interpretation Frege is committed to holding a redundancy theory of truth. His grounds are as follows:

Frege’s view of truth . . . is not a version of the redundancy theory. Frege does not use the equivalence of sentences of the form “ p ” and “it is true that p ” to argue that the truth predicate is eliminable, and to conclude from this eliminability that truth is not a property. Instead, I read Frege, in the context of his conception of judgment, to argue that were truth a property, then a truth-predicate would be required to make a predication implicit in every assertion explicit. The superfluity of certain uses of the word “true” shows that the word “true” does not function in this way.
(1996, p. 134)

It is hard to see that the argument which Ricketts ascribes to Frege could be anything but the following:

1. Suppose truth is a property
2. Then a truth predicate would be required to make explicit a predication implicit in every assertion.
3. Sentences of the form p and ‘It is true that p ’ have the same sense.
4. Hence the truth predicate does not make explicit a predication implicit in every assertion.
5. This contradicts the supposition that truth is a property.
6. Hence, if sentences of the form p and ‘It is true that p ’ have the same sense, then truth is not a property.

I do not understand how this argument is supposed to differ from that which Ricketts attributes to the redundancy theory in the quoted passage. The only suggestion that seems workable here is that Ricketts thinks Frege arrives at a redundancy theory from the regress argument, rather than assume the redundancy theory as a premise in the regress argument. But in any event it is hard to avoid the conclusion that, whatever the differences may be between Frege's position and the redundancy theory, they are committed to the same treatment of the truth predicate.

13. Ramsey's propositional quantification does not obviously apply, since the required quantifiers have to range, not over propositions, but syntactic constituents of sentences. Perhaps Grover and Belnap's theory of substitutional quantification into quotational contexts, together with their prosentential interpretation of truth predication, can handle the reinterpretation envisaged in the text. See Grover, 1992, for details. In any event, whether the prosentential theory successfully eliminates all uses of the truth predicate remains controversial.

14. See Stanley, 1996; Tappenden, 1998; Heck, forthcoming.

15. One further point about this rejection of psychologism is worth noting. It does not, and is not intended to, show that there can be no such field as the psychology of reasoning, which studies empirical generalizations about how we think. All it does reject is the claim that the laws discovered by such a field are the laws of logic.

16. The commitment to defending one's assertions is one of the dimensions of assertion described in Brandom, 1983. This commitment may be explained, from the present perspective, by the claim that failure to produce reasons for an assertion when challenged makes it unwarranted, and thereby incorrectly made.

17. In Geach's and Stoothoff's English translation:

Could we not maintain that there is truth when there is correspondence in a certain respect? But which respect? For in that case what ought we to do so as to decide whether something is true? We should have to inquire whether it is true that an idea and a reality, say, correspond in the specified respect. And then we should be confronted by a question of the same kind, and the game could begin again. So the attempted explanation of truth as correspondence breaks down. And any other attempt to define truth also breaks down. For in a definition certain characteristics would have to be specified. And in application to any particular case the question would always arise whether it were true that the characteristics were present. So we should be going round in a circle. So it seems likely that the content of the word "true" is *sui generis* and indefinable. (Frege 1984, p. 353)

See also Frege, 1897, pp. 139–40; English translation, pp. 128–29.

18. This response seems to be made by Ricketts (1996).

19. It seems to me that a reformulation of the regress argument is also possible based on the characterization of assertion as requiring a commitment to defending one's assertions. Here one would argue thus. Suppose that a defense of the assertion that *p* required showing that its content has the property of being true. Then this defense requires one to make the further assertion that *p* is true. But that assertion cannot be defended unless one showed that its content has the property of being true. And so now we have begun a regress.

20. Note that this account of how it is possible to make infinitely many judgments in the regress can explain why Dummett claims that, given his objection, the conclusion that Frege should have reached is this: any truth definition "should yield the result that, e.g., to enquire whether the statement 'Frege died in 1925' is true is to enquire whether Frege died in 1925" (Dummett, 1981a, p. 444).

21. This is clearly stated by Frege: "So scheint denn dem Gedankens dadurch nichts hinzugefügt zu werden, daß ich ihm die Eigenschaft der Wahrheit beilege" (Frege, 1918, p. 345). English translation: "It seems then that nothing is added to the thought by my ascribing to it the property of truth" (Frege, 1984, p. 354).

22. In particular, Ricketts, 1996, 1997.
23. I am indebted to an unpublished work of Ian Proops for this point.
24. English translation: “The content of the word ‘true’ is *sui generis* and indefinable” (Frege, 1984, p. 353).
25. One way to render these views consistent is this. Take the claim that truth is a primitive and unanalyzable property to be about the metaphysical status of the property. And take the claim that we recognize the presence of this property in virtue of having a capacity to make judgments to be a purely epistemological thesis. Note also that it is equally unclear whether the view that truth is a primitive indefinable property of thoughts is consistent with the view that truth is *not* a property of thoughts.
26. English translation: “The assertion is not to be found in the word ‘true,’ but in the assertoric force with which the sentence is uttered” (Frege, 1979, p. 251).
27. English translation: “The word ‘true’ seems to make the impossible possible; it allows what corresponds to the assertoric force to assume the form of a contribution to the thought” (Frege, 1979, p. 252).
28. This view of truth is a constant in Dummett’s writings; it was first expressed in Dummett, 1959. Its explicit attribution to Frege, however, is much more recent; so far as I know, the first instance is Dummett, 1993a.
29. See, e.g., Dummett, 1976, §§ I–II; and Dummett, 1978, preface, for his account of the notion of a theory of meaning.
30. Goodman’s argument is given in Goodman 1983, pp. 63–64.
31. See, e.g., Putnam, 1983, p. 322; Levine, 1996, pp. 152–54.
32. See, e.g., Ricketts, 1986b, § 3.
33. See also Dummett, 1981b, chap. 15.
34. Tappenden, 1998.
35. Tappenden should not, of course, be taken to endorse either this conception of semantics or my formulation of it.
36. I take this to be the right way to understand the question that Burton Dreben is reported to have asked, “Does model theory have anything to do with logic?” See Sacks, 1972, p. 1.

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Logical Objects in Frege's *Grundgesetze*, Section 10

Marco Ruffino

Section 10 of *Grundgesetze der Arithmetik* (henceforth *GGA* §10)¹ is the place in Frege's writings where truth-values are identified with particular extensions of concepts—and this means that the two main kinds of logical objects in the ontological basis of Frege's logic and arithmetic are actually one and the same. This is a relatively short section of the book, but it has generated a number of questions related to the understanding and evaluation of Frege's argument and procedure. Many influential Fregean scholars have raised doubts as to whether Frege's procedure is sufficient to guarantee that names of extensions have a reference (and hence, since Frege defines numbers as extensions, that numerals have a reference as well). These doubts are usually motivated by the conviction that, for all his efforts, Frege is not able to solve a persistent version of what is known as the Julius Caesar problem, that is, the problem of fixing the identity of extensions in relation to all other objects. Closely related to this doubt is the uncertainty as to which kinds of objects Frege thought to be included in the range of the first-order variables of *GGA*. On the other hand, some scholars tend to see a conflict between Frege's procedure in this section of the book and the Platonism that he advocates in several other places.

In this essay I shall address these three main questions. In my view, an adequate elucidation of what exactly Frege takes to be a logical object is the key to understanding his procedure in *GGA* §10. But before addressing these issues, I will briefly review the main steps of Frege's argument in *GGA* §10. This will be the subject of sections I and III of this essay. In section II, I will argue against Dummett's contention that Frege meant to apply a generalized form of the context principle in *GGA* §10. In sections IV and V, I will develop an alternative way of looking at Frege's procedure in *GGA* §10 that does not allow for any remaining indetermi-

nacy for the notion of value-ranges. Finally, in section VI, I shall argue that Frege's procedure in *GGA* §10 is consistent with his Platonism.

I. The Permutation Argument

Frege introduces his infamous Axiom V in *GGA* §3 and, with it, the notion of value-range (which is actually a generalization of the notion of extension of concepts). The axiom says that two functions have the same value-range if and only if they share the same value for all objects as arguments. In symbols:

$$\dot{\epsilon}f(\varepsilon) = \dot{\alpha}g(\alpha) = (\forall x)(f(x) = g(x))^2$$

(where ' $\dot{\epsilon}f(\varepsilon)$ ' and ' $\dot{\alpha}g(\alpha)$ ' are, in Frege's notation, the value-ranges of $f(x)$ and $g(x)$, respectively). In *GGA* §10 Frege develops an argument that shows that, despite the introduction of Axiom V, the reference of value-range terms is less than completely determined. I will follow Dummett in calling this the permutation argument. The argument goes as follows. Suppose that, in the universe of objects denoted by the proper names of the system of *GGA*, some function $\chi(x)$ is defined satisfying the following conditions:

1. For any two distinct objects a and b , $\chi(a) \neq \chi(b)$.
2. It is not the case that, for every object a , $\chi(a) = a$

(i.e., $\chi(x)$ is a bijection and is not just identity). Then the formal system of *GGA* is true of the objects of the image of $\chi(x)$ if it is true of the objects of its domain, since for any two value-ranges $\dot{\epsilon}f(\varepsilon)$ and $\dot{\epsilon}g(\varepsilon)$, $\chi(\dot{\epsilon}f(\varepsilon))$ and $\chi(\dot{\epsilon}g(\varepsilon))$ are the same just in case $f(x)$ and $g(x)$ agree for any object as argument. Hence, Axiom V cannot guarantee that the reference of value-range terms like ' $\dot{\epsilon}f(\varepsilon)$ ' is the intended one (i.e., the value-range of $f(x)$ and not $\chi(\dot{\epsilon}f(\varepsilon))$) for some $\chi(x)$ specified as above. The reference of value-range terms is, therefore, underdetermined by the formal system of *GGA*.

The way to solve the indeterminacy problem is indicated by Frege in the same section: "How can the indeterminacy be eliminated? By determining, when introducing any function, the value that it assumes for value-ranges as arguments, as well as for all other arguments."³ The basic functions introduced in the formal system up to this point are three: — x (i.e., the function that associates the truth-value true to the true as argument, and the false to any other object as argument), — $\neg x$ (i.e., negation), and $x = y$ (identity). If the value of — x is determined for any object as argument, then the value of — x is also thereby determined, since the latter always assumes the truth-value opposite to the one that — x assumes for the same argument. In addition, as Frege notices, — x assumes exactly the same values as $x = (x = x)$ for any argument, since the latter is true only for the true and false for any other object as argument. Hence, the solution for the indeterminacy problem requires only, according to Frege, fixing the truth-value of $x = y$ for value-ranges as arguments. For the case in which both arguments are value-ranges, the value of the identity is already given by Axiom V. However, for the case in which one of the arguments is a value-range and the other argument is a truth value, nothing said up to *GGA* §10 gives any hint regarding the truth or falsity of the identity.

In order to solve this problem, Frege adopts the following strategy. In the universe of objects denoted by the names of the formal language, he defines the function $\chi(x)$ as follows:

$$\chi(\dot{e}f(\varepsilon)) = T$$

$$\chi(\dot{e}g(\varepsilon)) = F$$

$$\chi(T) = \dot{e}f(\varepsilon)$$

$$\chi(F) = \dot{e}g(\varepsilon)$$

$$\chi(s) = s$$

where $f(x)$ and $g(x)$ are two arbitrarily chosen extensionally nonequivalent functions, T and F are the truth-values true and false, respectively, and s is any other object that is not one of the truth-values or one of the value-ranges $\dot{e}f(\varepsilon)$ or $\dot{e}g(\varepsilon)$. The objects denoted by ' $\chi(\dot{e}f(\varepsilon))$ ' and ' $\chi(\dot{e}g(\varepsilon))$ ' still have as their criterion of identity Axiom V, that is, $\chi(\dot{e}f(\varepsilon))$ and $\chi(\dot{e}g(\varepsilon))$ are the same just in case the values of $f(x)$ and $g(x)$ agree for all possible arguments. It is on the basis of this fact that Frege takes his next (and final) step for the resolution of the indeterminacy problem by stipulating that names of value-ranges are to denote the values of $\chi(\dot{e}f(\varepsilon))$, while all the other names denote exactly the same object as in the old interpretation. The result is that all names of the formal system (including the names of truth-values) refer now to exactly the same object as before, except for ' $\dot{e}f(\varepsilon)$ ' and ' $\dot{e}g(\varepsilon)$ ', which now refer to the truth-values true and false, respectively.

It looks as though Frege's claim is that we may choose any two nonidentical value-ranges and identify one of them with the true and the other one with the false.⁴ Or, to put it another way, the identification of a value-range with the true and of another value-range with the false is consistent with everything that was laid down before, including Axiom V.⁵ Frege's particular way of identifying the value-ranges with the true and the false seems to be intuitively appealing: he identifies the true with the extension of the concept under which only the true falls (i.e., $\dot{e}(\neg\neg\varepsilon)$), and similarly for the false (i.e., $\dot{e}(\varepsilon = \neg(\forall x)(x = x))$).⁶ Since two particular value-ranges are now identical with the true and the false, it is determined whether any value range is identical with the true or the false via Axiom V. With this supplement to Axiom V, Frege claims, value-ranges are as well determined as they can be, and the reference of value-range terms is assured.⁷ He would have been right if Axiom V were consistent.

II. GGA §10: Context Principle or Leibniz's Principle?

Before proceeding further, I want to pause for a moment and consider the content of Frege's claim that, in order to solve the indeterminacy problem, it suffices to fix the value of all functions for value-ranges as arguments. There is a suggestion made by Dummett that this claim should be seen as a generalized version of Frege's famous context principle originally formulated in *GLA* (from 1884), that is, the principle according to which only in the context of a sentence names (in particular names of abstract entities) have reference. Dummett explains the generalized version of the context principle as follows:

Stated generally, in respect of any range of proper names, rather than just of value-range terms, the principle is that the references of those names are fixed when it has been determined, for every (primitive) function, what the value of that function is to be for the referent of any of those names as arguments. . . .

This is, in effect, a generalization of the context principle. It is a generalization, in that it allots no distinguished place to sentences (names of truth-values). (1981, p. 408)

Contrasted with the original context principle as it was formulated in *GLA*, p. x, and used in *GLA* §62, this later doctrine is, according to Dummett, “not clearly coherent,” since it amounts to the claim that a proper name has reference only if all composed proper names containing it as part have reference (p. 409). The original principle attributes a unique status to sentences, but this status vanishes in the generalized version. Hence, according to Dummett, while there was a perfectly reasonable motivation for the first principle, there seems to be no such motivation for the latter (generalized) principle. This interpretation of Frege’s procedure in *GGA* §10 is one of the main reasons for Dummett’s claim that, in his mature philosophy, Frege abandoned the semantic primacy of sentences in questions of referentiality of abstract terms.⁸

If Frege’s claim in *GGA* §10 is indeed to be interpreted as echoing the original context principle, then this generalized version seems to be of reduced philosophical interest as compared with the original principle. But I do not share Dummett’s view that Frege meant to apply anything like a generalized version of the context principle at this particular point in the book. The most important reason not to regard the passage from *GGA* §10 quoted in the previous section as expressing a version of the context principle is that there is a more plausible way to understand Frege’s way out in *GGA* §10 that removes the flavor of philosophical extravagance and eliminates the apparent trivialization of the principle. In my view, Frege just meant to apply Leibniz’s principle of identity (which he endorses on several occasions) as a means of eliminating the indeterminacy in the identity of value-ranges as objects. I shall sharpen this point in the rest of this section.

Frege seems to be dealing with two different—though related—problems in *GGA* §10, without sharply distinguishing between both. One is the ambiguity of the reference of names like ‘ $\varepsilon f(\varepsilon)$ '; the other is the vagueness of value-ranges as entities: they do not have a clear identity. The first question introduced in *GGA* §10 (see quotation above) is clearly a formulation of the first problem, that is, Frege notices that there is no guarantee, up to this point in the text, that names of this form in fact refer to value-ranges or to some (different?) kind of objects univocally associated with them. (Incidentally, the only remaining possibility consists of truth-values.) Both possibilities are compatible with Axiom V.⁹ But Frege’s approach in *GGA* §10 seems rather to target the second problem. He seems to think that value-ranges qua objects are still vaguely specified entities, not clearly distinct from truth-values and not clearly identical with truth-values. His strategy outlined in the crucial passage quoted before (“By determining, when introducing any function, the value that it assumes for value-ranges as argument, as well as for any other argument”) is a specification of value-ranges qua objects. Of course

Frege's solution for the second problem (identifying truth-values and extensions) is, indirectly, a solution for the first as well, since by eliminating the alternative referents of names like ' $\dot{e}f(\epsilon)$ ' he also eliminates the possibility of ambiguity. The specification of value-ranges as logical entities leads to the disambiguation of the names in Frege's system. But had Frege adopted a different path of specification of value-ranges, keeping them distinct from truth-values, then the second problem would have been solved, but the first one would still remain, for in this case names like ' $\dot{e}f(\epsilon)$ ' could refer both to value-ranges and to something different from value-ranges, although univocally associated with them (i.e., to truth-values).

Now what Dummett takes to be an application of a generalized version of the context principle to the first problem is, in my view, an application of Leibniz's principle to the second problem. How can a value-range be distinguished from another object that is possibly associated with it by the function $\chi(x)$ as described in the previous section? According to Leibniz's principle, two objects are identical if and only if every function has the same value for both as arguments. But this requires that, if the identity of any two objects is determined in respect to each other, the values assumed by any function for these two objects as arguments are determined as well. (If the function has an undetermined value for some object, e.g., for a value-range, then there may be no answer as to whether this object is identical with another one, e.g., a truth-value.) Consequently, the reference of a value-range term is distinguishable from the reference of another name only if some first-order function assumes different values for both objects. This requires, at least in Frege's system, that for every first-order function, its values must be determined for value-ranges and for truth-values as arguments.

In the last paragraph of *GGA* §10, Frege states: "With this we have determined the value-ranges as much as it is possible here. If there is a further question of introducing a function that is not reducible to the functions known so far, then only in this case we must determine which value it must have for value-ranges as arguments; and this can be seen as a determination of value-ranges as much as of the function itself." The first sentence of this passage seems to support the view that Frege is primarily attacking the second problem in *GGA* §10, since he talks about the determination of the things, and not of the references (although, as I said, the latter follows from the particular way that he makes value-ranges more precise). The claim contained in the last sentence of the quotation seems to follow from an analogue of Leibniz's principle for concepts (functions) that Frege defends elsewhere. Identity is a first-order relation, and hence we cannot say that it applies to concepts. But, as he comments in "Ausführungen über Sinn und Bedeutung" (NS 131), from 1892–95, and in "Über Schoenflies: Die logischen Paradoxien der Mengenlehre" (NS 197), from 1906, there is a second-order relation that is the analogue of identity for concepts. This relation holds between two concepts (functions) $f(x)$ and $g(x)$ if and only if, for any object a as argument, $f(a)$ is true if and only if $g(a)$ is true ($f(a)$ is the same as $g(a)$). The identity or distinguishability of functions requires that they have determinate values for all objects. (Two functions that have indeterminate values for a particular object but have the same determinate value for any other object as argument are not clearly identical nor clearly distinct.) This would certainly be false if partial functions—that is, functions that have values only for a limited range of objects—were allowed. But

clearly partial functions are not functions in Frege's sense. Functions, like concepts, must be sharply delimited in his view, that is, they must have a determinate value for every object (*GGA* §2; *KS* 135, 230). If all admissible functions in the system have sharp boundaries, that is, a determinate value for all objects, it follows that for any object named in the formal system, the value of any function is determined for this object. So the requirement that an object must yield a determinate value when taken as argument by any function is the counterpart of the requirement of sharp delimitation for functions. Both requirements follow from Leibniz's principle and from the corresponding principle for functions.

In this reconstruction of Frege's solution for the indeterminacy problem, nothing like the context principle was invoked. The principles that, in my view, are behind Frege's solution for the indeterminacy in *GGA* §10 can be held quite independently of the context principle. There is no reason, therefore, for believing, as Dummett does, that Frege's context principle was transformed into something less interesting in *GGA* §10 because, as I see it, no context principle was meant to be applied here.¹⁰

III. Indeterminacy of the Notion of Extension?

There has been some debate in the Fregean literature as to whether *GGA* §10 contains all that is necessary to fix unequivocally the references of names of value-ranges. The importance of this question comes from the fact that Frege defines numbers as extensions of concepts, which in turn are just particular value-ranges. If there is some residual indeterminacy for the reference of names of value-ranges, then the reference of numerals is also indeterminate. This might undermine Frege's claim to provide a satisfactory reduction of arithmetic to logic and would be in conflict with many of his criticisms of the formalist view of arithmetic.

Why is this so? Could not a reduction of arithmetic to logic be considered as successful even if it still admits of a residual indeterminacy, provided that the indeterminacy is confined to logical objects? There are several elements of Frege's thought that would be incompatible with this tolerant attitude toward indeterminacy. First, he insists in *GLA* that we must consider the name of a number as referring to one and only one object. (This insistence is frequently based on the considerations associated with the use of the definite article.)¹¹ He makes use of this view to refute some rival theories of arithmetic. Now, the name of a number is defined as having the same reference as the name of an extension. If there is an intrinsic indeterminacy in the reference of the names of extensions, how can it still be maintained that names of numbers refer to a unique object? Second, a condition for the applicability of numbers to a concept is that the concept be sharply delimited (*WB* 163). But numbers certainly apply to concepts defined over logical objects, since they apply to everything (in particular, they apply to concepts derived from the series of finite numbers, which is a series of logical objects). This means that concepts like the one denoted by 'identical with the true' or by 'is the same as the value-range *so-and-so*' must be sharply delimited if arithmetic is to apply to them as well. That is to say, no indeterminacy should be allowed here, or so Frege's view requires.

In pursuing this problem of the indeterminacy of extensions, it may be helpful first to examine a possible analogy between Frege's reasons for rejecting two attempted definitions of numbers offered in *GLA* and the issue raised in *GGA* §10. This is what I shall do in the rest of this section. In section V, I will argue that the analogy breaks down at a fundamental point.

III.1 Indeterminacy of the Reference of Numerals in *GLA*

Frege considered, in *GLA* §55, the possibility of defining numbers contextually as second-order concepts. More precisely, he tried to fix the reference of predicates of the form ‘the number n belongs to the concept *so-and-so*’, of which the numerals ‘0’, ‘1’, and so on are an essential part. This first attempted definition comprises the following three sentence schemata (in which the expression ‘the number n belongs to the concept F ’ is abbreviated by ‘ $(\exists_n x)Fx$ ’):

$$\begin{aligned} (\exists_0 x)Fx &\leftrightarrow (\forall x)\neg Fx \\ (\exists_1 x)Fx &\leftrightarrow \neg(\forall x)\neg Fx \wedge (\forall x)(\forall y)((Fx \wedge Fy) \rightarrow x = y) \\ (\exists_{n+1} x)Fx &\leftrightarrow (\exists x)(Fx \wedge (\exists_n y)(Fy \wedge y \neq x)). \end{aligned}$$

In *GLA* §56 he raises an objection to this definition: “But we can never—to take a crude example—decide by means of our definition whether any concept has the number JULIUS CAESAR belonging to it, or whether that famous conqueror of Gaul is a number or not.”¹²

The objection expressed in the quotation is a combination of the following two complaints (the first one is only implicit in the passage):

1. The definition fails to characterize numbers as objects (since numerals do not show up as singular terms in it)
2. The definition fails to distinguish numbers from objects not given as numbers, like Julius Caesar. That is, it fails to determine the truth-value of identity statements between a numeral and names of ordinary objects.

Clearly complaint 2. is only legitimate if complaint 1. is legitimate (although the converse does not hold). For if numbers are not to be characterized as objects, there is no point in demanding that the definition should be able to determine the truth-value of identity statements between numbers and objects of other kinds, for such statements are simply meaningless. The second complaint is usually referred to in the Fregean literature as the Julius Caesar problem. Frege's main reason for making the first complaint seems to derive from certain grammatical features of the use of numerical terms in arithmetic, especially the fact that they can be preceded by the definite article.¹³

The second complaint is more relevant for our purposes. Frege had already argued in his discussion in *GLA* previous to §55 that numbers are not empirical objects of any kind. So we know already, before any definition is formulated, that the number that belongs to the concept F cannot be identified with the aggregate of objects that fall under F or any other aggregate, still less with something like Julius Caesar. However, what complaint 2. seems to reveal is that Frege does not consider the appeal to this previous knowledge as legitimate; rather, he wants the falsity of

identity statements between numbers and Julius Caesar to be something that we determine by means of the definition. That is, the definition has to play some role in determining the truth or falsity of identity statements involving the defined name. This seems to be a reasonable requirement since Frege has in mind numerical equalities and inequalities as the primary form of statements involving numbers. This requirement (and the corresponding complaint 2.) appears again when he criticizes the second attempted definition of *GLA*, which I will discuss next.

Frege opens *GLA* §62 by asking: "How, then, are numbers to be given to us, if we cannot have any ideas or intuitions of them?" And the answer starts with a formulation of the context principle: "Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs." In this particular place of the text the context principle serves as the main pillar of Frege's disagreement with Kant's view on arithmetic.¹⁴ The strategy Frege follows in his second attempt to define numbers is the following: If we can lay down the truth-conditions of some characteristic sentences in which numerical terms occur using only sentences in which they do not occur, then we can guarantee that these terms have a reference. The most characteristic numeral-containing sentences are, according to him, identity statements of the form ' $a = b$ ' where both ' a ' and ' b ' are numerical terms. Now Frege construes these numerical terms as having the form 'the number that belongs to the concept *so-and-so*' in order to stress the fact that numbers are always associated with concepts. (I will abbreviate these terms by ' $Nx:Fx$ '.) The second attempted definition (Hume's principle)

$$Nx:Fx = Nx:Gx \leftrightarrow F \text{ and } G \text{ are equinumerous}$$

is first considered in *GLA* §§62–63 and then explained in *GLA* §§64–65 by analogy with the geometrical definition of the direction of a line. After outlining this definition, Frege raises some possible worries about it and one of them, introduced in *GLA* §66, motivates his rejection of the definition. According to Frege, the definition can only account for the truth or falsity of identity statements between two terms of the form 'the number that belongs to the concept *so-and-so*', but it says nothing about the truth or falsity of identity statements in which at least one of the terms does not have this form. For example, we cannot decide, using the definition, whether 'the number that belongs to the concept $x \neq x$ is identical to Julius Caesar' is true or false. That is to say, the definition is able to play no role in the differentiation between numbers and objects of other kinds. Clearly Frege's criticism of this definition is not exactly the same as that of the definition in *GLA* §55. Against that definition two complaints were brought; but here only the second complaint is raised. The first complaint obviously does not apply any more because, in this second definition, numerical terms are already being treated as singular terms. Hence, the Julius Caesar problem is left unsolved by this second definition.

Importantly, there are two ways of understanding Frege's second complaint here. The first interpretation—which I shall call the strong one—is that the complaint corresponds to the demand that all identity statements between numbers and all negation of identity statements between numbers and anything else should follow only from the definition (Hume's principle) together with other relevant definitions and the laws of logic. The second interpretation—which I will call the weak one—regards the second complaint as a more modest demand, that is, that

the definition of numbers should be an essential part of the determination of all equalities and inequalities involving numbers, although the derivation of these equalities and inequalities may involve some other information besides Hume's principle, laws of logic, and relevant definitions. It is not clear which of these demands Frege has in mind in *GLA* §66 when he formulates the complaint. But the weaker one seems more reasonable if we are talking about numbers associated with nonlogical concepts. For example, the statement 'the number of atoms of hydrogen of any molecule of water = the number of protons of any atom of helium' cannot be derived unless we know that there is a one-to-one correspondence between the objects falling under the concepts *atom of hydrogen of a molecule of water* and *protons of an atom of helium*, which is an additional piece of information contained not in the definition of numbers or in the laws of logic but in chemistry.

For those numerical statements involving only logical concepts, the stronger demand looks more reasonable. The identity of numbers belonging to nonlogical concepts may depend on contingencies, but the identity of numbers belonging to logical concepts does not. Consequently, it should be determined only by logic and the relevant definitions whether or not there is a one-to-one correspondence between the objects falling under two logical concepts. The examples Frege uses in *GLA* are frequently of numbers associated with empirical concepts, so it seems plausible to assume that the second complaint corresponds to the weak demand. But in the context of *GGA*, in which no nonlogical concept at all plays any role in Frege's definitions and proofs, the demand on the definition seems to be more appropriately taken to be the stronger one.

If it cannot be decided, based on the definition of *GLA* §§62–65, whether or not the direction of the Earth's axis is identical with England, then, according to Frege, we cannot have a precisely delimited concept of direction and so no real concept at all. Similarly with the concept of number (*GLA* §68). As we know, Frege proceeds to propose instead the famous explicit definition of numbers as extensions in *GLA* §68.

III.2 An Analogous Problem for Extensions in *GGA*?

Frege does not make clear his notion of extension in *GLA*; the first complete account of it appears only in *GGA*. Our question now is whether there is a problem of indeterminacy for this notion in *GGA* analogous to the Julius Caesar problem of *GLA* for the notion of number (as we understood it in connection with Frege's criticism of his definitions of *GLA* §55 and *GLA* §§62–65). In this section I discuss some evidence that suggests a parallel between *GLA* and *GGA* in this respect. In section V, I argue that there is an important disanalogy between the cases that undermines the parallel.

As we saw already, Axiom V of *GGA* is supposed to lay down the criterion of identity for value-ranges. It has the same general form as Frege's definition of numbers in *GLA* §§62–65, despite the fact that Axiom V is not called a definition or treated as one in *GGA*. It is natural, therefore, to suspect that an analogous problem of indeterminacy will arise for the reference of the names introduced by Axiom V. Indeed, Frege begins *GGA* §10 with the following remark, which seems to be an analogue of his second complaint in *GLA* §66:

Although we have laid down that the combination of signs ' $\dot{\epsilon}\Phi(\epsilon) = \dot{\epsilon}\Psi(\epsilon)$ ' has the same reference as ' $(\forall x)(\phi(x) = \Psi(x))$ ', obviously the reference of a name like ' $\dot{\epsilon}\Phi(\epsilon)$ ' is by no means completely fixed. We have only a means of always recognizing a value-range when it is denoted by a name like ' $\dot{\epsilon}\Phi(\epsilon)$ ' by which it is already recognizable as a value-range. But so far we can neither decide whether an object that is not given as a value range is actually one, and to which function it might correspond, nor can we decide in general whether a given value-range has a given property unless we know that this property is connected with a property of the corresponding function.

As already discussed, the permutation argument (which follows the passage quoted) shows that Axiom V alone does not tell which kind of entity value-ranges are and, in particular, is of no help in deciding the truth-value of identity statements involving value-range names and other kinds of names. Also Frege's strategy for resolving the indeterminacy is, as we saw, to lay down the value of all functions introduced up to this point in his system when value-ranges are taken as arguments. As he argues, it suffices for his purposes to determine the value of identities in which one of the arguments is a value-range; and the only problematic case is the one in which the other argument is not a value-range, but some other object. Now the only kind of objects that Frege considers as possible arguments are truth-values: "Since up to now we have introduced only truth-values and value-ranges as objects, the question can only be whether one of the truth-values is a value-range."

As already remarked, what follows is the stipulation¹⁵ that the true should be identical with the extension of the concept *is the true*, and the false should be identical with the extension of the concept *is the false*. And, according to Frege, by means of these stipulations he has determined value-ranges as far as possible.

But is Frege correct in assuming this? Or should he also have determined the identity relation for the case in which one of the arguments is a value-range and the other one is an ordinary object (such as Julius Caesar)? In other words, could it be the case that even after the additional stipulation of GGA §10 there is still a remaining indeterminacy, since it is still unclear whether Julius Caesar is identical with a certain value-range or not? In other words, has Frege made any progress whatsoever regarding the Julius Caesar problem?¹⁶

Frege in fact considers in the second footnote to GGA §10 some strategies for incorporating ordinary objects as extensions in his system. The first possibility considered is to identify an ordinary object with the extension of the concept under which only this object falls. That is to say, for any object Δ , $\Delta = \dot{\epsilon} (\epsilon = \Delta)$. And by Axiom Δ we could check whether this extension is identical with any other extension. However, this stipulation would be inconsistent with Axiom V for the case in which Δ is already a value-range. For, let Δ be $\dot{\epsilon}f(\epsilon)$; then the stipulation would yield $\dot{\epsilon}f(\epsilon) = \dot{\epsilon} (\epsilon = \dot{\epsilon}f(\epsilon))$. Now, by Axiom V, the functions $f(x)$ and $x = \dot{\epsilon}f(\epsilon)$ would have to agree for any object of the domain as argument, but this is not necessarily the case for any function $f(x)$.¹⁷ And it is of no help to claim that the identification should only be carried out for objects that are not value-ranges, for this would already presuppose determination of what value-ranges are. But this is precisely what Frege is trying to attain with the stipulations.

In the same footnote Frege considers a second possibility. As he notices, the stipulation $\Delta = \dot{e}(\varepsilon = \Delta)$ is a particular case of $\Delta = \dot{e} \Omega(\varepsilon, \Delta)$, where $\Omega(x, y)$ is some binary function. If $\Omega(x, y)$ is the relation of identity, we know already that the stipulation does not work. The question is whether there is some other $\Omega(x, y)$ for which the stipulation could work (i.e., be consistent with Axiom V). The only further possibility that Frege mentions is the function $x \sim y$ (which will appear later in *GGA* §34). If a is any object and b is the value-range of a function $f(x)$, then the value of $a \sim b$ is $f(a)$. Hence the new suggestion is to identify any object Δ with $\dot{e}(\varepsilon \sim \Delta)$. This identification does not conflict with Axiom V for the case in which Δ is already a value-range. By the definition of the function $x \sim y$ in *GGA* §34,

$$a \sim \Delta = \dot{\alpha}(\exists \Phi(\Delta = \dot{v}\Phi(v) \wedge \Phi(a) = \alpha))$$

(where ‘ $\dot{\alpha}$ ’ designates the definite description operator and ‘ Φ ’ is a second-order variable).¹⁸ If Δ is a value-range of a function $g(x)$, then according to the definition of ‘ $\dot{\alpha}$ ’, $a \sim \Delta$ is the value $g(a)$. Hence $\dot{e}(\varepsilon \sim \dot{v}g(v))$ is identical to $\dot{e}g(\varepsilon)$, as we would expect. If Δ is not a value-range, then by the definition of the definite-description operator, $a \sim \Delta$ is identical to

$$\dot{\alpha}(\exists \Phi(\Delta = \dot{v}\Phi(v) \wedge \Phi(a) = \alpha))$$

This is the value-range of a function that associates the value false with each object (since there is no function that satisfies the condition above). Hence the identification is

$$\Delta = \dot{e}(\varepsilon \sim \Delta) = \dot{e}(\dot{\alpha}(\exists \Phi(\Delta = \dot{v}\Phi(v) \wedge \Phi(\varepsilon) = \alpha)))$$

The double value-range corresponds to a function that associates the value false to each pair of objects taken as argument (since, by hypothesis, there is no function $g(x)$ such that $\Delta = \dot{v}g(v)$). In other words, the extension represented by the above double value-range is empty. Hence, Frege’s suggestion seems to be to identify each ordinary object (i.e., each object that is not a value-range or a truth-value) with the extension of a relation that has no instances (in terms of classes, this would be the empty class). The value-range with which Δ is identified is the extension of the relation denoted by ‘ Δ ’ is the value-range of some function $g(x)$ and the value of this function for x as argument is y ’. Since the first part of the conjunction is false, the extension is empty.

The resulting stipulation is consistent with Axiom V. That is, when Δ is a value-range $\dot{\alpha}f(\alpha)$, it is always true that

$$\dot{e} \Omega(\varepsilon, \dot{\alpha}f(\alpha)) = \dot{\alpha}f(\alpha)$$

holds if and only if for every x ,

$$\Omega(x, \dot{\alpha}f(\alpha)) = f(x)$$

because

$$\Omega(x, \dot{\alpha}f(\alpha)) = x \sim \dot{\alpha}f(\alpha) = f(x).$$

But, despite its technical convenience, this strategy of identifying ordinary objects with the empty extension is rejected by Frege, on the grounds that the definition of $x \sim y$ already makes use of the notion of value-ranges, which is exactly what Frege is trying to determine in *GGA* §10. Frege ends the footnote of *GGA* §10 by

saying that the function $x \sim y$ cannot be of any help here, without expressing any opinion as to whether any other function could possibly serve.

IV. Logical versus Ordinary Objects

Why does Frege identify truth-values with certain extensions in *GGA* §10? Would it not have the same effect to stipulate that truth-values are distinct from extensions, so that the ontological basis of *GGA* contains two kinds of objects instead of just one? If the point of the discussion in *GGA* §10 is, as some scholars suggest,¹⁹ to solve the Julius Caesar problem, then there seems to exist no justification for preferring assimilation to separation. But there is a deeper justification for Frege's choice. As I argued elsewhere,²⁰ extensions have a special status for him as the paradigmatic case of logical objects. This is suggested in many passages, but especially in a crucial passage in a letter to Russell dated July 28, 1902: "I myself resisted for a long time the recognition of value-ranges, and with them classes; but I saw no other possibility of giving a logical foundation to arithmetic. What is at stake is the question: how do we grasp [fassen] logical objects? and I found no other answer for it besides the following: we grasp them as extension of concepts, or, more generally, as value-ranges of functions" (*WB* 223). Why is this so? Why do extensions have this special status as paradigmatic logical objects for Frege? This perspective has its roots, I think, in Frege's broader view on the nature of logic as a science. The primary task of logic is, for him, the uncovering of relations holding among concepts. (This is why his first and most fundamental work in pure logic from 1879 is a *Begriffsschrift*, that is, a language for the precise expression of conceptual contents.) Now, as the following passage from "Über Begriff und Gegenstand" (from 1892) suggests, in logic we are frequently required to "transform" concepts into their "representative objects":

In logical investigations one needs frequently to say something about a concept and to express this in the usual form for such predications, so that what is said about the concept becomes the content of the grammatical predicate. As a consequence, one would expect the concept as the reference of the grammatical subject; but the concept cannot appear so without any preparation due to its predicative nature; it must first be transformed [*verwandelt werden*] into an object, or, more precisely, it must be represented by an object, which we designate by prefixing the words "the concept"; e.g.: "the concept man is not empty." (*KS* 171)

Although I shall not discuss this point in detail here, there is strong textual evidence that Frege has extensions in mind when he mentions, in this passage, the objects that "represent" concepts.²¹ If this is correct, then extensions are the most basic logical objects; and if truth-values are to be seen as logical objects, then it is only natural that at some point they must be identified with extensions. This follows from Frege's view about the ontology of logic and from his related view about the epistemology of logical objects.²²

Now, what about the identity of extensions with other objects besides truth-values? It is common to find in the literature the assumption that Frege imposes the following two demands on the system of *GGA*:

1. The range of the first-order variables is not to be limited to some special kind of objects
2. The definition of a proper name (plus, possibly, some additional information)²³ should be able to distinguish the reference of this name from any other object included in the range of the first-order variables. That is, by using the definition we should be able to determine the truth-value of any identity involving the name being defined.

It is doubtful that any solution could ever be found for the indeterminacy problem if Frege were indeed imposing the two demands above. Hardly any definition in mathematics satisfies the second demand. For example, the definition of a particular filter in topology cannot tell whether Julius Caesar is identical with it or not—or, better, it plays no role at all in our knowledge that Julius Caesar is not a filter. The reason why this is not a problem in mathematics is that for theories like topology the first demand is not made, that is, the domain is not unrestricted. Within the restricted domain the definition can distinguish precisely between what is and what is not a filter. The two demands above can hardly be jointly satisfied, and that is why there can probably be no solution for the Julius Caesar problem as it was understood (or as I take it to have been understood), for example, by Dummett, Wright, and Parsons.²⁴

However, I think that there is some evidence that Frege was not imposing the first demand above. Although he does not mention what the first-order variables will range over when he introduces them in *GGA* (thereby giving the impression that their range includes everything), there is a passage in *GGA* §31 where he seems to imply that they are ranging only over extensions. In this paragraph, Frege tries to provide a proof of consistency for his system, which means for him showing that every name introduced so far has a reference. When he considers the names of first-order functions of one argument ‘—ξ’ and ‘——ξ’, he makes the following remark: “In order now to show, first, that the names of functions ‘—ξ’ and ‘——ξ’ refer to something, we have only to show that the names resulting from the substitution of ‘ξ’ by a name of a truth-value refer to something (we do not know other objects yet).” Only names of truth-values have to be substituted for the first-order variables because, as the sentence in parentheses suggests, the two truth-values (T and F) are the only two objects known to be in the domain of *GGA* (at least up to this point in the text). After *GGA* §31—that is to say, after Frege has shown that names for value-ranges have reference—the domain includes only truth-values and value-ranges. But since truth-values were already identified with value-ranges in *GGA* §10, it follows that the only objects in the range of the first-order variables are value-ranges. In fact not all value-ranges are included, but only those of the primitive functions or functions composed out of the primitive functions.

Some scholars²⁵ have pointed out that, when Frege explains in *GGA* §2 what he takes to be arguments of first-order functions, he does not mention any restriction to be made, which could mean that the range of first-order variables must be unrestricted as well. Frege says in *GGA* §2: “Objects are opposed to functions. Accordingly I consider as objects everything that is not a function.” This seems to go against my suggestion. But how unrestricted really is the realm of objects that Frege has in mind here? The sentence that immediately follows the above quotation is the following: “for example, numbers, truth-values and the value-ranges

that will be introduced below." Only logical objects are mentioned here as examples of objects. Furthermore, to the best of my knowledge, at no point in *GGA* does Frege mention any other objects that are suited as arguments for the functions in his formal system. Although this is not a decisive piece of evidence, the last passage above strongly suggests that the generalization of the notion of objects that Frege had in mind for the system of *GGA* is a limited one: He wants to argue for the recognition of other logical objects (truth-values and extensions or value-ranges) as possible arguments for functions besides those traditionally treated as such by traditional arithmetic (numbers). As it seems, Frege does not want to advocate the unrestricted allowance of objects, but primarily the recognition of truth-values and extensions (value-ranges) as arguments of first-order functions.

If this is correct, there is no remaining indeterminacy for the reference of value-range terms arising from the two incompatible demands above, simply because the first demand is not made by Frege. He has only to account for the truth-value of identity statements between logical objects, that is to say, between truth-values and value-ranges; and this is exactly what is done in *GGA* §10. I am not suggesting that Frege is absolutely clear on this point. Indeed, he does not seem to be very clear in delimiting the objects about which his system is talking. But as I see it, there are two reasons that may explain his silence about objects that are not extensions. First, it may have appeared obvious to him that, if the purpose of *GGA* is to show how numbers can be constructed out of logical objects, then these are the only objects we have to consider as elements of the ontological basis of the system. Second, Frege emphasizes on several occasions in his writings previous to *GGA* that it is not enough for the consistency of a system that no contradiction is derivable from it so far. What is necessary, according to him, is a proof that every name of the system has one and only one reference. This is, then, the limited purpose of *GGA* §10 and *GGA* §31. I suspect that Frege did not feel he had to talk about other objects in part because it is not necessary to consider them in order to guarantee consistency. The system of *GGA* is one in which the proper names are supposed to refer to logical objects. If other objects are to be included, then corresponding adjustments have to be made later, but these would come only after the identity of numbers is fixed in relation to other logical entities.

A parallel conclusion that can also be drawn from these considerations is the following: although value-ranges in general are logical objects in Frege's view (as he suggests, e.g., in *WB* 223), there seem to be some that are purely logical, namely, those value-ranges of functions employed in logic. In particular, there is the value-range of the horizontal, since it corresponds to the most fundamental function in logic, namely, the function expressed by 'is true'.²⁶ These are the only objects that are part of the primitive ontological basis of logic and arithmetic.²⁷ These objects can be distinguished from other (not purely logical) value-ranges by Axiom V. And how can they be distinguished from nonlogical objects? As I said before, there is something distinctive about logical objects in Frege's view: They are objects that relate in a special way to concepts. Logical theory frequently requires the "transformation" of concepts into their representative objects (i.e., the consideration of concepts through these representative objects), and these objects, which are the respective extensions, are the primary logical objects for Frege. Their primacy as logical objects is derived from the primacy of concepts as the fundamental material of logic. Names such as 'the concept *inhabitant of the German Empire*' or 'the

class of Russian czars' play an essential role in logical theory, and this indicates that what they denote are logical objects. But proper names such as 'Julius Caesar' play no essential role in such a theory—and hence denote a nonlogical object. A moral to be drawn from these considerations is that, on Frege's view, our grounds for considering one or another ontological basis for logic as correct is not independent from the development of logical theories themselves and from the methodological features of logical investigation. I will return to this point in section VI.

V. Breaking the Parallel between *GLA* and *GGA*

I will now reconsider the issue raised in section III.2. As we saw, there seemed to be a parallel between the second complaint formulated in *GLA* about the two contextual definitions for numbers and the problem brought up in *GGA* §10. I will argue in this section that the parallel breaks down at an important point. In my view, if we look at this matter in the appropriate light, keeping in mind some central aspects of Frege's notion of logical object, we will see that while it was a problem in *GLA* to distinguish numbers from Julius Caesar, it is not a problem in *GGA* to distinguish extensions from ordinary objects.

Instead of considering directly whether the discussion in *GGA* §10 solves the analogue of the Julius Caesar problem for extensions, I want to consider the background in *GLA* to Frege's second complaint. A crucial thing to notice about Frege's second complaint concerning the definitions of *GLA* §55 and *GLA* §§62–63 (the failure to solve the Julius Caesar problem) is that it occurs in a context in which no reduction of numbers to objects that could be uncontroversially regarded as logical has been provided so far in the text. Hume's principle

$$NF=NG \leftrightarrow F \text{ and } G \text{ are equinumerous}$$

does not define numbers by reducing them to any kind of object that could be regarded as logical beyond any doubt. Hence it does not make manifest the logical nature of numbers. Now, what distinguishes numbers from ordinary objects at this point? How can we know whether a number is indeed a logical object and not Julius Caesar? There is no answer at hand. Strictly speaking, Hume's principle only says whether or not two concepts have the same number, but it does not say *what* numbers are. The identity of numbers as objects remains vaguely specified under Hume's principle—and hence they are not sharply distinguished from anything else. In this sense, the definition of numbers as extensions from *GLA* §68 is a specification: it says which objects numbers are, namely, the equivalence classes of equinumerous concepts.

Frege tends to focus his criticism on the deficiency of Hume's principle qua definition, but the point underlying his rejection goes beyond the technical inadequacy of it. Even if the strict requirement imposed on the definition is abandoned, Hume's principle cannot fulfill the task of the book, namely, to make it clear that numbers are reducible to things that a considerable range of philosophers and mathematicians would consider to be essentially logical.

The situation changes completely when numbers are defined as extensions. For they are, as I mentioned before, the paradigmatic case of logical object for Frege.

This assumption was backed by a tradition among respectable logicians and mathematicians like Boole, Schröder, and Dedekind of employing names of extensions as proper names in logical theories. (This tradition is later mentioned by Frege, e.g., in *GGA*, vol. 2, p. 253 [in 1902] and *KS* 339 [in 1910], as a partial explanation for his former confidence concerning the existence of classes and their primacy as logical objects.) And this assumption was reinforced by the sort of consideration tied to Frege's broader views about logic mentioned before, that is, with the fact that, in logical investigations, concepts are almost always considered via their representative objects, which are the extensions. Once numbers are defined as extensions, there can be no doubt any more that, say, the number 3 is not Julius Caesar, for the latter lacks those crucial features that make extensions paradigmatic cases of logical objects.

If this reasoning is correct, there is a very important disanalogy between the Julius Caesar problem and the problem of indeterminacy in *GGA* §10. The Julius Caesar problem, as it appears in *GLA*, arises only insofar as the logical nature of numbers is not uncontroversially established according to Frege's standards. The indeterminacy discussed in *GGA* §10 is a different matter, namely, the problem of how logical objects relate to one another. There is no question concerning their distinctness from objects of any other kind. It is incorrect, therefore, to interpret *GGA* §10 as taking up "unfinished business" (as Dummett [1991, p. 159] puts it) from *GLA*. To be sure, there is a sense in which Frege was completing the work of *GLA*; he was sharpening the notion of extension, describing extensions as value-ranges, explaining the relation between extensions and truth-values, and so on. But none of this was meant to take care of a persistent, unsolved version of the Julius Caesar problem.²⁸

VI. *GGA* §10 and Frege's Platonism

In their paper "Grundgesetze, Section 10," Moore and Rein (1986), raise some doubts concerning the compatibility of Frege's Platonism with his solution for the indeterminacy in *GGA* §10 by means of a *stipulation*. Their worry is best expressed in the following passage:

However, the means Frege adopts for eliminating the indeterminacy introduced into his system by the stipulation in Section 10 gives rise to another puzzle, which we shall simply record. Frege stipulates that each truth-value is to be identified with its unit class. But with what right does he do this? For Frege, the truth-values and the value-ranges inhabit a Platonic realm, whose existence and structure are quite independent of our capacity to apprehend them. Surely, then, it is a matter of objective fact whether or not each of the truth-values is a value-range and thus not something which can be decided by stipulation. In short, Frege's stipulation identifying the truth-values with their unit class appears to be in direct conflict with his avowed Platonism. (p. 383)

The worry expressed in the quoted passage is certainly legitimate. On some occasions Frege indeed gives the impression that the identity of value-ranges depends largely on stipulations. One of these occasions is the following remark

in the final paragraph of *GGA* §10, which I quote again: “With this we have determined the value-ranges as much as it is possible here. If there is a further question of introducing a function that is not reducible to the functions known so far, then only in this case we must determine which value it must have for value-ranges as arguments; and this can be seen as a determination of value-ranges as much as of the function itself.” When a new function is introduced in the system of *GGA*, the values that it assumes for value-ranges as arguments have to be fixed. By doing so, we are completing the work of determining what value-ranges are. It is tempting to interpret the remark as suggesting that the value the new function assumes for a value-range as argument is entirely a matter of convention. It seems to follow that the identity of logical objects largely depends on our choices, which would be incompatible with Frege’s Platonism.

As I said, this may be the first impression one gets from this passage, and this impression seems to be the basis of Moore and Rein’s puzzle. But the impression is misleading. Frege’s procedure in *GGA* §10 is backed by considerations that he does not make explicit in the book. If we put together some other remarks scattered throughout his writings, we can provide a context in which his procedure can be better appreciated. First of all, the basis of Moore and Rein’s puzzle is a misleading impression caused by the English translation of ‘*Festsetzung*’ as ‘stipulation’ in its occurrences in *GGA* §10. Although most of the time Frege employs ‘*Festsetzung*’ in this sense, there are a number of occasions on which the term is employed not in the sense of an arbitrary stipulation, but rather in the sense of laying down something or establishing a fact. For example, in “Über Begriff und Gegenstand” (*KS* 170) Frege describes his distinction between concept and object as a *Festsetzung*. Obviously he does not mean to imply that the distinction is the product of any arbitrary stipulation or convention. This would be completely at odds with his statement in the closing sentences of “Funktion und Begriff” (published just one year earlier) that his doctrine of functions is something “founded deep in the nature of things” (*KS* 142). Another example can be found in *GGA*, vol. 2, §146, footnote, in which Frege refers to Axiom V as a *Festsetzung*. Obviously he does not regard—not before 1903 at any rate—this axiom as an arbitrary stipulation, although he does show traces of doubt about its correctness sometimes. Other passages to this effect can be found, but a detailed discussion of them would take us too long here. The conclusion I want to draw from this evidence is that it is far from obvious that, when Frege talks about *Festsetzung* in *GGA* §10, he means something like a mere convention or a free stipulation, and not something that is forced upon us by logic.

We can better see how Frege’s procedure in *GGA* §10 is compatible with his Platonism by paying attention to his attitude toward some of the main assumptions of his logic, especially those present in his 1890s writings. There is some evidence that Frege regarded some of his fundamental theses about the ontology of logic as conjectures which should be tested like any scientific hypotheses (although they do not relate to the empirical investigation of the universe, but rather to its logical structure). Although logical laws are supposed to enunciate the general and most basic laws of thought and, thus, should be more self-evident than any other laws, the discovery of these laws is in Frege’s view not a trivial task. Indeed, the discovery of logical laws should be even more difficult if they imply, as they in fact do in his view, the existence of a special category of object. In other

words, the realm of logical laws and objects has to be progressively uncovered by a process of approximations to the truth, in which some conjectures are put to proof. The most reliable guide in this process of approximation to the truth is, as I shall discuss shortly, the fruitfulness of logical theories, rather than the self-evidence of logical principles. To illustrate this point, let us look at some occasions at which this attitude is manifested.

Frege seems to have regarded the assumption that sentences refer to truth-values (and, hence, that the latter are objects) as a hypothesis that was well confirmed by the successful development of *GGA*. As he explains in the introduction of the book (from 1893), an early version of *GGA* in which truth-values do not show up as objects had been prepared before. However, as he says now, the recognition of truth-values as objects led him to an almost complete rejection of the earlier manuscript. The reason is explained in the following passage: “Only a detailed consideration of this book can show how much simpler and clearer everything becomes with the introduction of truth-values. These advantages alone put already a great weight in the balance in favor of my conception, which admittedly may seem strange at first sight” (*GGA*, p. x). That is to say, Frege’s acceptance of the hypothesis that truth-values are objects was predicated on its fruitfulness in terms of simplicity, clarity, sharpness, and so on, and not on its intuitive appeal or self-evidence.²⁹

Another indication that Frege saw this fundamental assumption as essentially conjectural can be found in “Über Sinn und Bedeutung” (from 1892). Both in *KS* 150 and in *KS* 151, Frege describes the assumption that truth-values are the reference of sentences as a conjecture (*Vermutung*). In *KS* 151 he says that this conjecture should be “put to further tests.” As Tyler Burge comments in discussing this passage, the rest of the article can be seen as advancing a sequence of tests for this conjecture.³⁰

In “Über die Begriffsschrift des Herrn Peano und meine eigene” (from 1896), Frege welcomes Peano’s adoption of the sign ‘ \wedge ’ (which designates or represents “the absurd” in Peano’s notation) as an implicit “confirmation” (*Bestätigung*) of his own doctrine of the objecthood of the true and false (*KS* 225). A few lines later he suggests that both the ontological doctrine of regarding the true and false as objects and the semantic distinction between sense and reference are connected hypotheses, so that any consideration that supports the former “corroborates” (*bekräftigt*) the latter as well.

Another sign that Frege regarded many of his fundamental logical assumptions as conjectures is that he sometimes seems to voice doubts about them. For example, he shows some traces of doubt regarding Axiom V even before the discovery of Russell’s antinomy. In the preface of *GGA* he admits that, if any fundamental critical point should be raised about his system, it should be about Axiom V. (He declares it to be a logical law but admits that it is not clearly so.) In the appendix of *GGA*, vol. 2 (from 1902), written right after he took notice of Russell’s antinomy, Frege confesses his old doubts about Axiom V, since it appears to be less self-evident (*einleuchtend*) than a logical law would require. There is also an indirect piece of evidence that he might have had doubts regarding the definition of numbers as extensions even after the work of *GLA* was completed. In the catalog of Frege’s *Nachlass* prepared by Scholz, there is the register of a fragment produced immediately after the publication of *GLA* (listed as N 47), which, according to Scholz’s description, deals with an attempt to define ‘the number that belongs to the concept F’ without using extensions of concepts.³¹

Frege held a Platonistic view of the fundamental concepts involved in human thought, especially the basic concepts of mathematics. The features and boundaries of these concepts are independent of our knowledge and beliefs about them. This view was already part of Frege's philosophical framework in his very early writings, as the following passage from *GLA* nicely shows: "What is known as the history of concepts is really a history either of our knowledge of concepts or of the meanings of words. Often it is only after immense intellectual effort, which may have continued over centuries, that humanity at last succeeds in achieving knowledge of a concept in its pure form, in stripping off the irrelevant accretions which veil it from the eyes of the mind" (*GLA* vii). (A quite similar contrast between our knowledge of a concept and the concept in its purity appears in *KS* 122.) Part of the task of logical analysis of mathematics (or of any other science) is to uncover its concepts and their interrelations, showing the real boundaries and connections among them. As the passage above suggests, this may be a very long and laborious process. In the case of mathematics, some of the concepts to be uncovered are the most fundamental ones, like number, successor, infinity, continuity, and so on. Frege regarded the different philosophical accounts of these concepts previous to his own as rough approximations to the real conceptual network behind pure arithmetical thinking.

In "Boole's rechnende Logik und die Begriffsschrift" (from 1880–81), we find an indication of how, in Frege's view, we can tell which attempts to describe the conceptual framework of arithmetic are closer to the truth than others: "All these concepts were developed in science and have proved their fruitfulness. What we discover in them can, therefore, have a far greater claim on our attention than anything that the everyday course of thought might show. For fruitfulness is the touchstone of concepts, and scientific workshops are the suitable field of study for logic" (*NS* 36–37).

This passage strongly suggests that Frege saw an analogy between the work of the logician and that of the empirical scientist, since he talks about concepts being put to proof and about a "field of study" (*Beobachtungsfeld*) of logic. He seems to imply that different conceptual schemes are to be seen as different attempts to describe the true, and they have to be evaluated on the basis of the fruitfulness of their main concepts, that is to say, on the basis of the potential that these concepts have for unifying theories and for yielding new and interesting results (theorems).³²

I think that this early view of fruitfulness as the crucial test for the adequacy of conceptual schemes permeates the development of Frege's logicism between 1884 and 1903. Fruitfulness is the crucial test that many fundamental assumptions in Frege's system are supposed to pass. The only elements of his system that Frege does not seem to have treated as conjectures are the propositional and quantificational calculus that were already present in the *Begriffsschrift*, which corresponds to the fragment of *GGA* that is independent of Axioms IV, V, and VI.

Something similar can be said of the stipulations in *GGA* §10. They are meant, like other fundamental assumptions, to be conjectures and thus approaches to the truth. These stipulations are supposed to manifest their fruitfulness in the rest of the book. Frege's derivation of virtually the whole of arithmetic and analysis (including several interesting results about infinity) showed that the stipulations, taken together with the other fundamental assumptions, were fruitful and hence approximate the truth. As I am describing them here, Frege's *Festsetzungen* in *GGA*

§10 are indeed to be seen as hypotheses that involve a certain conjectural element. But the conjectural element involved here should be seen as that necessarily involved in the framing of any scientific hypothesis which purports to describe some kind of reality existing independently of the knowing subject. Frege's stipulations in *GGA* §10 are not just artifacts of technical convenience, nor are they arbitrary posits, nor are they self-evident. Rather, their justification lies in the technical advantages that they guarantee. This is, for Frege, the proper indication that they are true, or at least good approximations to the truth.

To sum up: in Frege's view, it is not open to free stipulation whether there are in the true conceptual basis of arithmetic; nor is it open to free stipulation whether or not the extension of any of these concepts is identical with the true or the false. These are objective matters. But it is open to conjecture—this is entirely consistent with his Platonism. Now, the only way to evaluate how good these conjectures are is by checking how fruitful they prove to be in the technical development of arithmetic. There is a strong indication of this attitude in the closing statement of the preface of *GGA*:

For the whole Part II [of *GGA*] is actually a test for my logical convictions. It is *prima facie* unlikely that such a construction could be erected on an uncertain, faulty ground. Whoever has different convictions can try to erect a similar construction based upon them, and he will realize, as I believe, that it does not work, or at least it does not work so well. And I could only recognize as a refutation if someone actually shows that a better, more stable building could be erected based on different fundamental convictions, or else that my basic laws lead to clearly false consequences. (*GGA*, p. xxvi)

The remark is generic and does not specify which of the fundamental “logical convictions” are to be justified by Part II (although, as I indicated, I think that Frege is referring to the portion of *GGA* that goes beyond the *Begriffsschrift* of 1879). So there is no reason not to suppose that the stipulations of *GGA* §10 are included here. I take Frege to regard the truth of

$$\dot{\varepsilon}(__e) = (\forall x)(x = x)$$

and of

$$\dot{\varepsilon}(\varepsilon = (\neg(\forall x)(x = x))) = \neg(\forall x)(x = x),$$

like other fundamental assumptions in his logical system, as well-supported conjectures. And, again, this is entirely compatible with his uncompromising Platonism.

Notes

I am indebted to Glenn Branch, Tyler Burge, Oswaldo Chateaubriand, Erich Reck, and Matthias Schirn for their valuable comments on previous versions of this essay. I am also indebted to the *Fundação de Amparo à Pesquisa do Estado de São Paulo* (FAPESP, Brazil) for the grant 91–0388/8 that made my research possible. The translations of the passages quoted from Frege's works in the text are mine (unless otherwise indicated). I consulted, however, the existing English translations that are listed in the bibliography. The differences between my translations and the existing ones are, most of the time, just a matter of emphasis. All pages indicated in quotations are from the German editions.

1. See bibliography for all abbreviations of Frege's works used in this essay.
2. Here, as elsewhere in this essay, I translate Frege's notation for universal quantification into modern notation.
 3. I will say a little more about the significance of this claim in the next section.
 4. I will discuss in some more detail later how free we actually are, in Frege's view, to choose any nonequivalent functions $f(x)$ and $g(x)$.
 5. This second fundamental step of *GGA* §10 is called by Schroeder-Heister the *identifiability* thesis (Schroeder-Heister, 1987, p. 69), and its legitimacy is supported, according to him, by the permutation argument.
6. Moore and Rein point out that, though it is correct to say that the different interpretations yielded by the permutations on the domain still model Axiom V, it is incorrect to say that they leave untouched the truth-value of every sentence of the formal system. For there are sentences in the system that are still undecidable (before Frege's final stipulations), such as $\dot{e}(\varepsilon = \varepsilon) = (\forall x)(x = x)$ (Moore and Rein, 1986, p. 379).
7. I will shortly address what appears to be a remaining case, namely, the case in which one of the terms of the identity is a value-range and the other is an ordinary name.
8. This point appears again in Dummett, 1991, p. 210.
9. This worry seems to be part of the motivation for the following remark in Frege's letter to Russell from August 3, 1902: "When we talk about the identity of functions, we can only mean the identity of their value-ranges or the identity of something that is univocally associated [*eindeutig verknüpft*] with value-ranges" (WB 226).
10. In my view a more appropriate place to look for an implicit appeal to the context principle is the following passage from *GGA*, vol. 2, §147: "If there are logical objects at all—and the objects of arithmetic are of such a kind—then there must be also a means for their apprehension and recognition. For this purpose the basic law above [Axiom V] is useful to us, since it allows the transformation of a generality into an identity. Without such a device the scientific justification of arithmetic would be impossible." In *GLA*, the context principle (originally stated in *GLA*, p. x) appears in §62 as an answer to the question "How are numbers to be given to us if we have no representation or intuition of them?" The principle is behind the formulation of Hume's principle, in which a generality is converted into an identity. The passage quoted above indicates that the same principle is behind Axiom V of *GGA*. The point both of Hume's principle and of Axiom V is to provide a means of recognizing the existence and identity of logical objects, given the fact that we have no perception or other mental representation of them.
11. E.g., in *GLA* §38, §57, §68 n. 1. These considerations appear over and over again in Frege's later work, e.g., in *KS* 127, *WB* 195, *GGA*, vol. 2, §100.
12. Here, as elsewhere in this essay, I quote from Austin's translation of *GLA*.
13. See note 11. I will not discuss in detail here the motivation and soundness of Frege's claim that numerals must be treated as singular terms.
14. Frege wants to deny what Kant asserts, i.e., that we cannot talk about numbers if we do not have a corresponding intuition or mental representation of them. Frege appears to impute to Kant a psychologistic twist derived from the ambiguity of his use of the term *Vorstellung*. I shall forgo here any discussion of whether or not Frege's understanding of Kant is correct.
15. There are some important qualifications to be made about these stipulations, which I will discuss in section VI.
16. Dummett presents a long, detailed discussion of Frege's goals and achievements in the arguments advanced in *GGA* §10 (Dummett, 1981, chap. 19). He sees an almost exact parallel between Frege's opening remarks in *GGA* §10 and the second complaint in *GLA* §56 about the first attempted definition of *GLA* §55 (which is repeated in *GLA* §66 in connection with the contextual definition of directions of *GLA* §§62–65). His discussion about the indeterminacy of the reference of names of value-ranges in *GGA*

§10 is tied to the question of which kind of objects are included in the range of first-order variables. Dummett finds it odd that, while Frege does not seem to place any restriction on the range of first-order variables, he does not attempt to specify the identity of value-ranges compared with other objects besides truth-values either. Dummett concludes his discussion with the skeptical remark that there seems to be no “determinate conception that can with any confidence be ascribed to him [Frege] of how abstract objects are to be introduced” (p. 421). Wright interprets Frege essentially along the same lines in his discussion of the indeterminacy of numbers (Wright, 1983, section 14). According to Wright, the only explicit reason motivating Frege’s definition of numbers as extensions is the elimination of the Julius Caesar problem, but an analogous problem still persists for extensions as objects. Section 10 of *GGA* can eliminate only part of the indeterminacy, but, Wright concludes, the problem is left essentially unsolved by the move (Wright, 1983, p. 113). Parsons raises what can be seen as a variation of the same problem of indeterminacy concerning numbers and classes (Parsons, 1965, section 5). The indeterminacy in the abstract notion of value-ranges is, according to Parsons, part of a more general problem for the introduction of abstract objects. According to Parsons, Frege cannot complete the specification of numbers without making some assumptions that could in principle be denied (Parsons, 1965, p. 160). He seems to interpret this as a trace of conventionalism in Frege’s procedure, which is incompatible with his realist point of view.

17. To see this, consider the function $x = x$: it yields the value true for any object as argument, while $x = (\dot{e} \ (e = e))$ is only true for $(\dot{e} \ (e = e))$ as argument, and false otherwise.

18. According to the explanation in *GGA* §11, if a is an extension under which only the object Δ falls, then the value of $\backslash a$ is Δ . That is to say, for those concepts that have only one instance, $\backslash a$ singles out this instance. If a is not an extension or if there is no unique object that falls under it, then the value of $\backslash a$ is simply a .

19. E.g., Dummett, 1991, chap. 17; Wright, 1983, section 14.

20. See Ruffino, 1996, chap. 1; 2000.

21. This is actually not an uncontroversial point in the Fregean scholarship. Schirn vehemently denies that Frege identifies these “objects of a special kind” with extensions (Schirn, 1990; 1996b, esp. pp. 156–58). Sluga denies it as well (Sluga, 1980, pp. 142–43). As a rule, the textual evidence presented by both Schirn and Sluga is weak. On the other hand, Parsons (1984) and Burge (1984) defend the view that I am embracing. As I argued in detail elsewhere (Ruffino, 1996, chaps. 1, 2, 4; 2000), the relevant textual evidence overwhelmingly supports the interpretation that Frege has extensions in mind as representative objects for concepts, although he is not explicit about it.

22. My discussion here only shows that truth-values have to be identified with some extensions if they are to be logical objects. But it admittedly does not show why they have to be identified with the particular extensions that Frege chooses. For an account of Frege’s particular choice, see Burge, 1986, esp. section 3.

23. As I said before, there is a weak and a strong version of this demand as applied to numbers in Frege. The weak version allows for the use of additional information. The stronger version does not allow the use of any other information besides the definition(s) and the laws of logic.

24. See note 16 above.

25. E.g., Dummett, 1981, p. 407; Schirn, 1996a, introduction, p. 12.

26. A primary task of logic is, as Frege repeatedly states (e.g., in *NS* 3–4, 133, 139–40, 272), the unfolding of the laws governing this function.

27. Frege’s restriction on the kinds of value-ranges that are to be counted as belonging to the ontological basis of *GGA* is not entirely surprising. A consideration that may have motivated this restriction is the following: the existence of the extension of an empirical (i.e., nonlogical) concept is necessary according to Frege, provided that this concept has sharp boundaries. But the identity of the extension depends on con-

tingent facts. For example, whether the extension of the concept *creature with a heart* is identical with the extension of the concept *creature with a kidney* is a matter of contingency, although the existence of both extensions is not contingent. The identity of the extension of logical concepts and relations, on the other hand, is not dependent on any contingency.

28. There is an intriguing passage in Frege's correspondence with Russell in which something is suggested along the lines that I discuss above. Answering Russell's question of how to distinguish value-ranges from ordinary objects, Frege replies: "You ask how could it be known that something is a value-range. Indeed a difficult point. Now all objects of arithmetic are introduced as value-ranges. As soon as a new object to be considered is introduced not as a value-range, one has to answer immediately to the question whether it is a value-range, and the answer is apparently always no, because it would have been introduced as a value-range if it were one" (*WB* 225). Frege seems to be suggesting in the last sentence of the quotation that the differentiation between value-ranges and other objects does not offer any great difficulty. But the passage is ambiguous in that, just a few lines earlier, Frege described Russell's question as a difficult one ("Allerdings ein schwieriger Punkt"). There is no clear indication, however, that Frege meant by "difficult point" anything more than the difficulty involved in the general philosophical question of when entities of a specific discipline (arithmetic, geometry, physics) qualify as logical. Anyway, this intriguing passage cannot be taken as definite evidence for my interpretation because the procedure that Frege here describes seems to be subject to the same worries Frege himself expresses in *GLA* §67. In the above passage Frege seems to assume that the form in which an object is given is an unchangeable property of it, which, according to himself, is false.

29. I develop this point further in Ruffino, 1997. See also Burge, 1998, for a detailed discussion of the justification for believing the axioms according to Frege's standards.

30. Burge, 1986, p. 104.

31. Veraart 1976, p. 95. This fragment was lost in World War II. As far as I know, Burge was the first to call attention to this fragment in this particular way (Burge, 1984, pp. 12–13).

32. For a more detailed account of fruitfulness and some of the most fundamental assumptions of Frege's ontology of logic, see Ruffino, 1997.

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Section 31 Revisited

Frege's Elucidations

Joan Weiner

Section 31 of Frege's *Basic Laws* is titled "Our simple names denote something [*bedeuten etwas*]."¹ Any sophisticated contemporary reader knows, in advance of reading §31, that there must be something wrong with at least one part of the argument. For one of the simple Begriffsschrift names is supposed to name a second-level function whose value, given a first-level function as argument, is the course-of-values of that first-level function.² And there can be no such function. The assumption that each function has a course-of-values is the source of the inconsistency of the logic of *Basic Laws*. The reader who knows this but has not yet read §31 will expect to find an apparently good argument that the Begriffsschrift course-of-values function name designates a second-level function; that is, an argument that *would be* perfectly good were Frege's assumption correct. But such a reader will be surprised. Instead of an apparently good argument, the reader will find an apparently circular argument that, circularity aside, depends on assumptions that seem inconsistent with Frege's explicit statements about the nature of functions.³ The puzzle that immediately confronts the reader is not so much a puzzle about where Frege went wrong as it is a puzzle about how Frege can have believed that his proof works.

Attempts to solve this puzzle have generated a substantial secondary literature.⁴ Although there are disagreements in this literature about what mistakes or confusions led Frege to formulate his peculiar argument, there is also a large area of consensus. There is consensus that §31 is meant to carry out the basis case of an inductive metatheoretic proof that all Begriffsschrift expressions have Bedeutung.⁵ And there is consensus that there are multiple confusions and mistakes in Frege's argument. I believe that, in both cases, the consensus view is mistaken. It is the first view—that §31 is meant to carry out the basis case of an inductive metatheoretic proof—that forces most philosophers who have written about §31 to

attribute confusions to Frege. I will argue that Frege's writings provide ample straightforward evidence that this view is incorrect: the argument in §31 is meant neither as a metatheoretic proof nor as the basis case of an inductive argument. In this essay I will offer an alternative reading that takes seriously, as the standard reading does not, Frege's explicit remarks about the nature of his project and his logic. I will end with an explanation of how this reading solves at least one of the exegetical puzzles in §31: On my reading, there is no circularity in the arguments of §31.

The issue, as I have described it so far, may seem to be one of significance only to those interested in the exegesis of a small, isolated piece of text, one that is familiar only to specialists. In fact, however, what is also at stake is how to understand Frege's conception of logic. On the reading that until recently has been standard, one of Frege's contributions to the contemporary understanding of logic was the introduction, not just of a formal language, but of a metatheory. The most unequivocal support for this reading is supposed to be found in the early sections of *Basic Laws*, which include §31, along with "On Sense and Meaning." I will argue that, far from providing unequivocal support, these texts actually undermine the metatheory reading. When we look at these traditional sources of the metatheory reading with an eye toward explaining section 31, we will see that they provide a powerful argument for rejecting the metatheory reading.

One of the salutary consequences of this rejection, as I will show, is that it allows us to make sense of some of the initially puzzling features of §31. Moreover, when we attempt to make sense of §31, we will be steered toward passages that provide an outline of Frege's conception of logic and its relation to language. Although this conception differs from those popular today, it is, in my view, neither archaic nor inferior to contemporary conceptions. As I have argued elsewhere, some of the problems that bother philosophers today are best addressed by taking into account some of Frege's neglected insights about language and logic.⁶

I. Section 31 and the Metatheory Reading

Although there are a number of statements in §31 that are likely to puzzle any reader, §31 presents special difficulties for advocates of the metatheory reading. One difficulty is that Frege's statement of what is required for the proof in §31 conflicts with what, according to the metatheory reading, should be required of such a proof. To see this, let us consider some central features of the metatheory reading. Michael Dummett writes: "For Frege, the reference of an expression is an extra-linguistic entity, and, in the informal semantics, or model theory, which has been developed from his ideas, an interpretation associates with each individual constant, predicate, etc., of the language a non-linguistic entity of a suitable type" (Dummett, 1978, p. 123). On this reading, Frege means to be setting out an intended interpretation for the formal logic in the early sections of *Basic Laws*. To hold that Frege is introducing a metatheory in these sections, we need not go so far as to say that the intended interpretation actually is a model theoretic interpretation of his formal language. Even Dummett writes only that model theory was "developed from his ideas." But to hold this view we do need to regard Frege's introduction of his logical language as something more than an explanation of what

its symbols mean, of how to understand the new language. As Dummett indicates, we need to regard the introductions of the symbols as associating them with extralinguistic entities.

Of course, the mere association of symbols with extralinguistic entities need not constitute part of a metatheory. One does not enter the realm of metatheory by naming the family pet. What makes such statements a part of a metatheory is their use in proofs. On the metatheory reading the proofs of *Basic Laws* are not limited to those expressed in *Begriffsschrift*. They include, also, proofs about the logical language and its interpretation. These are proofs in which statements about his symbols' referring to extralinguistic entities play a role; proofs involving an ineliminable use of a truth predicate for the logical language. Such proofs are stated in natural language. On this reading, the task of §31 is to set out part of one of these proofs.

The proof in question, according to the metatheory reading, is an inductive proof, found in §§28–31, that the intended interpretation associates appropriate entities with each expression of *Begriffsschrift*. In the earlier sections Frege proves the inductive step: that any complex term constructed from terms that are associated with appropriate entities will, itself, designate an appropriate entity. And the task of §31 is to prove the basis case: that each of his primitive terms is associated with an appropriate entity. Supposing the metatheory reading is correct, how would we expect Frege to carry out the task of §31?

It is important to begin by noting that all primitive *Begriffsschrift* expressions are function signs. *Begriffsschrift* has no simple proper names. Thus there are no stipulations that particular proper names name particular objects.⁷ Since the primitive *Begriffsschrift* expressions are function-names, Frege should assign, by stipulation, a function to each primitive function sign. The burden of §31 is, it seems, to show that these stipulations are successful. But one would not expect this to be a difficult task and, in particular, one would not expect this task to require a proof. All that seems to be required is an explanation, in everyday language, of which function is assigned to each primitive function sign. If, for example, one were to decide to introduce a primitive term for the successor function on the natural numbers, it would seem sufficient to stipulate that the term in question designates this function.⁸ Given the requisite stipulation, no further proof should be required (although, if we are careless in our choice of axioms, the axioms may turn out to be false under this interpretation).⁹ Thus there is something peculiar about the activity of offering a proof that the intended interpretation associates the appropriate sort of entity with each primitive *Begriffsschrift* name. What is the point of offering such proofs in §31?

One obvious answer is that an attempt at stipulation could go wrong. After all, the function I mentioned in the above example, the successor function, is rather different from the functions that Frege assigns to his primitive symbols. Frege might have worried (and with good reason) that there are problems with defining these peculiar functions—he might have worried, that is, that his description of the functions did not suffice to specify a unique value for each argument. The real worry, of course, is about Frege's attempt to describe a particular function: the second-level course-of-values function. There may be no function that satisfies Frege's description. Thus Frege seems to need an argument that his attempt at a natural language description of this function really does pick out a function. If it

does, then the Begriffsschrift second-level course-of-values function name really picks out a function. Moreover, there is some support for the view that the point of §31 is to take care of this problem. For, while Frege discusses all his primitive function names in §31, most of his attention is devoted to the second-level course-of-values function name. On this reading, the important proof is one case of a general proof that all primitive Begriffsschrift terms have Bedeutung. But can we read §31 as an attempt to prove that all primitive Begriffsschrift terms have Bedeutung in this way—that is, by proving that the everyday-language definitions of the functions these terms are to designate really do pick out functions?

There are several problems with such a reading. One of these is that the worry does not seem the sort of worry that can be addressed by metatheoretic proof. The issue, in fact, is not a metatheoretic issue at all. Frege has attempted to describe functions in natural language, and he has stipulated that these are named by certain Begriffsschrift expressions. But the worry is not about the stipulation. The worry is about his attempts at descriptions of functions. The issue is whether these attempts succeed. If so, then, of course, all his primitive Begriffsschrift terms have Bedeutung. If not, they do not. It is difficult to see how metatheory has anything to contribute.

Another problem with this sort of reading is that Frege's account of what must be shown seems incorrect. He writes: "In order now to show, first, that the function names '—ξ' and '——ξ' denote something [*etwas bedeuten*], we have only to show that those names succeed in denoting [*bedeutungsvoll sind*] that result from our putting for 'ξ,' the name of a truth-value (we are not yet recognizing other objects)" (BLA, vol. 1, §31). There are several puzzles here. One of these is that Frege does not talk about associating entities with his symbols. This might seem reasonable were function names the only names at issue in this passage. For, one might argue, the association of a function with a function name does not require talk about entities or functions—it requires only a definition of the function, that is, an indication of what values it has for each argument. But Frege is not talking only about function names in this passage. Some of the names mentioned in the above passage are object names: the results of putting a name of a truth-value for "ξ" in "—ξ" and "——ξ." On the metatheory reading, what needs to be shown of an object name is that the interpretation associates it with an *object*. But, as his surprising choice of words indicates—there is no indication in the German expression *bedeutungsvoll sind* that any object or relation between a name and object is involved—Frege does not acknowledge this.¹⁰

One might suspect that this is just an odd choice of words. But this choice of words is entirely in line with Frege's explanation, in §29, of when an object name denotes something. He writes:

A proper name has a denotation [*hat eine Bedeutung*] if the proper name that results from that proper name's filling the argument-places of a denoting name [*eines bedeutungsvollen Namens*] of a first-level function of one argument always has a denotation [*eine Bedeutung hat*], and if the name of a first-level function of one argument that results from the proper name in question's filling the ξ-argument-places of a denoting name [*eines bedeutungsvollen Namens*] of a first-level function of two arguments always has a denotation [*eine Bedeutung hat*], and if the same holds also for the ξ-argument-places. (BLA, vol. 1, §29)

There is no explicit mention that there must be objects that the proper names denote (or functions that the function names denote). Even worse, the explanation of when a proper name has Bedeutung seems circular. A proper name has Bedeutung, provided certain other terms—terms formed by putting the proper name in the argument place of a first-level function sign that has Bedeutung—do. So it seems that the explanation of when a proper name has Bedeutung is parasitic on the explanation (which appears in the previous paragraph of §29) of when a first-level function sign has Bedeutung. Yet that explanation is “A name of a first-level function of one argument has a *denotation* (*denotes something, succeeds in denoting*) [*hat dann eine Bedeutung (bedeutet etwas, ist bedeutungsvoll)*] if the proper name that results from this function name by its argument-places’ being filled by a proper name always has a denotation if the name substituted denotes something [*etwas bedeutet*]” (BLA, vol. 1, §29). Here Frege indicates that a first-level function sign of one argument has Bedeutung provided certain proper names do.

The circular character of this explanation is especially puzzling because, if Frege means to be giving an interpretation of his language that associates names with extralinguistic entities, it seems both unnecessarily complicated and incorrect.¹¹

Frege’s treatment of the second-level course-of-values function sign differs, in exactly the same way, from what the metatheory reading leads us to expect. On the metatheory reading, Frege ought to argue that his second-level course-of-values function sign is, by stipulation, a *Begriffsschrift* name for the function that, given a first-level function as argument, yields its course-of-values as its value. For although he is somewhat uncomfortable with this, Frege does assume in *Basic Laws* that each first-level function has a course-of-values. And this assumption is, from the contemporary point of view, enough to establish that the sign in question is associated with the appropriate sort of entity. Yet Frege does not appeal to this assumption in §31. Nor does he attempt to prove that his assumption is correct—either in §31 or anywhere else.

Does §31 contain a different sort of argument that there is a second-level course-of-values function? Frege argues that his second-level course-of-values function sign has Bedeutung because the proper names formed by completing it—that is, by filling in the argument place of this second-level function sign with a sign for a first-level function of one argument—satisfy the condition described in the above quotation.¹² The fact that Frege was uncomfortable with his assumption that each first-level function has a course-of-values does not explain this odd argument. Even if his discomfort prevented him from relying on this assumption, why would he have thought that the strategy employed in §31 could work?

The evidence of these sections in isolation is that, whatever Frege means by the interchangeable expressions ‘has a denotation [*hat eine Bedeutung*],’ ‘denotes something’ [*bedeutet etwas*] and “succeeds in denoting” ‘*ist bedeutungsvoll*’, it is not ‘*associated by the interpretation with the appropriate sort of entity*’. Thus §§28–31 present a number of difficulties for the metatheory reading. This is no reason, in itself, to reject the metatheory reading. After all, these are notoriously difficult sections. Moreover, there are other passages from the early sections of *Basic Laws* that appear to provide direct support for the metatheory reading.

One apparent source of direct support is Frege’s introduction of his generalized notion of function in §§1 and 2. Functions, given Frege’s generalized notion, include not only recognizable mathematical functions but also concepts and re-

lations. Concepts and relations are functions that take arguments to truth-values. Truth-values are introduced in §2, where he writes: “I say: the names ‘ $2^2 = 4$ ’ and ‘ $3 > 2$ ’ denote [*bedeuten*] the same truth-value, which I call for short *the True*. Likewise, for me ‘ $3^2 = 4$ ’ and ‘ $1 > 2$ ’ denote [*bedeuten*] the same truth-value, which I call for short *the False*, precisely as the name ‘ 2^2 ’ denotes [*bedeutet*] the number four. Accordingly I call the number four the *denotation* [*Bedeutung*] of ‘ 4 ’ and of ‘ 2^2 ,’ and I call the *True* the denotation [*Bedeutung*] of ‘ $3 > 2$ ’”. In this passage Frege seems to be saying explicitly that, at least for expressions that are proper names (among which are sentences), having Bedeutung amounts to being associated with an extralinguistic entity. In a footnote he refers his readers to an earlier essay, “On Sense and Meaning,” a work in which he makes similar remarks about Bedeutung.

The supporter of the metatheory reading might argue that these passages show that the problem with §31 is not a problem with the metatheory reading; it is a problem with Frege’s discussions in §§28–31 of *Basic Laws*. It is true that Frege’s answer to the question “When does a name denote [*bedeutet*] something?” in §29 involves no allusion to extralinguistic entities. Yet, in §§1 and 2 Frege does seem to indicate that to have Bedeutung simply is to be associated with the appropriate sort of extralinguistic entity. The metatheory reading, one might argue, is not a *reading* of the above passage; it is simply a statement of what Frege explicitly says.

But this argument is too quick. It is central to the metatheory reading that statements about Bedeutung play a particular role in Frege’s project—that they are used in actual proofs. And the explicit statements from §§1 and 2 of *Basic Laws* do not go this far. The supporter of the metatheory reading might respond that these sections are really the wrong place to look for a detailed explanation of the role that the notion of Bedeutung plays in Frege’s project. As a footnote in §2 suggests, the reader who wants a more complete explanation of this notion must turn to the work in which it is introduced, “On Sense and Meaning.” Indeed, it is this paper that, on metatheory reading, sets out the background against which the early sections of *Basic Laws* must be understood—it is this paper that sets out the beginning of Frege’s theory of reference. Moreover, some of Frege’s comments in “On Sense and Meaning” have been taken to show why he felt a need for metatheoretic proof. Before we turn to the actual details of §31, it will help to begin with an examination of these earlier writings. Let us consider the apparent need for metatheoretic proof that is supposed to come out in “On Sense and Meaning.”

II. “On Sense and Meaning” and the Metatheory Reading

Although there is no explicit statement in “On Sense and Meaning” about the need for metatheoretic proofs, the supporter of the metatheory reading may argue that the need follows almost immediately from something that Frege does say explicitly. Frege writes: “A logically perfect language [*Begriffsschrift*] should satisfy the conditions, that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object, and that no new sign shall be introduced as a proper name without being secured a meaning [*Bedeutung*]” (Frege, 1892b, p. 41). He goes on to say: “The logic books contain warnings against logical mistakes arising from the ambiguity of expressions. I

regard as no less pertinent a warning against apparent proper names having no meaning [*Bedeutung*]. The history of mathematics supplies errors which have arisen in this way" (Frege, 1892b, p. 41). One might well expect this issue to come up in *Basic Laws*, which is, after all, a logic book. And it does. In *Basic Laws*, Frege explicitly identifies as a basic principle "that every correctly formed name denotes something [*etwas bedeuten*]" (BLA, vol. 1, p. xii).¹³ Moreover, if this is a basic principle, and a principle that has been violated both in logic books and in the history of mathematics, one might expect Frege to offer proof that he is not, himself, violating the principle. Michael Resnik writes:

No methodological principle was more important to Frege than the one at stake in these passages: *in a properly constructed scientific language every name (including function-names as well as object-names) must have a reference.* In his eyes the repeated failures of his fellow mathematicians to be certain of satisfying this tenet was one of their most grievous errors. Thus it was entirely in keeping with this that he proved that every name in his own system has reference. (Resnik, 1986, p. 177).

If Resnik is right about this, then the above passages from "On Sense and Meaning," as well as the passages from the first two sections of *Basic Laws*, seem to tell us what is required of such proofs. For these passages seem unambiguously to support the reading on which to say that a proper name has *Bedeutung* is to say something about a relationship between that name and an object.

But there are problems for Resnik's claim. We have already seen one of these. Frege simply does not do what he ought to do: he does not attempt to show that each primitive name is correlated with an extralinguistic entity. There is also another serious problem. The views from "On Sense and Meaning," along with Frege's oft-stated comments about primitiveness, commit him to denying that there is any way to give the sort of metatheoretic proof that, according to Resnik, Frege wants to give.¹⁴

To see this, we need to begin by looking more closely at Frege's formulation of the worry that the principle is supposed to allay. The first formulation of the principle appears in a passage about a page earlier than the passage Resnik quotes. Frege writes: "Now languages have the fault of containing expressions which fail to designate an object (although their grammatical form seems to qualify them for that purpose) because the truth of some sentence is a prerequisite" (Frege, 1892b, p. 40). There is an important difference between this passage and the later passage that Resnik quotes. Frege's worry, as he describes it in this passage, is not simply that languages contain proper names that fail to designate objects. It is, rather, that languages contain proper names that fail to designate objects *because the truth of some sentence is a prerequisite*. The example that Frege uses to illustrate the problem is the following sentence:

Whoever discovered the elliptic form of the planetary orbits died in misery.

The problem arises with the expression "whoever discovered the elliptic form of the planetary orbits." Whether or not this expression designates some object, Frege says, depends on whether or not the following sentence is true:

There was someone who discovered the elliptic form of the planetary orbits. (Frege, 1892b, p. 40)

But, assuming the sentence *is* true, why is there a problem? One reason, Frege tells us, is that if this is so, the denial of the original sentence is not,

Whoever discovered the elliptic form of the planetary orbits did not die in misery

but, rather,

Either whoever discovered the elliptic form of the planetary orbits did not die in misery or there was nobody who discovered the elliptic form of the planetary orbits.

In other words, the natural language sentence contains a tacit presupposition: that there was someone who discovered the elliptic form of the planetary orbits. This, Frege says, arises from an imperfection of language—an imperfection that is to be avoided in a logically perfect language.¹⁵

But why is this to be avoided in a logically perfect language? The answer lies in Frege's view of the purpose of logically perfect language. When he first introduces his logical notation in *Begriffsschrift*, Frege characterizes it as a tool invented for "certain scientific purposes," and, he adds, "one must not condemn it because it is not suited to others" (BEG, p. 6/BS, p. xi). The purposes for which it is not suited are the everyday uses that we make of natural language. What are the scientific purposes for which *Begriffsschrift* is suited?

Frege's *Begriffsschrift* is introduced as a tool for expressing proofs that can play a particular sort of role: proofs that give us "a basis upon which to judge the epistemological nature of the law that is proved" (BLA, vol. 1, p. vii). The particular laws with which Frege is concerned are, of course, the laws of arithmetic. He wants to show that the truths of arithmetic are analytic. The enterprise is, as he characterizes it in *Begriffsschrift*, to show that one can prove the truths of arithmetic "with the sole support of those laws of thought that transcend all particulars" (BEG, p. 5/BS, p. x). It is, as he says in the preface to *Basic Laws*, to show "that arithmetic is a branch of logic and need not borrow any ground of proof whatever from either experience or intuition" (BLA, vol. 1, §0). One problem with using natural language proofs to show this is that tacit presuppositions on which these proofs depend may require some ground either from experience or from intuition. Frege says that it is "precisely the presuppositions made tacitly and without clear awareness that obstruct our insight into the epistemological nature of a law" (BLA, vol. 1, §0). It is thus of the utmost importance that the logical proofs of the laws of arithmetic be stated in a presuppositionless language.

Hence the "first purpose" of the *Begriffsschrift*, as Frege says when he introduces it, is to prevent presuppositions from sneaking in unnoticed (BEG, p. 6/BS, p. x). The *Begriffsschrift* expression of each of his propositions, Frege says in *Basic Laws*, explicitly expresses "all of the conditions necessary to its validity"; there can be no "tacit attachment of presuppositions in thought" (BLA, vol. 1, p. vi). This explains the nature of Frege's worry in "On Sense and Meaning." It is, as Resnik claims, a problem if a logical language contains proper names that do not have Bedeutung. For a logically perfect language must not contain proper names with no Bedeutung. But the problem that Frege finds especially worrisome is that it might contain proper names whose having Bedeutung depends on the truth of

a thought. The discussion from “On Sense and Meaning” ends with the claim that, after the introduction of a logically perfect language, “Then such objections as the one discussed above would become impossible, because it could never depend upon the truth of a thought whether a proper name had meaning” (Frege, 1892b, p. 41, my emphasis).

This does not, in itself, undermine the metatheory reading. For Frege’s concern here is not to prohibit presuppositions of the sort expressed in metatheoretic statements. For example, Frege does not mean to prohibit the presupposition that a simple proper name designates something. Indeed, he writes, “If anything is asserted there is always an obvious presupposition that the simple or compound proper names used have meaning [*Bedeutung*]. If therefore one asserts ‘Kepler’ died in misery,’ there is a presupposition that the name ‘Kepler’ designates something; but it does not follow that the sense of the sentence ‘Kepler died in misery’ contains the thought that the name ‘Kepler’ designates something” (Frege, 1892b, p. 40). There are, however no such proper names in the *Begriffsschrift* of *Basic Laws*. These proper names are all complex. And the presuppositions that worry Frege, as his example indicates, are presuppositions that complex proper names have *Bedeutung*. Moreover, as we shall see, the sort of thoughts that worry Frege here—thoughts on which a complex proper name’s having *Bedeutung* might depend—are not metatheoretic at all. The problem for the metatheory reading arises, oddly enough, because Frege’s worry about course-of-values names is not really a metatheoretical worry.

III. “On Sense and Meaning” and Course-of-Values Names

If we take these remarks seriously, we have a partial explanation of one of the mysteries of §31. As we saw earlier, it seems mysterious that Frege does not appeal to the claim that each function has a (unique) course-of-values in his attempt to justify his claim that his second-level course-of-values function sign has *Bedeutung*. As the above discussion indicates, Frege thinks that whether or not a proper name of *Begriffsschrift* designates something cannot depend on the truth of a thought. In particular, then, whether or not a proper name of the form “ $\dot{e}f(\varepsilon)$ ”—that is, a name formed using the course-of-values function sign—designates something cannot depend on the truth of the claim that each function has a (unique) course-of-values.¹⁶

But there are respects in which this explanation is unsatisfying. One of these is that it may seem that Frege is simply wrong: whether or not the proper names in question designate objects *does* depend on the truth of the thought that each function has a (unique) course-of-values. In fact, however, this is far from obvious.

It helps to note, first, that we would not be inclined to say that whether or not a proper name of the form “the gobbledegook of $f(x)$ ” has *Bedeutung* depends on the truth of a thought: that each function has a gobbledegook. Or, at least, we would not be inclined to say this without first being told what it is to be a gobbledegook of a function. For, unless we have assigned an appropriate meaning to ‘gobbledegook’, the string of words ‘each function has a gobbledegook’ is not a sentence that has a truth-value. It is, rather, exactly the sort of nonsense it appears to be. But now consider the expression ‘each function has a course-of-values.’ Although ‘course-of-values’ is not known as a nonsense expression, it is also not a familiar

expression. What reason have we for thinking that the string of words set off above expresses a thought? The answer has to come from an account of the meaning of the term ‘course-of-values’. Frege’s introduction of the notion of course-of-values in §3 *Basic Laws* is:

I use the words

“the function $\phi(\xi)$ has the same *course-of-values* as the function $\psi(\xi)$ ” generally to denote the same as [*als gleichbedeutend mit*] the words

“the functions $\phi(\xi)$ and $\psi(\xi)$ have always the same value for the same argument.”

This equivalence, when stated in *Begriffsschrift*, is Frege’s Basic Law V. It also is, as he acknowledges in §10, insufficient to fix the sense of the term ‘course-of-values.’ But even in §10, which is titled “The course-of-values of a function more exactly specified,” he does not attempt to explain what he means by “course-of-values.” Rather, the section is devoted almost entirely to an argument that it can be determined “for every function when it is introduced, what values it takes on for courses-of-values as arguments” (BLA, vol. 1, §10)

How are we to know, then, what sorts of things courses-of-values are? Although Frege says very little about this in the first volume of *Basic Laws*, in volume II he takes up the issue again.¹⁷ He begins by rehearsing the introduction from §3 of volume I:

We said: If a (first-level) function (of one argument) and another function are such as always to have the same value for the same argument, then we may say instead that the course-of-values of the first is the same as that of the second. (BLA, vol. 2, §146)

As before, he claims that this must be regarded as a fundamental law of logic. However, in this discussion, unlike the earlier discussions, Frege gives us some information about why this should do as an introduction of the notion of course-of-values. He indicates that he is only appealing to our antecedent understanding that this is so. The transformation of the equality holding generally into an identity, he says, is implicitly exploited all the time both by logicians and mathematicians (BLA, vol. 2, §147).

But this is not a sufficient reason for not defining ‘course-of-values.’ After all, Frege insists on defining the concept of number, a far more familiar notion than that of course-of-values. Why does he not demand a definition of ‘course-of-values’? This, Frege suggests, is simply not possible. It is not just that, as he says, the above equivalence must be regarded as a fundamental law of logic. He also says: “This transformation must not be regarded as a definition; neither the word ‘same’ or the *equals* sign, nor the word ‘course-of-values’ or a complex symbol like ‘ $\epsilon \phi(\xi)$ ’, nor both together, are being defined by means of it” (BLA, vol. 2, §146).¹⁸ Although there is a technical reason why the transformation must not be regarded as a definition, there is also another reason. He adds, in a footnote, “In general, we must not regard the stipulations in volume I, with regard to the primitive signs, as definitions. Only what is logically complex can be defined; what is simple can only be pointed to [*hinweisen*]” (§146). The second-level course-of-values function sign is, of course, one of the primitive signs in Frege’s logical language. Thus the second-level course-of-values function sign cannot be defined.

How seriously should we take this answer? One might suspect that Frege is expressing an ad hoc view; a view introduced in the hope of avoiding awkward questions about the primitive second-level course-of-values function sign. However, that would be a mistake. The general view that primitive terms, or terms for something logically simple, cannot be defined appears throughout Frege's writings. He says this not only in the second volume of *Basic Laws*, but also in the first volume and also in numerous papers published both before the first volume and after the second.¹⁹

IV. Primitive Terms and Elucidation

One of the earliest of these statements is in “On Concept and Object.” Frege writes that what is simple cannot be defined and that, when introducing something logically simple, the only option is to “lead the reader or hearer, by means of hints, to understand the word as is intended” (Frege, 1892a, p. 193). The topic is also taken up in §0 of volume 1, where Frege explains the task of *Basic Laws*. He writes: “It will not always be possible to give a regular definition of everything, precisely because our endeavor must be to trace our way back to what is logically simple, which as such is not properly definable. I must then be satisfied with indicating what I intend by means of hints [*Winke*]” (BLA, vol. 1, §0). The notion of course-of-values, of course, is logically simple. And we have already seen that the sorts of hints by which Frege introduces this notion include no account of the nature of courses-of-values. We are given only some simple logical truths about courses-of-values. It seems to be Frege’s view that it is not possible to state what it is to be a course-of-values.

One upshot is that we can see why Frege might think that his *Begriffsschrift*, although flawed, is not open to the sort of objection he discussed in “On Sense and Meaning.” As we have seen, Frege claims that a logically perfect language will not be subject to such objections. For whether or not a proper name of a logically perfect language has *Bedeutung* cannot depend on the truth of a thought. But why should Frege think this? To answer this, consider the claim that proper names formed using the course-of-values function sign do have *Bedeutung*. Are there thoughts upon whose truth this claim depends? It would seem that the only candidates for such thoughts are those expressed by statements of what it is to be a course-of-values. And there can be no such thoughts.

But here, again, one might suspect that—even if this is Frege’s view—it is simply wrong. It is true that he does not supply an account of the nature of courses-of-values that can be viewed as expressing a thought. And, consequently, he offers no thought of this sort whose truth is required for certain proper names to have *Bedeutung*. However, one might suspect that there is a thought of another sort whose truth is required. One might think that Basic Law V is such a thought. After all, Frege introduces his course-of-values function sign with a natural language expression of Basic Law V. And one might think that the result of Russell’s proof of the contradiction also provides a demonstration that this thought is false.

But is there such a thought? It seems more accurate to say that Frege simply did not succeed, in *Basic Laws*, in assigning fixed content either to the (invented)

natural language expression ‘course-of-values’ or to the Begriffsschrift second-level course-of-values function sign. And this seems to have been Frege’s considered, postcontradiction view.²⁰ If so, the natural language expression that Frege translates into Begriffsschrift as Basic Law V does not express a thought at all. Whether or not a Begriffsschrift course-of-values name—that is, a proper name of the form “ $\dot{e}f(e)$ ”—has Bedeutung does not, then, depend on the truth of the thought that each function has a course-of-values. For there is no such thought.

We can now see why, on Frege’s view, there should be no way to give the sort of metatheoretic proof that, according to Resnik, Frege wants to give—that is, a metatheoretic proof that guarantees that all primitive terms have Bedeutung. The problem is simply that the primitive terms are terms for what is simple. Logically simple notions, Frege maintains in “On Concept and Object,” as well as in both volumes of *Basic Laws*, cannot be defined but only indicated by means of hints. Frege writes, in the second volume of *Basic Laws*, “In general, we must not regard the stipulations in Vol. i., with regard to the primitive signs as definitions. Only what is logically complex can be defined; what is simple can only be pointed to [*hinweisen*]” (BLA, vol. 2, §146). Moreover, in the discussions of primitive terms, Frege is not simply talking about primitive Begriffsschrift terms. Frege’s natural language expression ‘course-of-values’, as well as the second-level course-of-values function sign, is supposed to be a term for something logically simple and hence something that cannot be defined. Nor is the problem restricted to the simple notions of logic. Although Frege’s general project is to identify the foundations of arithmetic, he believes that such foundational enterprises are important for other sciences as well. One of the tasks involved in identifying the foundations of a science is to identify the primitive notions from which all its other concepts can be defined.

In a series of papers written shortly after the second volume of *Basic Laws*, Frege discusses a different science, geometry. Here, also, he considers the issue of what is necessary to introduce the primitive terms of the science. In any science, he claims, we need to make sure that we have the same understanding of our primitive terms. Since definition is not possible, the understanding is to be reached by figurative modes of expression, which he now calls “elucidations.” The task of these elucidations is to “make sure that all who use them henceforth also associate the same sense with the elucidated word” (Frege, 1906, p. 302). But because of the nature of elucidation, as Frege also acknowledges, there is no guarantee that it will be successful. He says, “we must have confidence that such an understanding can be reached through elucidation, although theoretically the contrary is not excluded” (p. 301).²¹ And, “we must be able to count on a little goodwill and cooperative understanding, even guesswork” (p. 301).

Of course, the originator of the elucidation must, Frege claims, “know for certain what he means” and “remain in agreement with himself” (Frege, 1906, p. 301). But Frege does not say how this certainty is to be achieved. And the evidence, as Frege surely knew when he wrote these words, is that one can go wrong even here. For Frege had already realized that he had gone wrong in his attempt to achieve this certainty with respect to the notion of course-of-values. Moreover, it is evident that, even when he was writing the first volume of *Basic Laws*, Frege was unconvinced that the hints by which he introduced his notion of course-of-values were sufficient.

V. Elucidation and Section 31

What does all this tell us about the official project of §31: the attempt to show that “Our simple names denote something [*bedeuten etwas*]”? Given this understanding of Frege’s introduction of the primitive terms of Begriffsschrift, there can be no proofs that the simple names designate anything. For the stipulations by which these terms are assigned meaning must be regarded as hints. And such hints cannot provide any guarantee that these terms do designate anything. Nor can hints figure as premises of proofs that these terms designate something. The moral, it seems, is that the arguments and discussions of §31 must have the status of elucidation rather than proof.

This is not the only reason for thinking that §31 must have the status of elucidation. Frege’s statements about the notions of concept and function give us another reason. Concepts and functions are unsaturated. He writes, “For not all the parts of a thought can be complete; at least one must be ‘unsaturated’ or predicative; otherwise they would not hold together” (Frege, 1892a, p. 205). One consequence is that what can be said about objects cannot be said about concepts and functions.²² In particular, as Frege writes in a letter to Russell, “We cannot properly say of a concept name that it means something [*dass er etwas bedeute*]” (Frege to Russell, June 29, 1902). And he continues, “strictly speaking, the expression ‘the meaning of a function name’ [*die Bedeutung eines Functionsnamens*] must not be used.” But Frege’s simple names are all function names. And the something that a name denotes is its Bedeutung.²³ It follows, then, that we cannot properly say of these names that they denote something. That is, the title of §31 “our simple names denote something [*bedeuten etwas*]” cannot be a statement of a literal truth to be proved in that section.

It may seem, at first, as if Frege must simply have made a mistake. Surely, one might think, Frege would not say that his simple names denote something unless he meant us to take this statement as expressing a literal truth. Yet Frege often does write sentences that he does not mean us to take as expressing literal truths. Although he does not explicitly say that the title of §31 is one of these, he does say, of other sentences that appear in his writings, that they cannot be regarded as statements of literal truths. He writes, in “On Concept and Object”: “By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me half-way—who does not begrudge a pinch of salt” (Frege, 1892a, p. 204). The expressions that, taken literally, miss his thought are some of the expressions (for example “concepts are unsaturated”) that Frege uses to introduce his notion of concept. The comment about relying on the reader who will meet him halfway is echoed in a statement from a later series of papers, where he writes, “For the decomposition into a saturated and an unsaturated part must be considered a logically primitive phenomenon which must simply be accepted and cannot be reduced to something simpler. I am well aware that expressions like ‘saturated’ and ‘unsaturated’ are metaphorical and only serve to indicate [*hinzuweisen*] what is meant—whereby one must always count on the cooperative understanding of the reader” (Frege, 1903b, pp. 371–72). Later in the same series, Frege says that, for elucidation to be successful, goodwill and cooperative understanding, even guesswork, may be

required. The expressions in question are meant to play an elucidatory role. And it is evident from Frege's remarks that this role can be played successfully even by expressions that cannot be regarded as statements of literal truths. Can the same be said of the title and many of the sentences of §31?

There is an obvious difference between the statements that appear in the arguments of §31 and the remarks about the nature of concepts that appear in Frege's papers. The latter are attempts to introduce new terms; they appear in informal discussions, and, when they appear, Frege hastens to disavow them as statements of literal truths. The arguments of §31, in contrast, are not attempts to introduce new terms. Moreover, these arguments look like actual proofs. And, while it is a consequence of some of Frege's remarks that the sentences of §31 do not express literal truths, this is not something that Frege explicitly acknowledges in §31. One might infer that this shows that Frege did not notice the consequence—that Frege regarded himself as proving literal truths in §31. But this inference is not warranted. To see why, it will help to consider Frege's notion of elucidation, as well as some of his actual elucidations, in more detail.

First, the introduction of primitive terms is not the only purpose for which elucidation can be required. Frege also characterizes elucidation as something belonging to the propaedeutic of a science and something that helps in "the communication of science to others" (1906, p. 301). Second, Frege seems to have regarded as elucidatory some of the sections of *Basic Laws* that appear to contain actual proofs. Frege divides the sections in part 2 of volume 1 of *Basic Laws* into those headed "Analysis" (*Zerlegung*) and those headed "Construction" (*Aufbau*). The construction sections are those that, Frege claims, contain his actual proofs—these proofs are expressed entirely in *Begriffsschrift*; they contain no German words at all. The Analysis sections have a different purpose. They "are meant to facilitate understanding by providing rough preliminary outlines of the proofs that follow them" (BLA, vol. 1, p. v). The role described fits the above characterization of elucidation. Other remarks in *Basic Laws* also indicate that he regards these sections as elucidatory.²⁴

Of course, the statements in the Analysis sections do not miss Frege's thought. They do seem to be statements of literal truths. Indeed, many of them are simply natural language statements of logical laws. But this is no reason to deny that they are meant as elucidations. For Frege's elucidations are by no means limited to expressions that miss his thought. In fact, even his discussions of the notion of concept include statements of literal truths. It is part of Frege's understanding of concepts as functions that each concept holds or not of each object. And this claim, far from missing his thought, being false or being nonsensical, is a natural language statement of a logical law. In a logical language of the sort Frege introduced, it can be expressed using the following sort of formula:

$$(F)(x)(F(x) \vee \neg F(x)).$$

This statement is elucidatory, insofar as it is used to communicate Frege's understanding of the term 'concept.' But it is also a natural language statement of a literal truth of the logical theory. It is also a statement that can be expressed in *Begriffsschrift*. Similarly, as we saw earlier, Frege uses what he takes to be a natural language statement of a literal truth of the logical theory (Basic Law V) in his introduction of the notion of course-of-values. We cannot, then, recognize a state-

ment as elucidatory from its content. Nor can we disqualify a discussion as elucidatory because it appears to set out a proof. Elucidation is recognizable by the role it plays in a science.

The analysis sections are not the only examples of elucidation that takes the form of proof. Frege explicitly identifies another discussion in *Basic Laws* as elucidatory. And this discussion, which appears in §34, has almost exactly the character of §31. In §34 Frege introduces a definition—the definition of his application-function sign. Immediately after introducing this definition, he says, “although the definiens contains only familiar notation, a few elucidations [*einige Erläuterungen*] are in order” (BLA, vol. 1, §34).²⁵ What follows is an elaborate argument that the function just defined has Bedeutung—an argument very like the argument in §31. It is, indeed, just the sort of discussion that is typically identified, on the metatheory reading, as a metatheoretic proof. Frege considers what is named by the complex object names that, according to his definition, result from inserting proper names in the argument spaces of the function-expression. He divides the possibilities into cases and argues that, for each case, the resulting object-expression will have a Bedeutung. The next section begins with the words: “Here we see confirmed, what we could gather from considerations already set out, that the function name ‘ $\xi \cap \zeta$ ’ has a denotation [*eine Bedeutung hat*]” (§35). As with the §31 arguments, the apparent conclusion of the §34 argument—that a particular function name has a Bedeutung—is, according to Frege’s remarks about the nature of functions, strictly speaking not something that can properly be said. But, as Frege’s next sentence indicates, he does not in fact mean us to take his elucidation as a proof whose conclusion is that the function name in question has a Bedeutung. For he continues: “Only this is fundamental to the forthcoming conduct of proofs; our elucidation could be wrong in other respects without placing the correctness of those proofs in question; for only the definition itself is the foundation for this edifice” (§35). The §34 argument, then, although it cannot be correct (because of the sort of claim that is made about a function) *need not* be correct. It is elucidatory. It is designed to get Frege’s readers to understand his application-function sign.

What does this tell us about §31? It is clear from a comparison of the arguments in §34 and §31 that nothing about the way §31 is written disqualifies it from playing an elucidatory role. Just as the arguments in §31 look like proofs from our perspective, the arguments in §34 look like proofs from our perspective. This also explains some of the remarks in the preface to *Basic Laws* that are surprising if we assume that §31 is meant to include proofs. Frege suggests, in the preface, that the reader skip some sections on a first reading. For, he says, “Some matters had to be taken up in order to meet all objections, but are nevertheless inessential to an understanding of the propositions of *Begriffsschrift*” (BLA, vol. 1, p. xii). Section 31 is among the sections that should be skipped for this reason. But he also suggests that the reader return to the sections after working through the book the first time. The reason, he adds, is that “In this way, I believe, the suspicion that may first be aroused by my innovations will gradually be dispelled [*So wird, glaube ich, das Misstrauen allmählich schwinden*]” (p. xii). This is odd as a characterization of what is to be accomplished by proof—it is much too weak. But it is exactly the sort of characterization one would expect for sections that contain elucidations, which, on Frege’s view, offer us no guarantees.

I claimed earlier that one result of a careful examination of §31 will be to undermine the metatheory reading. Nonetheless, there are many features of the metatheory reading with which I am in complete agreement. There is no question that, in the discursive sections of *Basic Laws*, Frege is writing about his language, Begriffsschrift—and no question that he is writing about its terms' having Bedeutung. Indeed, there is no question that Frege's aim, in these sections, is to convince his readers that his logical language does what it is supposed to do—in particular, that the language will not leave us with expressions that do not designate anything. The problems with the metatheory reading are problems with the status it accords the discussions in these sections. One problem is the insistence that what is required, on Frege's view, is actual proof. Another is the insistence that these proofs have a particular character. I have argued that what is required, on Frege's view, is not proof but elucidation. Above, I have concentrated on the first of these problems. I will turn, next, to the second.

VI. Section 31 and Circularity

My argument, so far, comes largely from an examination of Frege's general views. It is a consequence of his views about primitive terms that there are no premises to be used in proofs that the primitive Begriffsschrift terms have Bedeutung. It is a consequence of his views about the nature of functions that the apparent conclusions of such proofs are (as Frege says to Russell) imprecise expressions. It is not consistent with these views to take §31 as providing proofs that the primitive Begriffsschrift terms have Bedeutung. There is, however, another option. The purpose of §31 may be elucidatory. And there is nothing in Frege's account of the notion of elucidation that prevents us from taking the arguments of §31 as elucidatory.

But to say that §31 is meant to be elucidatory does not yet tell us how the elucidations of §31 are supposed to work or why Frege might have thought his readers would find them convincing. One of the problems that I have not yet addressed, the problem about the apparent circularity of the §31 arguments, is particularly vexing. This problem seems so obviously to undermine Frege's argument—whether or not it is regarded as elucidatory—that it is difficult to see how Frege could have thought that anyone would be convinced. Even if, as Frege claims, flaws in an elucidatory argument are acceptable, it is not as if any words will do, provided they are labeled 'elucidatory'.

I will try next to show that it is a mistake to regard Frege as offering an obviously bad or circular argument here. It may seem, however, that to engage in this sort of activity is to abandon the force of regarding the argument as elucidatory. If there is a perfectly good argument in §31, why not call the argument a metatheoretic proof? One answer is that Frege's elucidatory arguments cannot count as proofs because each is based on some sort of improper statement. But, as I will try to show below, there is also another answer, one that has to do, not with elucidation, but with the notion of metatheoretic proof. The metatheory reading does not tell us simply that Frege is offering proofs about the functioning of his logical language. The metatheory reading tells us that Frege is giving proofs of a certain sort. And one of the problems with the metatheory reading is that, even if we regard Frege as giving natural language proofs about the functioning of his logical

language, the proofs in question are not the proofs that, according to the meta-theory reading, Frege needs to give.

Let us return to the apparent circularity of §31. In §29, titled “When does a name denote something [*Wann bedeutet ein Name etwas?*]”, Frege gives a six-clause answer (one clause for each type of Begriffsschrift expression). He tells us, however, that this answer should not be taken as an account of what it is to have Bedeutung. For, as he says in the next section, the application of this six-clause explanation “always presupposes that we have already recognized some names as denoting [*als bedeutungsvolle*]” (BLA, vol. 1, §30). Such a claim seems perfectly reasonable, of course, if the explanation offered in §29 is designed to be used in the induction step of an inductive proof. For it is in the nature of an induction step to make use of such presuppositions. And most philosophers who have written about this part of *Basic Laws* agree that §29 is part of an inductive proof. Resnik writes that the clauses of §29

are to be used to show that names formed from names already known to denote also denote. Plainly, on pain of an infinite regress, the referentiality of some names must be established by other means. . . . Section 31 then opens with the remark that given what has been said before, one need only show that the simple names of the system denote in order to show all the names of the system do, and then turns to the proof that the simple names do denote. . . . To clinch the case, remember that simple names, with the notable exception of the abstraction function name, are directly assigned references. (Resnik, 1986, p. 184)

On this interpretation, §29 and §30 are both parts of the inductive step of the proof—the clauses of §29 are to be used only in §30. In §31, Resnik suggests, Frege must use other means to deal with the basis case; to show that “our simple names denote something [*bedeuten etwas*]”

The problem with this interpretation is that Frege does not do what he is supposed to do in §31. That is, Frege does not establish the referentiality of simple names by other means. Although basis cases of inductive proofs typically appear first, it is no accident that §31 does not appear before §29. This is, of course, explicable if, in addition to straightforward arguments that the unproblematic primitive terms have Bedeutung, Frege also offers a second inductive proof in §31—a proof by induction that his second-level course-of-values function sign has Bedeutung. And the consensus, in the secondary literature, is that he does. But there is something else that is not explicable. Frege’s employment of the §29 criteria is not limited to this purported inductive proof. He employs these criteria even in his discussion of the unproblematic primitive terms, including the horizontal. For he says that its having Bedeutung can be established by considering expressions that result from putting a proper name in its argument place. He makes similar statements in his characterizations of what must be shown for each of the other primitive Begriffsschrift signs. Some of the arguments are trivial and some are elaborate, but in each case there is no question that Frege is making use of the criteria introduced in §29. And it is implausible to suppose that he employed the §29 criteria in §31 without noticing that he was doing so. Yet, as we saw earlier, and Resnik indicates, the proof will be circular unless some parts of the §31 proof are accomplished without making use of the §29 criteria.

There are, I think, two keys to making sense of the arguments in §31. One, as I have already indicated, is that Frege means §31 to be elucidatory. The second is that Frege does not mean §31 to set out the basis case of inductive argument. It should be evident from my earlier discussion of elucidation that these two claims are independent. Given that the natural language statements of mathematical arguments in the analysis sections of *Basic Laws* are to be regarded as elucidatory, it is perfectly plausible that Frege might decide to give, in §§29–31, an elucidation that takes the form of an inductive argument. But there are important reasons for thinking that he did not regard §31 as part of an inductive argument—whether elucidatory or not. For Frege employs the §29 criteria in each of these arguments, and he says that the employment of these criteria “always presupposes that we have already recognized some names as denoting [*als bedeutungsvolle*]” (BLA, vol. 1, §30). Unless Frege has forgotten this statement from the previous section, he must regard the §31 arguments as resting on the presupposition that we have already recognized some names as denoting.

Moreover, Frege clearly does seem to be intentionally making this sort of presupposition in §31.²⁶ For, after listing all the primitive Begriffsschrift names but before discussing them individually, he writes: “We start from the fact that the names of truth-values denote something, namely, either the True or the False” (BLA, vol. 1, §31). The names in question, of course, cannot be primitive Begriffsschrift names, for the primitive names are function names and truth-values are objects. Truth-value names are complex expressions that contain the primitive names. If Frege is, as he claims, presupposing this, then §31 clearly is not meant to be setting out the basis case of an inductive argument. But what is he doing, and why does he think he can presuppose this?

Let us begin with the presupposition. The application of the §29 criteria requires us to presuppose, he says, that “we have already recognized some names as denoting [*das man einige Namen schon als bedeutungsvolle erkannt habe*]” (BLA, vol. 1, §30, my emphasis). This is not a presupposition about the symbols but about his readers. It is important, also, to note that the application of the §29 criteria does not require that we start with names that have been *proved* to have Bedeutung but, rather, with names that have been *recognized* as having Bedeutung (*der als bedeutungsvoll anzuerkennenden*) (§31). Why does Frege think that he can presuppose that his readers have already recognized some names as having Bedeutung? The answer is to be found by looking at the earlier sections of *Basic Laws*. We should have already recognized some truth-value names as having Bedeutung because Frege has already shown us this.

One of these names is “ $a \rightarrow (b \rightarrow a)$.²⁷ Frege has already tried to convince us that its Bedeutung is the True. Frege says in §18,

By §12,

$$\Gamma \rightarrow (\Delta \rightarrow \Gamma)$$

could be the False only if both Γ and Δ were the True while Γ was not the True. This is impossible; therefore

$$\vdash a \rightarrow (b \rightarrow a) \quad (\text{I})$$

(BLA, vol. 1, §18)

The name in question, of course, is also an expression of Basic Law I. On the interpretation I am offering, this argument must be viewed as elucidatory. But this is not to say that it is in any way problematic. It seems reasonable for Frege to suppose that his readers, having read §18, recognize that the expression “ $a \rightarrow(b \rightarrow a)$ ” is a truth-value name. Frege provides similar arguments for each basic law. Thus it seems perfectly reasonable for Frege to assume that, by the time his readers reach §31, they will recognize each Begriffsschrift expression of a basic law as a name of a truth-value (the True). In the argument in §31, as I interpret it, Frege means to exploit this assumption. To see the significance of this interpretation, it will help to contrast it with the metatheory reading.

The fact from which we start in §31, Frege says, is that the names of truth-values denote something. It is evident that Frege does not mean that truth-value names—if there are any—have Bedeutung. For he says next that we then gradually widen the sphere of names to be recognized as having Bedeutung. Talk of widening this sphere does not make sense unless we begin with names that we recognize as having Bedeutung. But it is difficult to see how, on the metatheory-reading, the proofs of §31 could start with this fact. The only Begriffsschrift candidates for truth-value names are complex. Since, on the metatheory-reading, compositionality is a central feature of Frege’s theory of reference, a truth-value name can have Bedeutung only if all its constituent expressions have Bedeutung. Thus to start with this fact is to presuppose that at least some simple function names do have Bedeutung.²⁸ Yet §31 includes, for each primitive name, an argument that it has Bedeutung.²⁹

The passage from §18 quoted above also presents another problem for the metatheory reading. On the metatheory reading, this passage must set out a metatheoretic proof. But it is difficult to see exactly what it is that the proof is supposed to show. It cannot be an attempt to offer a metatheoretic proof that shows, not that Basic Law I is true, but that the Begriffsschrift expression has the property of naming the True under Frege’s interpretation (or of expressing a true thought on Frege’s interpretation). For the passage contains no reference to Begriffschrift expressions at all. It contains no statement about the truth of thoughts or sentences. Nor does it contain a truth predicate. The predicate ‘names the True,’ which might plausibly be taken as a truth predicate, appears nowhere in §18. The only predicate that appears in the passage and that one might take to be a truth predicate is the predicate ‘is the True’. And it is a mistake to take this to be a truth predicate. For the True is an object and the ‘is’ in the predicate ‘is the True’ is the ‘is’ of identity. The True is neither a sentence nor a thought. Hence to say, of a sentence or thought, that it is the True is to say something false.

This may seem too quick. After all, the True is the extension of a concept, and, given that numbers are defined as extensions of concepts, there is no obvious reason that sentences and thoughts might not similarly be defined as extensions of concepts. A moment’s reflection shows, however, that if we take the True to be a sentence or thought, then ‘is the True’ is not a truth predicate. For, if ‘is the True’ is a truth predicate, it must hold of all true sentences. That is, each true sentence must be the True. Consequently, there is only one true sentence. Clearly, this will not do. And for similar reasons the True cannot be a thought.

Now Frege could certainly use, and sometimes does use, an expression that almost seems to qualify as a truth predicate: the expression “*das Wahre bedeutet*”.³⁰

But he does not use this expression in the passage set off above.³¹ Of course nothing prevents us from rewriting the passage so that it contains recognizable statements about sentences or thoughts. And the result would seem to be a metatheoretic argument. But to show that Frege's argument can be rewritten as a metatheoretic argument hardly provides support for the metatheory reading. If Frege meant to make a metatheoretic argument here, why did he not do so?

In fact, it is easy enough to see what Frege means the discussion of Basic Law I to do for his readers. These readers to whom his notation is new may not immediately grasp the sense of the Begriffsschrift expression of Basic Law I. The statement:

$$\Gamma \rightarrow (\Delta \rightarrow \Gamma)$$

could be the False only if both Γ and Δ were the True while Γ was not the True.

looks simply like a natural language gloss on how to understand Begriffsschrift expressions of the form " $\Gamma \rightarrow (\Delta \rightarrow \Gamma)$." And Frege explicitly recommends elucidation for the enterprise of showing us how to understand Begriffsschrift expressions. Moreover, once we understand the Begriffsschrift expression of Basic Law I, we can also see that it is true—without resort to any demonstration that its constituents are correlated with extralinguistic entities. Frege is introducing a new language, and these discussions help us to understand it.

One might object that this is not a plausible explanation of Frege's aim. Since, by §18, Frege has already introduced all the Begriffsschrift symbols that appear in Basic Law I, he should have no need to explain the expression—he should expect his readers already to understand the Begriffsschrift expression. But Frege typically does not expect this, nor should he. After all, Frege's §34 definition of the Begriffsschrift sign for the application function is formed from previously introduced Begriffsschrift expressions. Yet, as we saw earlier, he thinks it is also important to provide elucidation—including a natural language gloss explaining its content. It is perfectly reasonable to suppose that Frege would find it equally important to give such glosses for the Begriffsschrift expressions of his basic laws. In §18, and the other sections in which basic laws are introduced, Frege is not offering metatheoretic proofs that each Begriffsschrift expression of a basic law names the True. Rather, he is enabling his readers to understand what is meant by these expressions and, at the same time, to recognize each of them as naming the True.

The Begriffsschrift expressions of the basic laws are not the only proper names that Frege's readers should recognize as having Bedeutung by the time they reach §31. There are, in addition, two further names, which are discussed in §10. In §3 Frege introduced both his notion of course-of-values and a notation for the course-of-values function name. We can use the course-of-values function name to form a name for the course-of-values of a first-level function $\phi(\xi)$. The name looks like this: $\dot{\epsilon}\Phi(\epsilon)$. Frege uses only (a natural language version of) Basic Law V to introduce these names. But the use of Basic Law V is insufficient to establish what courses-of-values are. In particular, it does not commit us to recognizing truth-values as courses-of-values; nor does it commit us to recognizing truth-values as objects that are not courses-of-values. Now, supposing that " Δ " is a name of a truth

value, there is nothing that determines whether or not it is true that $\Delta = \dot{\epsilon}\Phi(\epsilon)$. But, assuming “ $\dot{\epsilon}\Phi(\epsilon)$ ” does name some object, it must be determinate whether or not the object named is Δ .

In §10, Frege offers us a solution to this difficulty. Without contradicting Basic Law V, he says, “it is always possible to stipulate that an arbitrary course-of-values is to be the True and another the False” (BLA vol. I, §10). He then proceeds to stipulate that the course-of-values of the horizontal function, $\dot{\epsilon}(_\epsilon)$, is to be the True and that another course-of-values,³² $\dot{\epsilon}(\epsilon = \sim(a)(a = a))$ is to be the False. The upshot is that two new Begriffsschrift names have been recognized as having Bedeutung, ‘ $\dot{\epsilon}(_\epsilon)$ ’ and ‘ $\dot{\epsilon}(\epsilon = \sim(a)(a = a))$ ’. These names, like the expressions for the basic logical laws—which we already recognize as having Bedeutung—are names of truth-values.

We can now see why the arguments in §31 are not circular. Frege is using the §29 criteria, and he has explained, in an earlier section, how these clauses can be used. He says: “their application always presupposes that we have already recognized some names as denoting [*als bedeutungsvolle*]. They can serve only in the extension step-by-step of the sphere of such names” (BLA, vol. 1, §30). The sphere to be extended, of course, is not the sphere of names that *have* Bedeutung; it is not the sphere of names that *have been proved to have* Bedeutung; it is, rather, the sphere of names that we *recognize* as having Bedeutung. In §31 Frege means to be using our antecedent recognition that the Begriffsschrift expression of the basic laws, as well as ‘ $\dot{\epsilon}(_\epsilon)$ ’ and ‘ $\dot{\epsilon}(\epsilon = \sim(a)(a = a))$ ’, have Bedeutung to get us to recognize that all the simple Begriffsschrift names have Bedeutung. His first step is to draw our attention to our antecedent recognition that these complex expressions have Bedeutung. All these expressions are truth-value names. Thus, after listing the primitive Begriffsschrift names, Frege writes, “We start from the fact that the names of truth-values denote something, namely, either the True or the False” (§31). He continues: “We then gradually widen the sphere of names to be recognized as succeeding denoting [*als bedeutungsvoll anzuerkennenden Namen*] by showing that those to be adopted, together with those already adopted, form denoting names [*bedeutungsvolle Namen*] by way of the one’s appearing at fitting argument-places of the other” (BLA vol. I, §31). There is no obvious circularity in this strategy.

But if there is no obvious circularity, it may seem that nonetheless there is a problem with Frege’s procedures. To see why, let us return to one of the passages quoted earlier from §31. Frege writes, “In order now to show, first, that the function names ‘ $_\xi$ ’ and ‘ $_\neg\xi$ ’ denote something [*etwas bedeuten*], we have only to show that those names succeed in denoting [*bedeutungsvoll sind*] that result from our putting for ‘ ξ ,’ the name of a truth-value (we are not yet recognizing other objects)” (BLA, vol. 1, §31). It should be evident that this procedure is in accord with his talk about gradually widening the sphere of names that are recognized as having Bedeutung. For the only Begriffsschrift names that have been recognized as having Bedeutung, at this point in *Basic Laws*, are truth-value names. But this should not be sufficient to satisfy the §29 criteria. Consider what the §29 criteria demand in order to show that the horizontal has Bedeutung. We must show that every proper name is *bedeutungsvoll* that is formed by inserting a *bedeutungsvoll* proper name in the argument place of the horizontal. And there are Begriffsschrift proper names (formed using the second-level course-of-values function name) that,

although this has not yet been shown, *do* have Bedeutung but are not truth-value names. Frege's official procedure is, it seems, simply wrong.

The situation is not as clear as it seems, however. Frege's aim is to have shown, by the end of the argument in §31, that all Begriffschrift names have Bedeutung. He begins by noting (very little, after all, is required for this) that the result of inserting a truth-value name into the argument place of the horizontal is a bedeutungsvoll proper name. Only later in the section are we confronted with Begriffschrift proper names that are not truth-value names. And the earlier discussion of the horizontal places a requirement on any attempt to show that these names have Bedeutung. Thus, immediately after introducing a category of proper names that includes some non-truth-value names (the fair course-of-value names), he writes, "We must examine whether a fair course-of-values name placed in the argument-places of ' $\lambda \xi$ ' and ' $\lambda \xi$ ' yields a denoting proper name [*einen bedeutungsvollen Eigennamen*][. . .]" (BLA, vol. 1, §31). In the discussion that follows, he argues that the discussion in §10 allows us to see that fair course-of-values- names satisfy the requirement.

There is a sense in which this is a kind of inductive argument, for the gradual widening takes place in stages and can be described only by introducing a hierarchy of Begriffschrift expressions.³³ On the first level are the proper names that have been recognized, as a result of earlier discussions, to have Bedeutung. These are truth-value names. Frege then proceeds to argue that the result of placing truth-value names in the argument place of first-level function names yields denoting proper names. The results, of course, are still truth-value names. Once these first-level functions are recognized as having Bedeutung (by the §29 criteria and insofar as we are limited to proper names that have been already recognized as having Bedeutung), Frege can proceed to his second- and third-level function signs. The quantifier comes next. The results of forming proper names of the next stage, using the quantifier function- name (a second-level function name) and first-level function names are also truth-value names. This is followed by the third-level function name and, finally, the most difficult case: the second-level course-of-values function name, which itself requires a complicated argument involving a sort of hierarchy. The importance of the hierarchy is to convince us that the procedure will ultimately encompass all Begriffschrift expressions. But it is also important to note that to say that Frege uses this hierarchy to structure his argument is not to say that Frege's argument is by induction on the complexity of an expression.

As we have seen, the expressions that we come to recognize, first, as having Bedeutung—the proper names used for basic laws and the two course-of-values names for the True and the False—are not primitive, but complex expressions. They are, in particular, more complex than the expressions that we are supposed to come to recognize, in §31, as having Bedeutung. It is only in the process of widening the sphere of names to be recognized as having Bedeutung that we come to recognize primitive Begriffschrift expressions as having Bedeutung. Indeed, this is merely another way of noting that Frege employs the §29 criteria in the arguments of §31. That is, he bases his arguments that the simple Begriffschrift names have Bedeutung on the assumption that we recognize *other* Begriffschrift names as having Bedeutung. Moreover, the §31 arguments that these primitive Begriffschrift expressions have Bedeutung involve arguing that still other even more complex expressions do. It should be evident at this point that, if we as-

sume that Frege is making an inductive argument on the complexity of an expression, circularity will be inevitable.³⁴

VII. Section 31 and Another Puzzle

The apparent circularity of §31 is one of two problems that almost immediately bother most people when they read the section. The other problem is also rooted in the §29 criteria. The §29 criteria consist of six clauses, one for each type of Begriffsschrift name. In each clause, we are told that a name of the type in question has *Bedeutung* just in case a certain group of expressions in which it appears have *Bedeutung*. The expressions in question are all proper names. Thus one salutary feature of employing the §29 criteria is that when Frege argues that a particular function name has *Bedeutung*, the substance of the argument need not actually contain the sort of talk that, he tells Russell, ought to be prohibited. He need not include talk about the meaning of a function name. Instead, he can limit his discussion to that of whether the members of a class of proper names in which the function name appears have *Bedeutung*.

The strategy has a less salutary—although not very surprising—feature as well. The employment of the §29 criteria will not suffice to show that a function name names a function. Frege is adamant that functions must have values for every argument of the appropriate type. The addition function, he tells us, must be defined not only for the numbers, but also for the moon (BLA, vol. 2, §64). Consider a first-level function name. Suppose Frege shows that every result of putting a Begriffsschrift proper name in its argument place is a *bedeutungsvoll* proper name. This seems only to show that the function name names something that has values for objects with Begriffsschrift names. Frege's own statements seem to require that, to show that the function name names a function, one must show that it has a value on the moon. Yet there is no Begriffsschrift name for the moon.

On the metatheory reading, this is a serious problem. For the employment of the §29 criteria cannot suffice to show that the primitive Begriffsschrift function names are associated with the appropriate sorts of extralinguistic entities. As I have argued, it is not Frege's aim in these sections to show that Begriffsschrift function names are associated with the appropriate sorts of extralinguistic entities. But this is not, itself, a solution to the problem. If it is not Frege's aim to show this in §§28–31, what is his aim? Frege says, in the introduction to *Basic Laws*, that the arguments of these sections are supposed to allay the suspicions aroused by his innovations. What suspicions are likely to be aroused? One obvious answer is that the reader will be suspicious of the second-level course-of-values function name—indeed, Frege himself alerts the reader to his own worry about this. Yet, as we have seen, this suspicion is not the sort that can really be addressed by the arguments of §31. There is, however, another suspicion that is likely to bother the reader and that can be addressed by these arguments.

Frege has argued, in earlier sections, that his expressions of logical laws express truths and that only truths follow by his rules of inference from truths.³⁵ Each line of each proof gives us a proper name preceded by a judgment stroke and horizontal. Frege needs his readers to be confident that each of these proper names names the True. The earlier arguments should convince the reader that it

suffices for each of these proper names to satisfy the following condition: it must either come from earlier lines by application of a rule of inference, or it must be a logical law or definition. By their construction, these names that appear in Frege's proofs seem to satisfy the condition. But there could still be a worry. Suppose one were to replace the variables of a basic law with Begriffsschrift proper names that have no Bedeutung. The result would appear, by its construction, to satisfy the condition, yet would not.

This can be avoided if every Begriffsschrift proper name has Bedeutung. Should Frege expect his readers to find it obvious that every Begriffsschrift proper name does have Bedeutung? Many of the proper names that appear in Frege's proofs are either rife with defined abbreviations or immensely complex. And the definitions are themselves immensely complex. Thus Frege may well have wanted to convince the reader that nothing goes wrong in the process of forming these complex names. And the satisfaction of the §29 criteria by all Begriffsschrift expressions is exactly what is required for this. This will show that any result of embedding Begriffsschrift expressions in more complex proper names will have Bedeutung. If Frege has succeeded in convincing the reader that the §29 criteria have been satisfied, he will not have succeeded in convincing the reader that all Begriffsschrift expressions are associated with extralinguistic entities. But he will have convinced the reader that all lines of his proof are in order.

There may still seem to be a problem here. For the upshot of Frege's use of the §29 criteria may seem to be that, at least in some cases, we have received the wrong verdict about what the Bedeutung of an expression is. To see this, consider Frege's discussion of the universal quantifier. Frege argues that a name ' $(a)\Phi(a)$ ' has Bedeutung just in case ' $\Phi(\xi)$ ' does.³⁶ Moreover, ' $\Phi(\xi)$ ' has Bedeutung just in case every proper name that results from inserting a bedeutungsvoll proper name in its argument place has Bedeutung. Frege goes on to say, in §30, that if ' $\Phi(\xi)$ ' has Bedeutung and if every proper name that results from filling in the argument place with a bedeutungsvoll proper name names the True, then ' $(a)\Phi(a)$ ' names the True. And this is where Frege seems to have given us the wrong account. For it would seem that every *object* has to satisfy ' $\Phi(\xi)$ ' to make the universal generalization true. Yet, according to Frege's claim in §31, the universal generalization is true provided all objects that have Begriffsschrift names satisfy it. And not all objects have Begriffsschrift names. Moreover, Frege comes close to giving the right account elsewhere in *Basic Laws*. In §8, for example, he suggests that ' $(a)\Phi(a)$ ' names the True just in case "for every argument the value of the function $\Phi(\xi)$ is the True".

The problem with the §31 account can be made more vivid by considering the statement that every object is a course-of-values—a statement that is easily expressible in Begriffsschrift by something like: $(x)(\exists F)(x = \dot{e}F(e))$. The problem is that each Begriffsschrift proper name formed from the symbols introduced in *Basic Laws* is a name of a course-of-values. Either it will have been formed using the second-level course-of-values function name or it will be a truth-value name; hence it will either name the True, which is $\dot{e}(\neg\neg e)$ or the False, which is $\dot{e}(e=\sim(a)(a=a))$. Thus the Begriffsschrift statement that every object is a course-of-values names the True—it is, it seems, a logical truth that every object is a course-of-values.

There is, however, a flaw in this reasoning. For the so-called logical truth cannot be proved in Begriffsschrift. It can be shown only by a metatheoretic argu-

ment. And the metatheoretic argument depends on facts about particular objects (in this case, the logical language). Thus the so-called logical truth does not qualify as a logical truth—it cannot be shown with the sole support of those laws of thought that transcend all particulars. Of course, it need not be a logical truth to cause problems. It is a considerable problem if this is true at all. But here, again, the problem is a result of placing our faith in metatheory and on natural language. If we take Frege to be giving truth conditions for *Begriffschrift* statements in which the quantifier appears, we must simply accuse him of making a mistake. But as I have argued, it is a mistake to do this. Moreover, it is a mistake to think that there are cases in which we can use natural language, but not *Begriffschrift*, to evaluate a claim.

Nonetheless, it will not suffice simply to say that it is not Frege's aim to provide a metatheoretic account of the contribution the first-level quantifier makes to the truth values of *Begriffschrift* expressions in which it appears. After all, we can see that there is (at least in *Basic Laws*) no *Begriffschrift* name for the moon. Hence we can see, it seems, that what he says in §31 is wrong. And while Frege does say, of some of his elucidations, that they go wrong of necessity, this is not one of them. It may seem that the only way to save Frege is to interpret the logic of *Basic Laws* as ranging over a particular, restricted domain—a domain consisting of courses-of-values of logical functions with *Begriffschrift* names.³⁷ This interpretation may save Frege's statement about the first-level quantifier symbol, but the price is high. There are a number of problems.

One of these is that there is no direct textual support for the interpretation. And it is mysterious that Frege should introduce a universe of discourse yet never tell us that this is what he is doing. But the problems with this interpretation, are not merely exegetical. The more serious issue is that, on this interpretation, the logic introduced in *Basic Laws* can not be used for the purpose for which it was intended. Frege's aim is to show that the truths of arithmetic are analytic, that they can be given gapless proofs using only definitions and laws that govern all thought—that is, laws that apply to the objects of the empirical sciences, to the objects of geometry, to everything thinkable. If Frege has only shown that his *Begriffschrift* laws apply to the objects of a limited universe of discourse, then derivations of truths of arithmetic from these laws will show that these truths are analytic.

There is another sense, as well, in which the logic of *Basic Laws* does not, on this view, do what it is supposed to do. The *Begriffschrift* expression of each of his propositions, Frege says in *Basic Laws*, explicitly expresses “all of the conditions necessary to its validity”; there can be no “tacit attachment of presuppositions in thought” (BLA, vol. 1, p. vi). Yet, if the laws of *Basic Laws* are to be regarded as holding only for courses-of-values, this is a tacit presupposition. And it is a tacit presupposition that could have been made explicit. To do this, we need only replace Frege's statement of the law (' $a \rightarrow(b \rightarrow a)$ ') with

$$a \rightarrow(b \rightarrow a) \text{ or } \sim(\exists\phi)(a = \dot{\epsilon}\phi(\varepsilon)) \text{ or } \sim(\exists\phi)(b = \dot{\epsilon}\phi(\varepsilon)).$$

The assumption that it is appropriate to talk of domains in the metalanguage, but not to eliminate tacit presupposition from *Begriffschrift* makes sense on the contemporary view of logical notation as a purely formal notation that is interpreted in natural language. But this is an un-Fregean view of logical notation. For Frege, *Begriffschrift* is a real language. Its expressive powers are different

from natural language. And its expressive powers come into their own when the purposes are scientific and the aim is the expression of gapless proof. To make use of a metatheoretic background language that states presuppositions that remain unexpressed in *Begriffsschrift* is to give up on one of the central purposes of *Begriffsschrift*. It is not plausible to suppose that Frege made such a momentous change without acknowledging it explicitly. It is especially implausible in light of Frege's statements about definitions in §§56–67 of the second volume of *Basic Laws*. In these sections Frege criticizes mathematicians for defining functions only over limited domains. The addition function, he says there, must be defined even for the Moon as argument. Moreover, he objects to the strategy of 'piecemeal definition'—beginning with a definition of a term over a limited domain and then later defining it over a wider domain. Were Frege's intent, in the first volume of *Basic Laws*, to show only that his logical functions are defined for courses-of-values as arguments, this work would be subject to both criticisms. It is, of course, possible that Frege simply changed his view. Yet there is no evidence of this. The later discussions contain no acknowledgement of any such change. They are, moreover, entirely in line with the (earlier) view of *Begriffsschrift* as a means for eliminating tacit presupposition.

What, then, are we to make of Frege's offending statement: that if ' $\Phi(\xi)$ ' has Bedeutung and if every proper name that results from filling in the argument place with a bedeutungsvoll proper name names the True, then ' $(a)\Phi(a)$ ' names the True? The question is too difficult to be addressed adequately here. It will be worthwhile, however, to make a few brief comments. Frege does say several times (in §§8, 17 and 20) what, it seems, he ought: that ' $(a)\Phi(a)$ ' names the True provided the value of the function $\Phi(\xi)$ is the True for every *argument*. Since this is correct, and he is willing to say it, why should he have made the offending statement at all? There is no explicit answer in Frege's writings, nor does Frege acknowledge the difference between the two sorts of statements. However, it is not difficult to see at least one reason for the appearance of the offending statement. It is forced on Frege by the employment, in §31, of the §29 criteria for having Bedeutung. The statement is also useful for drawing attention to the relation that holds between *Begriffsschrift* expressions when one expression is a universal generalization and the other is an instance of the first. Moreover, it is also not obvious that the offending statement is as bad as it may seem at first. Frege does, after all, envision adding names to *Begriffsschrift* for the purposes of evaluating inferences in other sciences. Provided we can identify an object, we can stipulate that a proper name is to designate this object. Frege may, then, be considering also the bedeutungsvollen proper names that can, in principle be added to *Begriffsschrift*. Thus, unless Frege thinks there are objects that are, in principle, unnameable, his description is more or less correct.³⁸

VIII. Conclusion

In this essay I have tried to show that some of the apparent difficulties with §31 can be addressed by taking Frege's remarks about the nature of his project more seriously than they are usually taken. In particular, I have tried to show that many

of the apparent difficulties with §31 are consequences of the metatheory reading. I would like to end with a few remarks about this reading.

Frege's task, as he characterizes it in *Foundations*, is to prove the truths of arithmetic from primitive truths. He writes:

The aim of proof is, in fact, not merely to place the truth of a proposition beyond all doubt, but also to afford us insight into the dependence of truths upon one another. After we have convinced ourselves a boulder is immovable, by trying unsuccessfully to move it, there remains the further question, what is it that supports it so securely? The further we pursue these enquiries, the fewer become the primitive truths to which we reduce everything. (FA, §2)

What are the primitive truths to which Frege means to reduce the truths of arithmetic? I have argued that these truths are the basic logical laws. Frege states these laws in *Begriffsschrift* and, using these laws and his definitions, derives some of the primitive truths of arithmetic. On Frege's view, as long as we understand *Begriffsschrift*, we can see that the basic *Begriffsschrift* laws are true. And, as he also indicates, we can see this without relying on evidence of the senses or intuition. We thus have only to work through Frege's proofs to see that the truths of arithmetic are analytic.

What would be gained by introducing a metatheory? Some have argued that a metatheory would enable Frege to show that his logical language does what it is supposed to do—that the language will not leave us with expressions that do not designate anything. But, as we have seen, from Frege's point of view, this is something that cannot be proved. For *Begriffsschrift* satisfies Frege standards only if all its terms, including all its primitive terms (that is, all its terms for things that are logically simple) have *Bedeutung*. And any attempt to prove that a term for something logically simple has *Bedeutung*, whether it is a term of natural language or a term of *Begriffsschrift*, is doomed to failure. Natural language, Frege is the first to admit, has advantages that *Begriffsschrift* does not. In a famous passage, he compares natural language to the eye—which is more useful for most of our purposes than a microscope, to which he compares *Begriffsschrift*. But to suppose that natural language is also superior for the purposes for which *Begriffsschrift* is formulated—to suppose that natural language is superior for purposes of explaining the primitive logical notions—is to give up on Frege's logical enterprise. It is not that it is impossible, given Frege's views, for there to be a science of semantics—although there are barriers—but that such a science is a special science not deserving of the appellation “logic.” And it is not a special science to which Frege devotes himself in *Basic Laws*.

Notes

My thanks to Mark Kaplan for helpful discussions and criticisms of earlier drafts and to Thomas Ricketts for correspondence about some of the topics discussed in this essay.

1. Citations of Frege's published work, with the exception of *Begriffsschrift*, are all to page numbers or section numbers of the original publication. For *Begriffsschrift* and the unpublished work, both English and German citations are included.

2. "Course-of-values" is Montgomery Furth's translation of *Wertverlauf*. Since I have chosen to use Furth's translation in my quotations from *Basic Laws*, I will, except where explicitly noted, be using his translations in my own prose as well.

3. Although circular arguments may not invariably be problematic, there is consensus that the circularity in this case is problematic.

4. See, e.g., Parsons, 1965; Resnik, 1986; Thiel, 1996; Heck, 1997.

5. In this essay I will be discussing not only the early sections of *Basic Laws* but also some of Frege's papers, including "On Sense and Meaning" and some of the secondary literature. Where Montgomery Furth's translation uses "denotation" and its cognates, the translators of "On Sense and Meaning" use "meaning" and its cognates, and Michael Resnik, whose interpretation I will be discussing, uses "reference." Moreover, the choice of translation in many cases carries with it a view about how Frege's writings should be interpreted, and one of my aims is to address the issue of which view is correct. Because of this, I have chosen not to alter any of the translations of these terms in the passages that I quote. In order to minimize confusion, however, I have included the German expressions as well. In my own prose, I have sometimes used the translation used by a writer I am discussing, and other times I have chosen not to adopt any of the translations but to use the expression "has Bedeutung" as if it were an English expression. I apologize for any confusion or infelicity that results.

6. I have suggested this in "Understanding Frege's Project," forthcoming in *The Cambridge Companion to Frege*. More recently, I have tried to show how issues concerning vagueness are best understood by incorporating Frege's insights into the relation of logic and language in "Science and Semantics: The Case of Vagueness" (in preparation).

7. In fact, it is probably also worth noting that, in *Begriffsschrift*, unlike contemporary logical notations, there are no nonlogical constants—although it is evident that Frege thought that nonlogical constants should be added in order to bring the tools of logic to bear on other disciplines. But only once *Begriffsschrift* is supplemented in this way would it be possible to give the sort of interpretation that is part of contemporary logic.

8. I choose this example for its familiarity. But I do not mean to suggest that Frege might introduce a primitive logical sign for this function. He would not because he does not regard the successor function as a primitive logical function. Frege's primitive logical functions are less familiar, not only to contemporary readers, but also to Frege's intended audience. However, it is evident from the discussions in which he introduces his primitive function signs (see *Basic Laws*, vol. 1, §§5–10) that, except for the second-level course-of-values function, he does not regard this unfamiliarity as a reason for requiring special proofs.

9. There is, in particular, no role for such proofs on the contemporary understanding of logic. An interpretation assigns each nonlogical constant an appropriate entity by stipulation. No proof is required. Nor is there any role for a proof that an interpretation assigns an appropriate entity to the sorts of *Begriffsschrift* signs that are of concern in §31. For these are not assigned entities at all by contemporary model theoretic interpretations. Their contribution to the truth-values of sentences in which they appear is provided by a definition of truth under interpretation.

10. Although it is possible that this use of words is an oversight, it seems unlikely, for in his discussions of the notions of function and concept, Frege exploits the different significance of various uses of *bedeuten* and its cognates. In some unpublished notes that are estimated by the editors of Frege's *Nachlass* to have been written sometime between 1892 and 1895, the following appears: "Indeed we should really outlaw the expression 'the meaning of the concept-word A' [*die Bedeutung des Begriffsworts A*] because the definite article before 'meaning' [*Bedeutung*] points to an object and belies the predicative nature of a concept. It would be better to confine ourselves to saying 'what the concept word A means' [*was das Begriffswort A bedeutet*]" (PW, p. 122/

NS, p. 133). In a later letter to Russell, Frege writes: “We cannot properly say of a concept name that it means something [*dass er etwas bedeute*]; but we can say that it is not meaningless [*dass er nicht bedeutungslos sei*]” (PMC, p. 136/BW, p. 219).

Moreover, in §31 Frege in fact *does not* argue that each proper name is associated with an object. He shows this for proper names that are truth-value names but not for proper names of courses-of-values that are not truth-values.

11. It seems unnecessarily complicated, since there is no obvious reason why Frege should not simply say that a first-level function name has a Bedeutung just in case it names a first-level function. It also seems incorrect because a first-level function must be defined for all objects, not just those that have *Begriffsschrift* names. This issue will be examined in more detail shortly.

12. Actually, this is a bit oversimplified. He does not argue that this holds for every first-level function name that has Bedeutung, but only that it holds for the primitive first-level function names.

13. This principle, of course, is more general than the principle introduced in “On Sense and Meaning,” for Frege uses ‘name’ in *Basic Laws* to include the sorts of expressions that we refer to today as logical constants.

14. I say that Frege is committed to denying this, not that he actually did deny it. It is no surprise that he did not actually deny it. After all, it is not as if there were a pre-“On Sense and Meaning” articulation of this view available for Frege to attack. It is only our post-Tarski sensibility that makes this seem an obvious issue.

15. One might think that this imperfection is not avoided in Frege’s *Begriffsschrift*, since, as I have indicated above, it seems that whether or not the second-level course-of-values function name has Bedeutung depends on the truth of a sentence: every function has a course-of-values. But this is not quite the imperfection mentioned here; Frege’s description only applies to proper names. Moreover, the situation is very different in this case. As I will argue shortly, one lesson of the contradiction seems to be that “every function has a course-of-values” is not a meaningful sentence.

16. Note that this claim does not belong to the metatheory. It can easily be expressed in *Begriffsschrift* using no symbols other than those introduced in *Basic Laws*.

17. Frege rarely says much about the notion of courses-of-values. When he introduces the notion of extension in *Foundations*, he simply says, “In this definition the sense of the expression ‘extension of a concept’ is assumed to be known” (FA, p. 117). In the earliest introduction of the notion of course-of-values, in “Function and Concept,” he simply says that if functions have the same value for each argument, “here we have an equality between value-ranges [*Werthverläufe*]” (Frege, 1891, p. 10). This, of course, is simply another way of describing Basic Law V. As I will argue shortly, there is a perfectly legitimate reason for this.

18. In the interest of consistency of terminology, I have substituted “course-of-values” for “graph” in Geach’s translation of *Werthverlauf*.

19. See, e.g., “On Concept and Object” (Frege, 1892a), p. 195; “What Is A Function?” (Frege, 1904), p. 665; “On the Foundations of Geometry: Second Series” (Frege, 1906), pp. 301–6.

20. Although, in his first response to Russell on June 22, he claims that Basic Law V is false, this view is not entirely consistent with the strategies for circumventing the difficulty that Frege discusses in his later letters. These strategies involve introducing new conceptions of the notions of courses-of-values and extensions (see, e.g., Frege to Russell, September 23, 1902). Similar strategies are discussed in the afterward to *Basic Laws*. He considers, for example, regarding what he now calls “class-names” as naming improper objects (*uneigentlichen Gegenständen*). And he considers regarding these names themselves as “pseudo-proper names [*Scheineigennamen*] which would thus in fact have no denotation [*Bedeutung*]” (GGA, p. 255). When he discusses the strategy that he proposes to use for solving the difficulty, he notes that “this simply does away with extensions of concepts in the received sense of the term” (GGA, pp. 260–61).

21. I have altered Kluge's translation here by using "elucidation," rather than "explanation," as the translation of *Erläuterung*.

22. It would take us too far afield to explain why this, and some of Frege's other peculiar statements about concepts and functions, are consequences of his understanding of concepts and functions as unsaturated. It is worth mentioning, however, that many philosophers think that these statements are both deeply problematic and easily separated from Frege's important views. On their view, these statements should be regarded simply as mistakes on Frege's part. I have argued elsewhere (Weiner, 1990, chap. 5; 1999, chap. 6) that these statements are integral parts of Frege's overall view and also that, understood correctly, they are not problematic at all.

23. Frege's understanding of the notion of function creates another difficulty for the metatheory reading. Because, on Frege's view, what can be said of objects cannot be said of functions, we cannot say that objects and functions are the entities to which linguistic (or Begriffsschrift) expressions refer. That is, there cannot be a "refers-to" relation that holds between proper names and objects and that holds between function-names and functions. Although this may seem to be a trivial problem for the metatheory reading, it is not. For a discussion of the significance of this issue, see Weiner, 1995a, 1995b.

24. In particular, Frege says in footnote to §2 that the symbol " ξ " will not occur at all in the development of the Begriffsschrift itself, but "I shall use it only in the exposition of it [*Darlegung der Begriffsschrift*], and in elucidations." This symbol does occur in the Analysis sections. Since part I is labeled "Exposition of the Begriffsschrift," and the Analysis sections appear in part II, which is labeled "Proofs of the Basic Laws of Number," it seems reasonable to assume that Frege regards the Analysis sections as providing elucidations.

25. I have slightly altered the translation here. Furth's translation is "a few explanatory remarks."

26. Christian Thiel (1965, p. 80) notes this but suggests that the presupposition is warranted by earlier proofs.

27. This is a bit oversimplified. Given the way Frege's notation works, both a and b are univerally quantified. A more revealing contemporary representation of Frege's law might be:

$$(a)(b)(a \rightarrow (b \rightarrow a)).$$

The difference is not significant for our purposes here.

28. Heck (1997) offers a different account of this part of §31. Heck asks what Frege's assumption (that the names of truth-value denote something) amounts to and answers: "I see no option but to suppose that what Frege means is that, if an expression denotes a truth-value, then it denotes *something*. So the assumption amounts to the stipulation that the domain of the theory is not empty, that, in particular, it contains the two truth values" (Heck 1997, p. 449). This much is certainly true: Frege does assume that the two truth-values are among the things over which his quantifiers range (I resist talking of a domain here, for reasons described later in this essay). But this does not exhaust the content of Frege's assumption. Frege's assumption, as stated, is an assumption about *names*.

What are these names? As I argue below, it is perfectly reasonable for Frege to assume in §31 that we have already recognized some Begriffsschrift expressions as names of truth-values. The Begriffsschrift statements of basic laws have the status of complex Begriffsschrift names, and one upshot of Frege's discussions should be to convince us that they are names of the True.

On Heck's interpretation, however, Frege's §31 assumption is not an assumption about Begriffsschrift names. Heck argues that we should not understand Frege to be talking about filling in argument places with actual expressions but, rather, with what Heck terms "auxiliary expressions." He writes: "The term ' Δ ' is not supposed to be a name in Begriffsschrift *at all*: it is a formal device, a *new name*, added to the language,

subject only to the condition that it should denote some object in the domain" (Heck 1997, p. 442). Although I do not have room to discuss all the difficulties with this interpretation, it will be useful to mention, briefly, its significance for the passages that I have been discussing in this essay. Frege begins §30 by saying that the application of the §29 provisions presupposes "that we have already recognized some names as denoting." On Heck's interpretation, however, this is misleading. For while Frege thinks that we *can* apply the §29 provisions, he does not think that we really have already recognized actual names as denoting. What Frege should have said, on Heck's interpretation, is that we can apply the §29 criteria because we recognize that we can stipulate that a capital Greek letter is to denote something and add it to Begriffsschrift. Frege's next comment—that the §29 provisions can only be used in the extension of the sphere of names we recognize as denoting—is also misleading. The auxiliary-name technique allows us to start with a situation in which *no* names are recognized as already denoting. We then use the §29 provisions to show that, because we can assign auxiliary symbols a denotation, we can recognize actual names as already denoting. Frege is, given Heck's interpretation, similarly misleading us when he goes on, in §31, to say that we start with the fact that "the names of truth-values" denote something and that we then "widen the sphere of names to be recognized as succeeding in denoting." Again, on Heck's view, there is no sphere of names that are already recognized as denoting. Instead, on Heck's view, Frege means to be saying that we start with the fact that, should we stipulate that ' Δ ' denotes a truth-value, given that there *are* truth-values, ' Δ ' denotes something. There is no sphere of already recognized-denoting names to widen. Why should Frege want to mislead his readers in this way? It is not particularly difficult to express the views that, according to Heck, Frege actually holds. What Frege *ought* to say, were Heck's interpretation correct, is: we start from the fact that *it is possible* to add non-Begriffsschrift proper names that denote truth-values, and, from this, we can show that *actual* Begriffsschrift expressions have Bedeutung. It is plausible that someone might hold the sort of view Heck attributes to Frege. But there is no textual evidence that Frege held it. It is very odd that Frege should have adopted this elaborate and ingenious strategy to make his proofs work yet never explained it to his readers. In particular, no such discussion appears in Frege's explanation of his use of the capital Greek letters in §5 of *Basic Laws*.

Moreover, if Heck is right, then capital Greek letters are misused in the passage from §18 quoted above. On Heck's interpretation, Frege uses capital Greek letters, not as metalinguistic variables, but as actual symbols that can be assigned a denotation and added to Begriffsschrift. Thus in §18 Frege means to be saying that, whatever values are assigned to ' T ' and ' Δ ', ' $T \rightarrow (\Delta \rightarrow \Gamma)$ ' names the True. He could have made this clear in the passage quoted by writing either

if both ' T ' and ' Δ ' named the True

or

if both ' T ' and ' Δ ' were assigned the True.

Instead, Frege writes something that makes no sense at all, on Heck's view. He writes:

if both Γ and Δ were the True while Γ was not the True.

Such uses of capital Greek letters, which are improper according to Heck's interpretation, appear throughout the early sections of *Basic Laws*. Again, given the ease with which Frege could have done this correctly, had he held the views that Heck attributes to him, it is difficult to see why these sections are written as they are.

29. One might argue that the reason is that Frege thinks he has already shown that some of his primitive names have Bedeutung. Most of the arguments in §31, after all, are fairly perfunctory. But this will not help. For the most perfunctory arguments are for the horizontal and negation stroke—he simply says, "This follows immediately from our explanations" (BLA, vol. 1, §31). Yet he precedes this statement with the

claim that we need to show that putting a name of a truth-value in the argument place of these function names results in names that are *bedeutungsvoll*. That is, we are supposed to make use of the assumption that there are Begriffsschrift names of truth-values even for the purpose of arguing (however perfunctorily) that the horizontal and negation stroke have Bedeutung.

30. It is, however, also worth noting that *das Wahre bedeutet* is not exactly a truth predicate in any contemporary sense. For a truth predicate, presumably, holds only of true sentences or thoughts. And this predicate holds of an expression, “ $\hat{\epsilon}(\underline{\hspace{1cm}}\epsilon)$,” which is not a true sentence.

31. It is interesting, in this context, to note that, in a paper devoted to trying to show why Frege must have had a metatheory, Jason Stanley writes: “Now a central thesis of this paper is that one way in which Frege uses his truth-predicate is in giving a justification, in the metalanguage, of the rules of inference of the *Begriffsschrift*” (Stanley, 1996, p. 47). And he continues: “The justification of the inference rule of Modus Ponens lies in the fact that if the conditional is true, and its antecedent is true, then so is the consequent. This fact gives some justification to accepting Modus Ponens as a rule of inference in a logical system by pointing to its truth-preserving nature.” But Frege’s justification includes no use of a truth predicate. The justification of *modus ponens* is (BLA, vol. 1, §14; I have substituted some modern notation for Frege’s here):

From the propositions ‘ $\vdash (\Delta \rightarrow \Gamma)$ ’ and ‘ $\vdash \Delta$ ’ we may infer ‘ $\vdash \Gamma$ ’; for if Γ were not the True, then since Δ is the True, $(\Delta \rightarrow \Gamma)$ would be the False.

But the predicate ‘is the True’, as we have seen, is not a truth predicate. Stanley also writes, “In assertions of the validity of inference rules, the truth-predicate occurs *ineliminably*” (Stanley, 1996, p. 53). But, while this is certainly true in contemporary discussions of logic, as the above indicates, on Frege’s conception of sentences as names of truth-values there need be no ineliminable appearance of a truth-predicate in assertions of the validity of inference rules. Indeed, as we can see from the passage set off above, in Frege’s own discussion of the validity of *modus ponens*, there is no appearance of a truth-predicate at all.

32. This course-of-values is the extension of the concept *is identical to the False*.

33. Due to limitations of space, I am not able to give an account of how this argument works and, in particular, Frege’s procedure in arguing that the second-level course-of-values function name has Bedeutung. I hope to cover this in a sequel to this essay, currently in preparation.

34. There are, in the secondary literature, a number of other arguments that §31 is circular, most of which concern the details of the purported inductive proof. See, e.g., Parsons, 1965; Resnik, 1986; Thiel, 1996. Although I do not have room to discuss these arguments here, it is instructive to see the role played in one of these discussions by the assumption that Frege is making a proof by induction on the complexity of an expression. Using an example that also appears in Thiel, 1965, Resnik writes that “following the model of the rest of Frege’s proof, the way to show that an abstract A is a referential abstract is to show that ‘ $(g)(g(A))$ ’ is referential. . . . However that is shown by showing that $F(A)$ is referential for every referential first-level function name $F(\xi)$ and ‘ $(g)(g(\xi))$ ’ is one of these; so we have gone in a circle” (Resnik, 1986, p. 187). Although I am not convinced that Thiel and Resnik are right about what must be shown to show that A is a referential abstract, what is most interesting for our purposes here is what Resnik says is required to show that “ $(g)(g(A))$ ” has Bedeutung. Why should this require us to show, *first*, that $F(A)$ has Bedeutung for every first-level function name $F(\xi)$ that does? Supposing, as Resnik does, that in §§29–31 Frege is offering an inductive proof on the complexity of an expression that all Begriffsschrift names have Bedeutung, then this proof should be applicable to show that particular names have Bedeutung. And the application of this sort of inductive proof to show the expression “ $(g)(g(A))$ ” has Bedeutung requires a demonstration that proceeds from simple signs to more complex signs. Thus the circularity Resnik finds is dependent on his regard-

ing Frege's proof as an inductive proof on the complexity of an expression. One might think, however, that there is a problem here in any case. How can Frege show that the expressions in question have Bedeutung?

To find an answer, we need to look for a place in which Frege actually discusses a proper name that has a second-level quantifier. The natural place to look is in Frege's discussion of Basic Law IIa. Frege writes: "We now understand by '(f) ((a)f(a) → (f(Γ))' the truth-value of one's always obtaining a name of the True, whatever function-name one may substitute in place of 'f' in '(a)f(a) → f(Γ).' This truth-value is the True, whatever object 'T' may denote" (BLA vol. I, §20; I have substituted some contemporary notation for Frege's here (emphasis is in the original)).

Here we have an argument that a complex Begriffsschrift expression beginning with a second-level quantifier has Bedeutung. It is an argument in which Frege does not follow the procedure that Resnik outlines. We can easily make a similar argument that "(g)(g(A))" names the False, whatever object A is. For, using Frege's words, we understand by "(g)(g(A))" the truth-value of one's always obtaining a name of the True, whatever function name one may substitute in place of "g" in "g(A)." And we know that, since one of the first-level function names is $\sim\xi=\xi$, this truth-value is the False. If, as I have argued, Frege's view is that a complex Begriffsschrift expression can be shown to have Bedeutung without an antecedent demonstration that its constituents have Bedeutung, the circularity problem, as Resnik states it, does not arise.

35. It is probably worth reminding the reader that to say that Frege makes these arguments is not to say that he has a metatheory in anything like the contemporary sense. In particular, as we have seen in the above discussion of §18, where Frege discusses Basic Law I, and of §14 (32 n.), where Frege discusses *Modus Ponens*, there is no use of a truth predicate. That is, no truth predicate is required for Frege to show that Basic Law I names the True and that modus ponens is truth preserving. A more complete discussion of this issue appears in my forthcoming, "Understanding Frege's Project."

36. It is worth noting here that, strictly speaking, ' $\Phi(\xi)$ ' is not a function name. Function names, like functions, are unsaturated or incomplete. When Frege introduces the use of ' ξ ' here, it is after noting that the 'x' in our usual attempts to name functions is not actually part of the function name. Rather, the function name is "the part of the expression which is there over and above the 'x'" (BLA, vol. 1, §1). The ' ξ ' is used in the place of 'x', he tells us, in the exposition of Begriffsschrift and in elucidations.

37. Heck (1997) adopts a view very close to this one.

38. On this issue, my interpretation bears some similarities to that in Heck, 1997.

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PART III

FREGE TO EARLY WITTGENSTEIN

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Wittgenstein's Understanding of Frege

The Pre-Tractarian Evidence

Warren Goldfarb

It is beyond argument that the two great influences on the young Wittgenstein were Frege and Russell. In the preface to the *Tractatus* he writes, “to the great works of Frege and the writings of my friend Mr. Bertrand Russell I owe in large measure the stimulation of my thoughts” (Wittgenstein, 1922, p. 3), and there are over fifteen explicit references to each in the text. However, it is far less clear what the relative weights of those two influences were.

About Russell's impact on Wittgenstein, there is much documentary evidence. Wittgenstein worked closely with Russell for the two academic years he spent at Cambridge. From Russell's accounts we know that Wittgenstein was involved in the details of various Russellian views, a notable example being the multiple-relation theory of judgment on which Russell was hard at work in 1913, and which Wittgenstein sharply and effectively criticized. In both the *Tractatus* and the works leading to it, Wittgenstein writes numerous remarks that exhibit a close knowledge of the positions of Russell's *Principles of Mathematics*, or at least of part I, the foundational part of Russell's philosophical logic.

Matters are more difficult to puzzle out with respect to Frege. Direct evidence for a deep engagement with Frege's thought is lacking. Wittgenstein's explicit mentions of Frege almost always concern fairly gross features of Frege's logical theory, and some of the most central aspects of that theory receive no attention at all. Wittgenstein's criticisms of Frege are compressed and cryptic, and therefore difficult to assess as indications of his level of involvement with Frege's views. His meetings with Frege were consequential, but few and skimpily documented; at best we can arrive only at a very general sense of the content of their discussions.

Nonetheless, there are those who claim for Frege an influence far surpassing that of Russell. Clearly Wittgenstein rejects most, if not all, of Russell's views; but, it is claimed, much of the *Tractatus* actually has its roots in Frege's philosophy.

Thus Peter Geach: “a great deal of the *Tractatus* is best understood as a refashioning of Frege’s function-and-argument analysis” (Geach, 1976, p. 64). Cora Diamond approvingly cites this remark of Geach’s and goes on to speak of Wittgenstein’s treatment of sentences and of the say/show distinction as “radically different from Frege’s, but . . . nevertheless . . . deeply Fregean in spirit and inspired by Frege” (Diamond, 1988, p. 180). The picture presented here is of Wittgenstein as rethinking Fregean views *from within*. He makes one major shift, undoing Frege’s assimilation of sentences to proper names, a feature of Frege’s thought that he had criticized from the very start. That shift helps make for the radical differences between Wittgenstein’s and Frege’s views, but, on this picture, the central ideas of the function-argument analysis—the distinction between saturated and unsaturated entities and the general treatment of what categorial differences amount to—are important and direct inspirations that Wittgenstein drew from Frege. Thomas Ricketts makes a similar claim about the relation of Wittgenstein’s thought to Frege’s, although with a somewhat different characterization of the continuities:

There are deep tensions between Frege’s official construal of the content of the axioms of the *Begriffsschrift* and his view of judgment that underlies the identification of the *Begriffsschrift* as logic. Wittgenstein’s *Tractatus* is, in large measure, a response to these tensions in Frege’s thought.

Wittgenstein largely shares Frege’s underlying view of judgment. . . . In the *Tractatus* he aims to develop a conception of logic that remains true to it. (Ricketts, 1985, p. 3)

Obviously, these claims of influence are based not on the explicit evidence, but rather on similarities between the views Wittgenstein on the one hand and Frege on the other are read as espousing. In particular, over the last twenty years there has grown up a school of thought about Frege that opposes the entrenched view of him as an arch-realist and as concerned with truth-conditional semantics. On this line of interpretation, Frege is not providing a semantic theory, not trying to link language to a world of independently given objects (and functions). In his denial that logic needs any prior grounding, Frege is denying that there is a stance from which to do such linking. Along with this goes a construal of the categorial differences in Frege’s logical theory: they are unearthed through logical analysis of statements; they do not register features of reality, in the sense of properties that some entities have and others lack; rather, they are distinctions that are embodied in the way that language functions.¹

Readings of the early Wittgenstein that are in important respects parallel have also been framed over the last few years. In Wittgenstein we find explicit remarks about the unavailability of a stance from which to do semantic theorizing. Earlier readings of the *Tractatus* de-emphasized this by treating the showing of things that cannot be said as though it were a type of “unofficial saying.” In contrast, some recent commentators—most notably Cora Diamond—have urged that “official saying” is the only kind of saying, and what cannot be said is plain nonsense, rather than something that can be “unofficially said.” As a consequence, such readings take the seemingly ontological remarks in the *Tractatus* as at best presenting us with matters that have the same status as the categorial distinctions in Frege, on the sort of interpretation of Frege alluded to above.²

These parallel readings of Frege and Wittgenstein can be argued to provide support for each other, if we can take Wittgenstein to have been deeply enough engaged with Frege's thought to recognize the features in it on which the above reading of Frege rests. If we can portray Wittgenstein's position as the result of deeply thinking through the view Frege has, on the interpretation I have mentioned, it becomes more plausible to ascribe that view to Frege, despite the long tradition that exists of taking Frege to be doing semantic theorizing and despite the texts of his that seem to support this imputation. Cora Diamond puts it just a little differently: "If the reading of Frege which I take for granted helps us to see the relation between Frege's thought and that of Wittgenstein, that would be some evidence that the reading was similar to Wittgenstein's" (Diamond, forthcoming, p. 26).

Now, although I would argue in favor of these sorts of readings of both Frege and Wittgenstein, I want nonetheless to cast doubt on the assessment of Frege's influence on Wittgenstein that Geach, Diamond, and Ricketts express. Thus I hold that Wittgenstein and Frege arrive at views—subtle and complex ones, with many features that are not at all intuitive or obvious—that have many similarities, and that they do so rather *independently*. That is a noteworthy phenomenon, to which I will briefly return at the close.

The issue is of consequence for the understanding of both figures. It is probably noncontroversial to say that one important strategy for gaining more insight into the *Tractatus* is to figure out what was moving its author: what he thought the problems were, what wanted explanation that his predecessors Frege and Russell did not correctly explain or even try to explain. Indeed, a good bit of the recent progress in *Tractatus* studies comes from advances on precisely this way in. (For instance, see the treatment of the development of the picture theory in relation to Wittgenstein's criticisms of Russell's multiple-relation theory of judgment in Ricketts, 1996b.) The problem, as I see it, then, is that ascribing the kind of influence to Frege that Geach, Diamond, and Ricketts do may constrain our perspective on the explanatory agenda that Wittgenstein is working with. It may well be that Frege's work, thought through from a certain direction, winds up having parallels to the views of the *Tractatus*; but it will not be healthy for our detailed understanding of the *Tractatus* to take it that the route *through* Frege must have been the route Wittgenstein took to those views.

The question of explanatory agendas also concerns our understanding of Frege. A too ready ascription to Wittgenstein of a deep engagement with Frege's philosophy lets us think too quickly that Wittgenstein is uncovering tensions intrinsic to Frege's thought; that is, that the criticisms Wittgenstein is launching come from within an enterprise that Frege is undertaking. Yet it may well be that Wittgenstein's agenda is distinct from Frege's; and that he is raising problems that from within Frege's scheme of ideas need have no answers. Thus, to blind ourselves to this possibility is to limit the depth of our understanding both of Frege and of Wittgenstein.³

In this essay I investigate what we can tell of the influence of Frege on Wittgenstein, that is, of Wittgenstein's understanding of Frege, when we prescind from taking as evidence the alleged similarities in the views of the two. After looking at the historical record of interchanges between Frege and Wittgenstein and reviewing some major contrasts between Frege's and Russell's logical theory, I shall

focus on the concerns Wittgenstein expresses in the period leading up to the composition of the *Tractatus*, as far as they can be surmised from his early writings, the 1913 “Notes on Logic,” the 1914 “Notes Dictated to G. E. Moore,” and the 1914 and 1915 *Notebooks*,⁴ with an eye to seeing the extent to which they might evidence a deep engagement with Frege’s views.

My conclusions will be largely negative. Wittgenstein clearly had a decent familiarity with Frege’s doctrines. Moreover, no one can doubt the enormous respect Wittgenstein had for Frege. As Diamond (forthcoming) emphasizes, Wittgenstein felt an enormous temperamental affinity with Frege. What I find no evidence for, though, is a really *internal* appreciation on Wittgenstein’s part of Frege’s thought: a sense that Wittgenstein tries to live within it, to see what questions it encourages and what questions it does not leave space for.

I. First Contact

Apparently, Wittgenstein started reading Frege’s work while he was a student in Manchester or even earlier, most probably with the *Grundgesetze*, not exactly the easiest entrée to the corpus. In the summer of 1911 he visited Frege to ask advice about his continuing to study and write philosophy; Frege “wiped the floor” with him, as he later told Geach, but was nonetheless encouraging, and so Wittgenstein went to Cambridge.⁵ Sixteen months or so later, Wittgenstein once again visited Frege, reporting on the visit in a letter to Russell: “I had a long discussion with Frege about our Theory of Symbolism of which, I think, he roughly understood the general outline. He said he would think the matter over” (Wittgenstein, 1995, p. 21). McGuinness thinks that it was on this occasion that Frege put to Wittgenstein an objection to Russellian talk about “complexes” (McGuinness, 1988, p. 164). This visit had a significant impact on Wittgenstein, as we shall see in section III. The next contact seems to have taken place about a year later. Wittgenstein wrote in October to ask for a visit, and in November to present objections to Frege’s views; the letters have not survived, but Heinrich Scholz’s brief characterizations of them from the 1930s are published in Frege, 1976. The visit occurred in December 1913 and was, apparently, the last time the two met. Their correspondence during the war does not seem to contain any further remarks by Wittgenstein on Frege’s views, judging from Scholz’s lack of comment on Wittgenstein’s letter to Frege and on the lack of philosophical content in Frege’s letters to Wittgenstein.⁶

There is some evidence that Wittgenstein discussed Frege’s work with others. I know of no record of such discussion with Russell. Particularly in 1912–13, Wittgenstein and Russell worked on their joint “theory of symbolism,” which is pervasively Russellian in its formulation and aims. However, in that year Wittgenstein also met Philip Jourdain, who had published his account of Frege’s logical theory in 1912 and was interested in producing translations of Frege’s writings. The two of them got on well, it appears, and surely must have talked about Frege’s work. Jourdain wrote Frege in March 1913, saying that Wittgenstein and he agreed that Frege should not try to redo the theory of real numbers from *Grundgesetze* because *Principia Mathematica* had done it “splendidly,” and in January 1914 Jourdain reported to Frege that Wittgenstein would be looking over Jourdain’s translation of excerpts from *Grundgesetze*.⁷ Through Jourdain, Witt-

genstein would have had access to copies of most, if not all, of Frege's publications. It thus seems most reasonable to surmise that Wittgenstein did spend time thinking about Frege's views, most particularly in 1912–13 and the autumn of 1913. This is just the period when Wittgenstein started writing; the *Notes on Logic* stem from October 1913 and presumably draw on material from notebooks Wittgenstein had been working on for at least a few months.

However, it gives one pause to see Jourdain asking Frege the following questions in a letter of January 1914:

(1) [If] your theory of functions "erster bzw. zweiter Stufe" is not the same as Russell's theory of orders. . . . (2) whether you now regard assertion (\vdash) as merely psychological; (3) whether, in view of what seems to be a fact, namely, that Russell has shown that propositions can be analyzed into a form which only assumes that a name has a "Bedeutung," & not a "Sinn," you would hold that "Sinn" was merely a psychological property of a name. (Frege, 1976, p. 126)

These are curious questions, coming from a supposed expert on Frege's philosophy. As for (1), Frege's theory of functions of the first and second level is a theory of extensional entities, and completely impredicative; Russell's theory of orders, which is fundamental to the ramified theory of types, is intensional and predicative. With respect to (2), there is no hint in Frege's writings that he took assertion to be psychological; why should he have come to believe it in 1914? The question of whether there was a logical sense of assertion as distinguished from a psychological one was Russell's question, going back to *Principles of Mathematics* §52, and reappearing from time to time. Finally, in (3) Jourdain assumes that Frege would simply accept Russell's theory of descriptions as the analysis of proper names. This overlooks the privileged role of names in Frege's logical theory, and the way Frege's understanding of the quantifier is lodged in the relation between a quantified statement and its instances. But there is an even more serious misunderstanding here, for the question presupposes that Frege has a category of propositions in which the analysis provided by the theory of descriptions would be exhibited. Of course, the only Fregean category that could serve is that of thoughts, and the constituents of thoughts are senses. Apparently, Jourdain is misreading Frege as wildly as Russell did in "On Denoting," namely, as having essentially the *Principles* view of propositions, with senses corresponding to denoting concepts.

Of course, this is Jourdain writing, not Wittgenstein, and we need not think that these questions reflect any discussions Jourdain had with Wittgenstein, who by this time had been gone from Cambridge for half a year. But it is a warning flag, telling us to watch out for what we might call a Russellian lens through which Frege's theories may have been pervasively viewed in Cambridge.

II. Frege versus Russell

Frege's philosophical logic and Russell's have many points of difference. To assess their influence on Wittgenstein, it is necessary to keep these differences in mind. In this section I shall lay out some contrasts between them concerning categorial distinctions in ontology, the nature of analysis, and the notion of truth.

On the reading of Frege with which I am concerned, “ontological categories are wholly supervenient on logical ones” (Ricketts, 1986, p. 66). Diamond emphasizes that the distinction between objects and concepts is not a difference in properties of different sorts of things; its cash value is simply the way object-expressions and concept-expressions behave differently in our language, particularly in our inferences (Diamond, 1984; see also Diamond, 1997, p. 75). In this way, Frege’s ontological categories are seen as arising out of the analysis of whole judgments. Indeed, Frege repeatedly emphasizes this direction. In 1919 he writes, “I do not begin with concepts and put them together to form a thought or a judgment; I come by the parts of a thought by analyzing the thought” (Frege, 1979, p. 253). In 1880 he says, “instead of putting a judgment together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgment” (Frege, 1979, p. 17). In the course of a comment on his differences with Russell’s *Principles*, he notes that a concept “can be distinguished within, but that it cannot be separated from the context in which it occurs” (Frege, 1903, p. 372). On Frege’s view there can be no question of how universals inhere in particulars, or how a concept attaches to an object, or how the parts of a judgment adhere to each other. There can be no question because we only recognize the concept *in* the judgment.

Now Frege does arrive at the “parts” of a thought by analysis. But on Frege’s conception, the parts of a thought are not determinate: a thought can be broken up in different ways. Thus, there is no intrinsic structure to the thought. This aspect of his view is evidenced when he remarks that there is no such thing as an intrinsically negative thought, and is strikingly evidenced in his claim that the same thought is expressed (although carved up differently) when we say two functions yield the same values for every argument as when we say the two value ranges are identical (Frege, 1918b, p. 50; 1891, p. 11).

Now Frege’s logical theory has two levels: that of Bedeutung and that of sense. The *judgment-based* nature of his view, if I may call it that, is operative on both. This is clear for senses, given the possibility of equally good different analyses of the same thought. On the level of Bedeutung, the crucial fact is that the referring parts of a sentence are located only given the inferential relations of the sentence with others. All this, I take it, is part of the import of his context principle, “never to ask for the meaning of a word in isolation, but only in the context of a sentence” (Frege, 1884, p. x).

Another feature of the level of Bedeutung is its extensionality. Frege’s logic is entirely extensional. In rendering a thought in logical symbolism, we show its referring parts and how they are put together. The parts so displayed can be replaced *salve veritate* by any other extensionally equivalent ones (coreferential proper names, coextensive concept expressions, etc.). Those referring parts are what instantiate quantifiers; that is, the quantifiers range over the extensional entities to which those parts refer. Of course, thoughts, on the level of sense, are intensional. But the important point is that logic concerns the level of Bedeutung. The laws of logic do not operate at the level of sense; rather, they are general laws about all objects, functions, relations, and so on.⁸

These views of Frege’s can be contrasted, in almost every detail, with Russell’s. To be sure, Russell agrees that analysis is the method of getting from judgments to their constituents. But he views the process as the discovery of the uniquely

determinate primitive parts of propositions, which subsist in and of themselves, are put together in a particular compositional way into the given proposition, and are recognizable outside the confines of that proposition. They are so recognizable outside the context of a proposition, in fact, that a predicative constituent of one proposition can be seen to be the same as a nominative part of another—and he invokes this as an objection to Frege's sharp concept-object distinction (Russell, 1903, §49, §481). Thus Russell's view of propositions and their constituents is *object-based*, not judgment-based. Moreover, for him, a proposition has an intrinsic structure: it is a complex constructed in a unique way from its constituents. The subject matter of logic is these propositions; the truths of logic are truth governing these intensional complexes and their relations with each other.

The contrast becomes even more marked in the period following "On Denoting." For, given the theory of descriptions, Russell is able to dispense with any notion of function but that of propositional function: a function that takes an argument to a proposition. Ordinary functions like " $x + 2$ " or "the father of x " are explained away in terms of descriptions and propositional functions.⁹ Propositional functions have the feature that their arguments will be constituents of the propositions that are their values. That feature is the consequence of the intensionality of Russell's conception and underlies the argument from the vicious circle principle to the restriction of his logic to predicative propositional functions only (Goldfarb, 1989).

Another important contrast between Frege and Russell concerns the notion of truth. This is a difficult issue in Frege, since for Frege the notion of truth has an anomalous status. In "Thought," he presents a regress argument to show that any attempt to define truth must fail, and concludes "the content of the word 'true' is *sui generis* and indefinable" (Frege, 1918a, p. 60). Both the argument and his subsequent considerations show that he does not mean simply that the notion of truth is a primitive notion, not to be defined in terms of anything more basic. After reflecting that "I smell the scent of violets" and "It is true that I smell the scent of violets" have the same content, so that the ascription of truth adds nothing, he concludes: "The meaning of the word 'true' seems to be altogether *sui generis*. May we not be dealing here with something which cannot be called a property in the ordinary sense at all?" (Frege, 1918a, p. 61). In unpublished writing, he goes so far as to say that to ascribe truth to a thought is not to subsume the thought under a concept (Frege, 1979, p. 194); in "On Sense and Meaning," he says that truth is not a predicate of thoughts (Frege, 1892, p. 35). Now what is the point of these remarks? Ricketts explains it thus:

The recognition-as-true of a thought is what an assertion of a sentence that expresses the thought publicly manifests. The word "true" is an attempt to formulate as a predicate the import of asserting something, by splitting off "true" from "recognize-as-true" and treating "true" simply as a predicate of what is judged. Were this possible, given Frege's characterization of judgment and assertion, every assertion would then ipso facto implicitly predicate truth of a thought. The contentless character of the word "true" shows the failure of this attempt to formulate as a predicate, as a condition, the goal of judgment. (Ricketts, 1996a, p. 133)

That is, Frege is committed to the objectivity of truth and its independence from anyone's recognition of that truth, of course; but the conception of truth he has is

immanent within our making of judgments, assertions, and inferences—our recognitions of truth.

The denial of a coherent separable notion of truth is linked to Frege's conception of analysis. For a separable conception of truth would provide a notion of things "out there" and of their behaviors or configurations that exist independent of our knowledge, where those behaviors or configurations would be that which renders our thoughts true or false. That would then immediately subvert the notion that analysis proceeds, starting with whole judgments, being responsible only to inferential patterns: for it would introduce a different factor entirely to which analysis would have to be responsible, namely, those behaviors or configurations. It thus would subvert the supervenience of the ontological categories on logical ones. In fact, it would force on Frege a conception much closer to Russell's.

In this way Frege's view depends on taking "recognition-of-truth" as primitive, and not based on a prior notion of truth *simpliciter*. This is just Frege's notion of judgment. Hence Frege's notion of judgment cannot be taken in the way familiar since Russell, as simply one attitude an agent can have toward a propositional content, on a par with belief or doubt or wish. Rather, it has to be recognition of truth in a factive sense, for it rather than a notion of truth carries the weight of Frege's insistence on objectivity. And with judgment in this sense comes Frege's notion of assertion, the expression of judgment.

All this is completely foreign to Russell. In the period of the *Principles*, Russell has a robust notion of truth, as a property of propositions. Later on, after he abandons propositions, he still has a robust notion of fact. Now in the *Principles* Russell invents a notion of "asserted proposition," but this has nothing to do with Frege's idea of assertion as the expression of a recognition-of-truth. Russell's notion seems to have the point of distinguishing a true proposition as a proposition, from the "propositional concept," since for Russell "Caesar died" and "the death of Caesar" are in constant danger of collapsing into the same item, a complex of Caesar and death. (This clearly evidences what I called the object-based nature of Russell's metaphysics. See Russell, 1903, §52.) However, Russell worries about whether there really is such a notion of asserted proposition, in the logical sense, or whether there is nothing more than assertion "in the psychological sense," a matter merely of an agent's relation to the propositional complex. This nest of issues has no analogue in Frege's theory; conversely, the notions of judgment and assertion central to Frege's immanent conception of truth have no analogues in Russell's.

III. Wittgenstein's Criticisms

The priority for Frege of the notion of recognition-of-truth to that of truth did not register on Wittgenstein, or at least there is no evidence that it did. This should occasion no surprise. Frege elaborates the point only in "Thoughts," which was not published until 1918, and in unpublished writings. The one remark cited above from "On Sense and Meaning," that the relation of thought to truth is not that of subject to predicate, would not naturally be read as having the import I have ascribed to it, without the more insistent and fuller treatments of the later writings, especially since Frege freely indulges in invoking the notion of truth when he

introduces the distinction between *Sinn* and *Bedeutung*, and in all the opening sections of the *Grundgesetze*.

Nevertheless, of the topics on which Wittgenstein explicitly criticizes Frege, truth and assertion figure prominently. "The verb of a proposition is not 'is true' or 'is false,' as Frege believes, but what is true must already contain the verb" (NL 100, TLP 4.063); "Every proposition must *already* have a sense; assertion cannot give it sense, for what it asserts is in the sense itself" (TLP 4.064). Understanding these and Wittgenstein's other remarks on the topic is a complicated matter that I cannot unravel here.¹⁰ It does appear likely that Wittgenstein's view of Frege's notion of assertion does not accurately capture what Frege was doing after 1890. It is an imputation to him of a view that is in part that of the *Begriffsschrift* and in part that which Russell was trying to formulate in the *Principles* as assertion "in the logical sense." One aspect of Wittgenstein's imputation, it appears, is that for Frege assertion plays the role of taking a proposition (or thought) and making it into something that represents how things are. This is reminiscent of Russell's attempt to use assertion to distinguish between "Caesar died" and "the death of Caesar." Now, it can appear that the same job has to be done in Frege's system, since for Frege sentences are names, and so one has to somehow transform a name into a depiction of a situation. Hence, it would be quite natural for Wittgenstein to read Fregean assertion in this way. What this ignores, though, is precisely Frege's denial of truth as a property of thoughts. That is, if recognition-of-truth is basic and there is no notion of truth apart from this, then there is no notion of "situation," and no notion of what the name has to be converted into in order to judge it. Later Frege became explicit that he has no notion of situation, in the sense of configuration of objects; in "Thoughts" he explains "fact" as a thought that is true (Frege, 1918a, p. 74). This suggests that Wittgenstein's work in the pre-Tractarian period is quite pervasively un-Fregean, since much of his writing in that period takes facts for granted and asks how they are represented in language. Moreover, questions about the realm of facts, such as whether there are negative facts, loom large. Such questions make no sense in Frege's framework.

Implicit in what I have just said is a claim about Wittgenstein's criticism of Frege on the doctrine that a sentence is a name—the central criticism, according to Geach and Diamond. I noted that it would be natural, given Wittgenstein's outlook, to think that, if sentences are names, something further would have to be done to get sentences to do more, that is, to present situations. My claim is that Wittgenstein's criticisms get credibility by overlooking Frege's denial of a notion of truth and ignoring the place, as primitive, of the notion of recognition-of-truth instead. This is an issue that needs to be explored further. But I do believe that there may be an explanatory burden that Wittgenstein is placing on Frege that Frege might have the means to shrug off. In that way, Wittgenstein's criticisms would not be purely internal. Of course, the evasion of the explanatory burden may be unsatisfactory for all sorts of other reasons; but it might be *coherently* done within the Fregean framework.

Wittgenstein's own use of the notion of truth certainly does not come from Frege. For Frege, the fact that "p is true" and "p" say the same thing shows that truth is not a property; rather, our concern with truth is given only through our judgmental and assertoric practices. This shows the importance and centrality to Frege of the notions of assertion and judgment. Wittgenstein discards assertion as psycho-

logical (NL 95); rather, the equivalence for him of “*p* is true” and “*p*” motivates the notion that it is intrinsic to propositions that they present situations.

Let us turn to the other Fregean theme I mentioned, the judgment-based nature of analysis in Frege. In Wittgenstein’s pre-Tractarian writings there is no hint of this way of looking at things; all discussions presuppose an opposite orientation. Throughout, Wittgenstein’s concern is on how propositions are constructed from their simpler parts, that is, how the parts are put together into judgments. In “Notes on Logic,” he talks of indefinables and of showing that every proposition about a “complex” is just a description of its constituents. He also says, “If we formed all atomic propositions, the world would be completely described if we declared the truth or falsity of each” (NL 103). In “Notes Dictated to G. E. Moore,” he says, “We want to explain the relation of propositions to reality. The relation is as follows: Its *simples* have meaning = are names of simples; and its relations have a quite different relation to relations; and these two facts already establish a sort of correspondence between a proposition which contains these and only these, and reality” (MN 112). Propositions are taken to have a unique structure, built up from undefinables or simples. “But it is clear that components of our propositions can be analysed by means of a definition, and must be, if we want to approximate to the *real structure* of the proposition” (NB, May 9, 1915, emphasis mine). He goes on, “And can it not now be asked whether this process comes to an end?” His answer underscores the object-based nature of Wittgenstein’s thinking: “When the proposition is just as complex as its reference, then it is *completely* analyzed.” This conception also underlies Wittgenstein’s occasional worry about whether there are subject-predicate propositions. This simply could not be a question for Frege because it presupposes a notion of the intrinsic structure of a proposition. In an entry of September 20, 1914, Wittgenstein writes, “the point in the above cases is to say how propositions hang together internally. How the *propositional bond* comes into existence” (cf. TLP 4.221(b); cf. Russell, 1903, §54). As I have mentioned, this is a question that Frege’s conception of analysis precludes.

To be sure, Wittgenstein’s thinking develops and changes on the question of simples. In the *Notebooks* entries of June 1915, we see an extended consideration of the idea of simple object, and a reciprocal process, in which the simplicity is not assumed as a feature of ontology, as the remark I cited above would seem to have it, but rather is a matter of linguistic phenomena. That is, Wittgenstein comes to see that features of language are what guide analysis and provide the criterion for when simplicity is reached: “It can be seen that a name stands for a complex object from an indefiniteness in the proposition in which it occurs” (NB, June 21, 1915; cf. TLP 3.24). This is one of the few remarks in the *Notebooks* that hints that the notion of simple object is not given ontologically, is not a feature of the world assumed as given, but rather is arrived at through the analysis of our propositions.¹¹ This, then, has a ring reminiscent of Frege—but only reminiscent, since Frege has no place in his view for simple objects, or for the notion that “A proposition has one and only one complete analysis” (TLP 3.25). The pervasiveness in Wittgenstein’s early writing of the assumption of simple elements and of the uniqueness of structure of propositions makes it extremely implausible to claim that a Fregean tinge to the final view is a result of Wittgenstein’s coming to that view via Frege, that is, by thinking through Frege’s views on the logical basis of ontology and coming up with the need for simples and uniqueness of analysis

from that process. Rather, it seems clear that Wittgenstein starts with a view straight out of Russell and thinks *it* through, in particular, thinks through the question of how we could attain the requisite notion of simplicity on logical grounds, avoiding Russell's epistemological characterization. As it happens, he winds up with a position that has a resemblance to Frege's about ontology.

There is another resemblance between Wittgenstein's ontological thought and Frege's, and that is in its notion that objects can have different logical forms, that is, different possibilities of combinations with each other. This is a more abstract version of Frege's distinction between saturated object and the various sorts of unsaturated function (such as concepts and relations); if the distinction is inflected as one that comes out of logical analysis of propositions, rather being than ontologically given, the resemblance is greater still. Now here we can see direct influence, stemming from Wittgenstein's visit to Frege in December 1912. "The complex problem is now clearer to me," Wittgenstein reports to Russell in December, and then, in January, "I now think that Qualities, relations, etc. are all copulae! That means I for instance analyse a subject-predicate proposition, say 'Socrates is human' into 'Socrates' and 'Something is mortal'. . . . It now becomes impossible to substitute the wrong way round" (Wittgenstein, 1995, p. 24). Of course, this goes through much development in his writing, and even here it is framed as an idea, Wittgenstein says, about "atomic' complexes." One major difference should not be overlooked. Frege's functions (including concepts and relations) took objects as arguments and returned objects as values, whereas Wittgenstein's qualities and relations join into complexes with objects.

In all his writing, Wittgenstein makes no explicit mention of Frege's extensional conception of function. Wittgenstein himself assumes an opposite stance, the stance of Russell after "On Denoting": that all complexity is propositional complexity and that all functions are propositional functions. This is clear in the passage in the *Tractatus* on the theory of types in a nutshell (3.333): "The reason why a function cannot be its own argument is that the sign for a function already contains the prototype of its argument, and it cannot contain itself." This and the ensuing explanation demand an intensionalist view of the functions, that is to say, demand the idea that any argument to a function will occur in the value of the function. Moreover, as is pointed out in Hylton, 1997, only if Wittgenstein is understood as using the word "function" exclusively for Russellian propositional functions do his remarks in the 5.2s that distinguish between functions and operations make even *prima facie* sense, for example, 5.25: "The occurrence of an operation does not characterize the sense of a proposition. . . . (Operations and functions must not be confused with each other.)" Generally, all the discussions in "Notes on Logic," "Notes Dictated to G. E. Moore," and the *Notebooks* seem to presuppose this Russellian view.

Although Wittgenstein does not deal with Frege's notion of function in the pre-Tractarian writings, he does bring it up at one place in the *Tractatus*:

The arguments of functions are readily confused with the indexes of names. . . . For example, when Russell writes ' $+_c$ ' the ' $_c$ ' is an index which indicates that the sign as a whole is the addition-sign for cardinal numbers. But the use of this sign is the result of arbitrary convention and it would be quite possible to choose a simple sign instead of ' $+_c$ '; in ' $\sim p$ ',

however, ‘p’ is not an index but an argument: the sense of ‘ $\sim p$ ’ cannot be understood unless the sense of ‘p’ has been understood already. . . .

A confusion between argument and index lies, if I am not mistaken, at the bottom of Frege’s theory of the Bedeutung of propositions and functions. For Frege, the propositions of logic were names, and their arguments were the indexes of those names. (5.02)

Cora Diamond (forthcoming) treats this passage in detail. She takes Wittgenstein to be expressing an elaborate criticism of Frege’s idea that sentences are names of truth-values, by way of showing that such a conception conflates the truth-conditions of a sentence, say, “ $5 > 4$,” with a different condition, namely, “The object $5 > 4 =$ the True.” I do not find her reading convincing, on several grounds. It takes Wittgenstein to be simply assuming that the sense of “ $5 > 4$ ” is distinct from the sense of “ $5 > 4 =$ the True,” and it is not clear that Frege has to believe this. Moreover, the criticism presupposes that there are two ways of using a sentence, namely, to express the thought that something is the case and “merely as an expression which has as its reference some object.” To be sure, that presupposition is, I think, the heart of any Wittgensteinian criticism of Frege’s identification of sentences as names; but, as a result, the talk of indexes does not seem to add anything to the criticism.

The reading I would give of the passage is simpler. First, note that Wittgenstein is concerned with Frege’s view of the Bedeutung not just of sentences, but also of functions (a word in the passage that Diamond’s reading does not at all pick up on). What he has in mind, I take it, is that in the Bedeutung of a sentence, there will be no trace of the Bedeutungen of the constituents of the sentence—that is the essence of Frege’s notion of function. In that case, the occurrence of the particular subsentential parts in the sentence is not essential to the identity of the value of the sentence; those parts thus occur as indexes. Now, Diamond develops her more elaborate reading to forestall a defense of Frege that relies on the distinction of sense and Bedeutung. After all, even though the Bedeutung of the sentence contains no trace of the Bedeutungen of its parts, the sense of the sentence *will* contain the senses of its parts; sense, in a word, is compositional. I take it, in contrast, that Wittgenstein is simply dismissing two-level explanations. The last sentence of the passage suggests as much in bringing up the propositions of logic as a particularly pointed example (the sentence would presumably remain true if “of logic” were omitted). For Frege, logic has to do with the realm of reference, with the behavior of functions and objects. A proposition of logic therefore will not show anything about the propositions that are its constituents. “ $p \vee \sim p$ ” names the True, and there is no particular character it has that is the result of its being just the disjunction of a proposition and its negation; thus, in Frege’s theory, its being a logical truth cannot show that “ $\sim p$ ” negates “ p ” and “ p ” negates “ $\sim p$.”

Indeed, I think that Wittgenstein took Frege’s two-level view of sense and Bedeutung to be a nonstarter. Frege’s view separates what logic is about from sentential senses, whereas Wittgenstein took it for granted, I think, that logic is about propositions, about the sense of sentences. Moreover, Frege’s view separates the having of sense from being true or false, and certainly after the summer of 1913 Wittgenstein would not think there could be such a separation. (It is to that summer that I would date Wittgenstein’s formulating the idea that he calls the “bipolarity” of propositions: “Every proposition is essentially true-false. . . . Thus every

proposition has two poles, corresponding to the case of its truth and the case of its falsehood. We call this the *sense [Sinn]* of a proposition" (NL 98–99.) Finally, Wittgenstein's acceptance of the theory of descriptions makes subsentential Sinn unnecessary.

This last issue is reflected in Wittgenstein's never using the word *Sinn* in connection with anything but sentences. Now, in the pre-Tractarian writings he does appropriate Frege's terminology and frames a Sinn-Bedeutung distinction for propositions. (In the *Tractatus*, Wittgenstein does not use the word *Bedeutung* when speaking solely about propositions, but neither does he deny that propositions have Bedeutung, at least in some broad sense. In some contexts, when he is talking about arbitrary expressions, a class that includes both names and propositional signs, he allows that any of them can have Bedeutung. See, for example, 3.33.)

In "Notes on Logic" and "Notes Dictated to G. E. Moore," the Bedeutung of a proposition is the fact that corresponds to it, that is, makes it true or makes it false; thus the Bedeutung is in the world, and p and $\sim p$ have the same Bedeutung. (In the *Tractatus*, Wittgenstein calls this the reality that corresponds to p , and to $\sim p$; see 4.0621.) The sense of the proposition, on the other hand, is the circumstance under which the proposition would be true, and hence is what is known when the proposition is understood, even if it is not known whether the proposition is true. The separation of what corresponds in reality to the proposition and what we know just when we understand a proposition is Fregean in spirit and shows Wittgenstein appreciating an aspect of Frege's distinction. For Frege, it was a distinction between the things a proposition is about and a whole other level, or realm. Wittgenstein's point is that it is not a matter of two levels at all.

Conclusion

We have excellent documentation of Wittgenstein's thinking from October 1913 through 1916, and this period includes the writing of most of the *Tractatus*. In this documentation, although one sees a significant amount of concern with Frege, there is little evidence of a full appreciation of Frege's views, and no evidence of Wittgenstein working through those views from within. Rather, the basic framework and the basic stance are thoroughly inherited from Russell; and the working through of them, from within—perhaps the *Überwindung* of them—is visible.

As to the whole idea of working through Frege's views from within, there are two possibilities. It may be that "tensions" various commentators have seen in Frege are not truly there; their appearance is only due to underestimating the resources Frege can marshal to avoid certain questions. I do not think this is fully correct, but I do believe a fuller understanding of Frege's views about truth makes it a far subtler matter to bring out what the tensions really are. If the tensions are there, then thinking them through will lead, I expect, to conclusions that are reminiscent of the *Tractatus*. In this case we might say, appropriating what Wittgenstein says about solipsism and realism (TLP 5.64), that the program of Fregean judgment-based analysis, when its implications are followed out strictly, coincides with the end product of Russellian object-based analysis, when its implications are similarly followed out. But it was the latter route, I maintain, that was Wittgenstein's.

Notes

I am grateful to Cora Diamond and, especially, Thomas Ricketts, for helpful discussions on the topic of this essay, and to Ricketts and Ian Proops for useful comments on a draft.

1. See, e.g., Diamond, 1984; Ricketts, 1986; Weiner, 1990.
2. This reading is formulated in Diamond, 1988, and analyzed in Goldfarb, 1997.
3. Burton Dreben long urged the notion that great philosophers always misunderstand each other because they bring their own explanatory agendas to philosophical problems, and that reshapes the problems in such a way that they cannot see accurately what their predecessors or contemporaries are doing. I am interested in whether this is true of Wittgenstein vis-à-vis Frege. I do not, by the way, believe it is true of Wittgenstein vis-à-vis Russell; but, as Dreben put it when I mentioned this thought to him, that case is different, since Russell did not understand what he himself was doing—it took a Wittgenstein to figure it out.
4. These are all published in Wittgenstein, 1979; below they are cited as “NL,” “MN,” and “NB,” respectively.
5. See McGuinness, 1988, p. 83; and Monk, 1990, p. 36. I do think Monk’s claim that Wittgenstein “devoted much of his first two terms at Manchester to a close study of both Russell’s *Principles* and Frege’s *Grundgesetze*” somewhat hyperbolic (Monk, 1990, p. 33).
6. It is, however, interesting to note Scholz’s précis of Wittgenstein’s letter to Frege of August 25, 1915: “This military postcard is the beginning of Wittgenstein’s correspondence with Frege about Wittgenstein’s ‘Abhandlung,’ later called the *Tractatus*. Aside from that, Wittgenstein communicates his new address” (Frege, 1976, p. 266). I believe this provides support for McGuinness’s early dating of the *Prototractatus* (McGuinness, 1989).
7. Frege, 1976, pp. 124, 126. I am grateful to Cora Diamond for reminding me of the Wittgenstein-Jourdain connection.
8. For elaboration of Frege’s conception of logic, see Goldfarb, forthcoming.
9. For more detail, see Hylton, 1994.
10. For a detailed discussion, see Proops 1997.
11. Kremer (1997) provides an excellent account of Wittgenstein’s progress in coming to a logical view of the simples, supplanting the epistemological criterion of simplicity found in Russell. Kremer, moreover, connects that progress to the importance for Wittgenstein of the context principle, which Wittgenstein lifts verbally from Frege.

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Frege and Early Wittgenstein on Logic and Language

Danielle Macbeth

In the preface to the *Tractatus*, Wittgenstein claims to be “indebted to Frege’s great works.”¹ Themes from those works are directly addressed in seventeen numbered passages, but in only one (3.318) does Wittgenstein express wholehearted agreement with Frege. In the remaining sixteen, his agreement with Frege is either qualified (3.325, 4.431, 5.451, 5.4733) or altogether absent. Wittgenstein argues, for instance, that neither the identity sign (6.232) nor the judgment stroke (4.442) is of any use in logic, that sentences are not names for the True (3.143, 5.02), that the truth-functional connectives are not relations (5.42). His criticisms are further developed in other passages, and they have been defended in turn by many recent commentators.² Frege, it would seem, was simply wrong on many fundamental points. It is, furthermore, hard to understand why he was. Why did Frege hold the views for which Wittgenstein criticizes him? Why did he maintain those views even after being made aware of Wittgenstein’s criticisms? Set against the backdrop of the standard reading of the relationship between Frege and Wittgenstein, answers have not been forthcoming. Perhaps it is time to entertain the possibility that the familiar differences between their views are themselves the fruits of a quite unfamiliar difference.

Frege and Wittgenstein, I will argue, have radically different conceptions of meaning. Whereas Wittgenstein takes a sentence to be contentful in virtue of its picturing a state of affairs, takes meaning to be given by truth conditions, Frege (after 1890) rejects this conception of meaning in favor of an inferentialist one. On Frege’s considered view a sentence is contentful in virtue of its displaying or depicting an inferential role, whether or not that sentence can be given an analysis relative to which truth conditions are formulable. It is reasoning, not representing, that is foremost on Frege’s mature conception of meaning. Should we follow Frege in demanding that our logic include both the judgment stroke and a

sign for identity, or should we instead follow Wittgenstein in deeming them useless? Should we take the laws of logic to be substantive truths as Frege urges, or are they *sinnlos* as Wittgenstein thinks? Can general propositions serve in the expression of causal laws as Frege claims, or is Wittgenstein right to hold that “the only necessity . . . is *logical* necessity” (6.37), that “outside logic everything is accidental” (6.3)? Such questions cannot be answered until it is determined what conception of meaning we ought to endorse, whether Frege’s or Wittgenstein’s.³

I. Wittgenstein’s Tractarian Account of Language

Wittgenstein first visited Frege in Jena in the summer of 1911. He wanted Frege’s advice regarding his study of philosophy. Wittgenstein returned to Jena to talk with Frege in December 1912, having spent the intervening months in Cambridge studying with Russell. Shortly after that second trip, in January 1913, Wittgenstein wrote to Russell: “I have changed my views on ‘atomic’ complexes: I now think that qualities, relations (like love) etc. are all copulae! . . . The reason for this is a very fundamental one: I think that there cannot be different Types of things! In other words whatever can be symbolized by a simple proper name must belong to one type. And further: every theory of types must be rendered superfluous by a proper theory of symbolism.”⁴ The seed of Wittgenstein’s Tractarian conception of language was sown.

Its first fruits were the “Notes on Logic,” dictated in October 1913. Already in these notes Wittgenstein develops the contrast between object names, on the one hand, and expressions for qualities and relations, on the other, into what is essentially the picture theory of the *Tractatus*. “In ‘aRb’ it is not the complex that symbolises but the fact that the symbol ‘a’ stands in a certain relation to the symbol ‘b.’ Thus facts are symbolised by facts, or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world” (NL 96; cf. 3.1432).⁵ “Symbols,” Wittgenstein has discovered, “are not what they seem to be.” In particular, “in ‘aRb,’ ‘R’ looks like a substantive, but is not one. What symbolizes in ‘aRb’ is that R occurs between a and b” (NL 98). “R” is not, Wittgenstein tells us, a designating symbol. As the point is put in the “Notes on Logic,” propositions consist of names (of objects) and forms, where “there is no thing which is the form of a proposition, and no name which is the name of a form” (NL 105). Signs such as “R” as it occurs in “aRb” have, in other words, no meaning in isolation; they are not significant bits of language in their own right. They have only a use. The nature of language so conceived is perspicuous in the symbol system Sellars calls Jumblese, a language containing no quality or relation words at all.⁶ Whereas in English one represents an object *a* as red, say, by concatenating the sign “is red” to a name for the object, in Jumblese one represents that *a* is red without the use of an auxiliary sign design, by writing “*a*” (i.e., a name for *a*) in, say, bold: ***a***. In Jumblese, an object is represented as being a certain way by representing a representation of it a certain way; a relation between two objects is represented by relating representations of those objects, say, one above the other, and so on. Such a language shows the function of, and the relationship between, referring and predicative expressions. Given such a language, one will not be much tempted to think that predicative expressions

designate anything in their own right. Their function in language is only to characterize objects otherwise given.

According to the conception of language developed in the 1913 "Notes on Logic," names and predicative expressions are distinguished as having fundamentally different logical roles: names refer to objects, and predicates are used to characterize them as thus and so. Names function as representatives of objects, and the form or manner of their presentation is isomorphic in some way to the state of affairs depicted. This isomorphism, for Wittgenstein, is the ground of the truth or falsity of the proposition. "I say that if an *x* stands in the relation *R* to a *y* the sign '*xRy*' is to be called true to the fact and otherwise false" (NL 95). It follows, Wittgenstein sees, that "in my theory *p* has the same meaning as *not-p* but opposite sense. The meaning is the fact. . . . In *not-p*, *p* is exactly the same as if it stands alone" (NL 95; cf. 4.0621).⁷ "This point," he adds, "is absolutely fundamental": "the chief characteristic of my theory is that, in it, *p* has the same *meaning* as *not-p*" (NL 103). It is one and the same fact that determines the truth of *p* and the falsity of *not-p*, or conversely, the falsity of *p* and the truth of *not-p*:

Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely? No! For a proposition is then true when it is as we assert in this proposition; and accordingly if by "*q*" we meant "*not-q*," and it is as we mean to assert, then in the new interpretation "*q*" is actually true and *not* false. But it is important that we *can* mean the same by "*q*" as by "*not-q*," for it shows that neither to the symbol "*not*" nor to the manner of its combination with "*q*" does a characteristic of the denotation of "*q*" correspond. (NL 97–98; cf. 4.062, 4.0621)

A certain sign design—say, in Jumblese, an object name written in bold—can be used either to express the fact that that object is, say, red or (in a different language) to express the fact that it is not red: "I must first determine under what circumstances I call a proposition true, and thereby I determine the *sense* of a proposition" (NL 99; cf. 3.11). If I call the (Jumblese) proposition "***a***" true just in case *a* is red, then the sentence has one sense; if I call that same proposition true just in case *a* is not red, it has another. In this way, a proposition divides the possibilities into two. But the presence of "*not*" does not by itself indicate which is which: "it is only from both, the negation and the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition" (NL 97). Thus, "*p*" and "*not-p*" are not pictures of two different facts, the one true and the other false. They have the same meaning. Their difference lies instead in their senses, the one saying, in effect, that this is how things are, namely, like this, the other that this is how things are, namely, not like this.

A sentence conceived as such a picture itself "asserts" that things are thus and so: "that a certain thing is the case in the symbol says that a certain thing is the case in the world" (NL 96): "A proposition *shows* its sense. A proposition *shows* how things stand *if* it is true. And it *says that* they do so stand" (4.022). In language so conceived there are not two separable moments, the presentation of a sense and the assertion of it as true, but only one: to present a sense in a picture *is* to present it as true; to characterize an object as thus and so is, in virtue of the nature of characterizing, to characterize it as *being* thus and so. The only role left

for the notion of assertion, then, is a “psychological” one (NL 95). Assertion can concern itself only with the question whether this or that person does indeed assent to the proposition in question. In Wittgenstein’s picture language, “Frege’s ‘judgement stroke’ ‘ \vdash ’ is logically quite meaningless” (4.442).

All these Tractarian themes are developed already in the “Notes on Logic.” What is absent from that account is the context principle which Wittgenstein inherits from Frege: the principle that, as Wittgenstein formulates it, “only propositions have sense; only in the nexus of a proposition does a name have meaning” (3.3).⁸ A proposition, Wittgenstein claims, “shows its sense . . . shows how things stand if it is true” (4.022). What he now sees is that since a proposition’s having sense is a matter of its presenting things as thus and so, a mere object name cannot on its own play its referring role.⁹ Only in the context of a proposition does an object name have meaning; referring is correlative to characterizing. It follows that names do not function logically to bring the objects named to mind. Suppose that they did bring objects to mind. Since objects in fact have various properties, names would in that case bring them to mind with all their various properties. One could not, then, picture an object as other than it in fact is, the object *a* as green, say, if it is red. But of course we can picture things as other than they are: “in a proposition a situation is, as it were, constructed by way of experiment” (4.031); “what a picture represents it represents independently of its truth or falsity” (2.22). If we are to be able to characterize objects as thus and so independent of the facts, then the function of a name must be *only* to refer to an object, as it were, to bring it to mind but not *as* it is: “Names are like points” (3.144), and objects—that is, objects as they matter to logic, as referred to by names—are “simple” (2.02); they “contain the possibility of all situations” (2.014); they are “in a manner of speaking . . . colourless” (2.0232). This is not to say that there are bare particulars. Of course objects have properties and stand in relations; they *are* propertied and related. But logically such entities (i.e., propertied and related objects) have the form of facts: “The world is the totality of facts, not of things” (1.1). The point of the context principle for Wittgenstein is that the logical notion of an object is the notion of an object as referred to in the context of a judgment, in the context of a picture of it as thus and so.¹⁰ The claim is not that the object itself has its properties as components of it and so is in itself only a bare particular; it is that in the fact that the object is thus and so the object is at once referred to and also characterized.¹¹ The smallest unit of cognitive significance is the judgment: not only are “thoughts without content . . . empty,” that is, mere forms in the absence of reference to any object, “intuitions [object names] without concepts [characterizations] are blind”—just as Kant says (A51/B75).

According to Wittgenstein’s Tractarian account, object names function referringly in a way that contrasts logically with predicative expressions. Object names so conceived have no descriptive content, no cognitive significance. They are, logically speaking, blind. It follows, as Wittgenstein argues, that “the identity sign . . . is not an essential constituent of conceptual notation” (5.533). What could it mean to say that $a = b$ where “*a*” and “*b*” are names of the same thing? “Roughly speaking, to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing at all” (5.5303). If names have a referring function as contrasted with a predicative role, then “ $a = b$ ” can have no role to play in the language. One can only show that one is talking about

the same thing again by using the same name again: “Identity of object I express by identity of sign, and not by using a sign for identity. Difference of objects I express by difference of signs” (5.53; see also 6.232).

A judgment on Wittgenstein’s picture theory is a representation of something as something. Predicating is thus already asserting; to present a picture is to present it as true. What account, then, can be given of the conditional, which would seem to present one proposition as true only on the condition that another is true, which would seem to present both without presenting either as true? On Wittgenstein’s picture theory, this is unintelligible. One cannot present a proposition without by the same token presenting it as true, cannot show how things stand if a proposition is true without also thereby saying that they do so stand. A conditional cannot, then, be a relation among propositions.¹² Truth-functions, on the Tractarian view, are not functions at all. They are, Wittgenstein says, operations. What he means is that truth-functions do not contribute to the sense of a proposition—where “the sense of a proposition is its agreement or disagreement with possibilities of holding or non-holding of atomic facts” (4.2)—but instead serve in the presentation of facts, as a kind of punctuation (5.4611). What a conditional expresses, on this view, is a “truth possibility,” that either this is not thus and so or that is thus and so. Such a proposition can be pictured by way of a standard truth table read as a disjunction of pictures (of those assigned the value True in the table), at least one of which must agree with reality in order for the whole to be true. A truth possibility of pictures is in this way also a picture; it shows how things stand if it is true, and it says that they do so stand. In a negated proposition, for instance, one presents a picture: this is how things are, namely, not like this. In a disjunction similarly one presents a picture: this is how things are, namely, either like this or like that. In general, as Hylton puts the point, “the sense of a non-elementary proposition . . . is that one of a number of combinations of elementary propositions obtains, while all of the other combinations do not obtain” (Hylton, 1997, p. 105). A truth-functional compound, representable by way of a truth table, is itself a picture of atomic facts. It is not a picture of secondary propositions in the Boolean sense, a picture of relations among propositions.

Given that a sentence is contentful in virtue of being itself a fact, a kind of picture of a possible state of affairs, it follows that tautologies and contradictions are empty. “They do not represent any possible situations. For the former admit *all* possible situations, and the latter *none*” (4.462). Both are sinnlos; they say nothing (4.461). “Tautologies and contradictions are not, however, nonsensical,” according to Wittgenstein (4.4611). “They are part of the symbolism, much as ‘0’ is part of the symbolism of arithmetic” (4.461). Tautologies and contradictions symbolize a kind of absence of sense, the first because it admits all situations, the second because it admits none, yet are nonetheless ineliminable in logic. One needs the truth-functional operations of negation and disjunction to express the truth possibilities of elementary propositions, even though in a picture language some combinations of these (e.g., “*p* or not-*p*”) do not represent any states of affairs.

Wittgenstein’s picture theory as it is set in place by the fundamental logical distinction between referring and predicating furthermore entails that “the only necessity that exists is logical necessity” (6.37), that “outside logic everything is accidental” (6.3). It follows that causal laws cannot be recognized in a Tractarian language. They cannot be said because only what is accidental can be said. And

they cannot be shown because only what is logically necessary can be shown. Since there is no third option, Frege's claim (e.g., in the *Begriffsschrift* §12) that causal connections can be expressed as generalized conditionals must be rejected. Generalized conditionals cannot express lawful relations among concepts; "to be general means no more than to be accidentally valid for all things" (6.1231). A generalization, say something of the form "all A is B," may in fact be true in virtue of a lawful relation between being A and being B, but what "all A is B" says (assuming that it is not analytic, and hence tautologous and therefore sinnlos) is, on Wittgenstein's view, only that for all x, either x is B or it is not A. Given also that the sense of a sentence must contain already the sense of any sentence that follows from it,¹³ it furthermore follows that the sense of a generalization in a Tractarian language is given by a conjunction of all its instances. Like any truth-functional compound, a Tractarian generality expresses a truth possibility.¹⁴

The fundamental insight underlying Wittgenstein's Tractarian conception of language, the seed from which it grows, is the thought that referring and predicative expressions play fundamentally different and complementary roles in language, the one to designate objects as that about which the judgment is made and the other to describe or characterize those objects as thus and so. A sentence on this view is itself a kind of a fact: an object is represented as being thus and so by means of a particular form or manner of representing it, two objects as related thus and so by means of a form or manner of representing those objects. It follows that to predicate, which (on this view) just is to show how things stand if the object referred to has the property ascribed, is to present a sentence as true. The judgment stroke, then, can have no logical role in such a language. Inference, too, is of merely psychological significance.¹⁵ Identities are logically meaningless, and truth-functional compounds are pictures of truth possibilities. It follows, finally, that the propositions of logic are not properly speaking truths, and that the very idea of a causal law is unintelligible.

II. Frege's Early Conception of Language

Wittgenstein argues that the judgment stroke is logically meaningless. Identities are either useless or nonsense. The only necessity, he argues, is logical necessity, and what is logically necessary is sinnlos. In each case, his view is traceable to his commitment to the fundamental logical distinction between referring and predication. What I want now to suggest is that Frege rejects each of these conclusions and so comes also to reject the conception of meaning on the basis of which they are made.

Both in the early *Begriffsschrift* logic and in his mature logic, Frege holds that the judgment stroke is essential. Nonetheless, Frege's account of the judgment stroke in the early logic fails to reveal any essential role for it. He writes in the *Begriffsschrift* (1879):¹⁶

A judgement will always be expressed with the aid of the symbol \vdash which stands to the left of the symbol or combination of symbols giving the content [*Inhalt*] of the judgement. If we omit the small vertical stroke at the left end of the horizontal one, then the judgement is to be transformed

into a mere combination of ideas [blosse Vorstellungverbindung] of which the writer does not state whether or not he acknowledges its truth. (BGS, §2)¹⁷

So described, the judgment stroke seems merely to mark a writer's assent to a proposition, merely to indicate (as Wittgenstein puts it in the *Tractatus* 4.442, speaking of Frege's and Russell's writings) "that these authors hold the propositions marked with this sign to be true." The impression is reinforced by Frege's suggestion in §3 of the *Begriffsschrift* that his concept script is a language that has only a single predicate, that "*the symbol ⊢ is its common predicate for all judgements,*" as if judging were a matter of predicating "is true" or "is a fact" of a judgeable content, thereby marking a speaker's assent to it, a speaker's view of the matter. Frege's early thought that it is legitimate to draw an inference from a false or merely unasserted proposition reinforces the point.¹⁸

In the 1891 essay "Funktion und Begriff," Frege offers a very different account of the judgment stroke. Now we are told that "the assertion sign cannot be used to construct functional expressions; for it does not serve, in conjunction with other signs, to designate an object. '⊢ 2 + 3 = 5' does not designate anything; it asserts something" (CP, p. 149, n.7). The point is made again in *Grundgesetze* §5:

in a mere equation there is as yet no assertion; "2 + 3 = 5" only designates a truth-value, without its being said which of the two it is. Again, if I wrote

"(2 + 3 = 5) = (2 = 2)"

and presupposed that we knew 2 = 2 to be the True, I still should not have asserted thereby that the sum of 2 and 3 is 5; rather I should only have designated the truth-value of "2 + 3 = 5"'s denoting the same as "2 = 2." We therefore require another special sign to be able to assert something as true.

Predicating "is true" or "is a fact" serves only to generate another judgeable content Frege now thinks: "by combining subject and predicate, one reaches only a thought, never passes from sense to meaning, never from a thought to its truth-value" (CP, p. 164). But judgment is precisely such a passage: "In every judgement, no matter how trivial, the step from the level of thoughts to the level of meaning (the objective) has already been taken" (CP, p. 164); "Judgements can be regarded as advances from a thought to a truth-value" (CP, pp. 164–65). The judgment stroke is needed, Frege now thinks, in order to distinguish judgments from thoughts. It cannot, then, function merely predicatively; for predicating yields only a thought, the sense of a sentence. "A proposition uttered without assertoric force can be logically useful" (PMC, p. 79); it can, for example, serve as the antecedent of another proposition. But inference requires premises that are true and whose truth is acknowledged. Their truth must be marked by a judgment stroke.

In a letter of January 15, 1914, Jourdain asks Frege "whether you [Frege] now regard assertion (⊢) as merely psychological" (PMC, p. 78). The text of both a draft and the letter Frege sent in reply (dated January 28, 1914) have survived. In the draft Frege writes first of Russell's *Principia*, then responds to Jourdain's question about the assertion stroke.¹⁹ He writes: "Judging (or recognizing as true) is certainly an inner mental process; but that something is true is independent of

the recognizing subject; it is objective. . . . If in representing an inference in my conceptual notation one were to leave out the judgement strokes before the premised propositions, something essential would be missing. . . . What is essential to an inference must be counted as part of logic" (PMC, pp. 78–79). Frege's thought seems to be this. There is, first, an objective difference between the case in which two thoughts are both true but have no logical relation one to the other and the case in which one thought follows on the basis of the other. Correspondingly, there is a difference between asserting that p and then asserting that q, on the one hand, and asserting that p then inferring on that basis that q, on the other. The difference between the cases can be marked, for instance, by putting a comma between the sentences 'p' and 'q' in the first case: p,q, and putting a turnstile between them in the second case: p ⊢ q. Suppose, now, that someone mistakenly thinks that q follows from p when it is merely the case that both p and q are true. In such a case, we can say, while one meant to be judging that q on the basis that p, one in fact only assents to p and then assents to q. In that case, one's assenting that p only causes one to assent that q; it does not justify the latter assertion; one has not inferred q from p.²⁰ One can, then, be mistaken in thinking that one infers that things are thus and so; perhaps one is merely assenting to a claim after one has assented to another. Inferring so conceived is not of merely psychological significance. It is a cognitive achievement that depends in part on objective logical relations. Similarly, in the case of freestanding judgment one can take oneself to acknowledge the truth of a thought when in fact one fails to do so since the thought is not true. There is, that is, an objective difference between thinking mistakenly that a certain sentence expresses a true thought and (successfully) acknowledging the truth of a thought. Judgment, like inference, is a cognitive achievement that depends in part on how things actually are. We can make mistakes. It follows, Frege thinks, that the judgment stroke is necessary in logic. One can mistakenly attach it to a sentence just as one can mistakenly conjoin two sentences with a turnstile. If assent to a proposition can be mistaken because the sentence is false, then assent is not of merely psychological significance, an expression of the fact that one takes the proposition to be true. It is logically significant and so ought to be marked in a properly logical language.

On Wittgenstein's conception of meaning in terms of the fundamental logical distinction between referring and predicative expressions, a sentence shows how things stand if it is true and it says that they do so stand. It follows, we have seen, that the judgment stroke can have no logical significance in language so conceived. Frege rejects the conclusion. As he writes to Jourdain, though judging is a mental process, acknowledging the truth of a thought is not. For "that something is true is independent of the recognizing subject; it is objective" (PMC, p. 78). So, he thinks, the judgment stroke is essential in logic. If it is, then a sentence cannot have the sort of meaning Wittgenstein supposes it to have. If the judgment stroke is necessary in logic, then a sentence cannot itself say that things are thus and so.

Although it is not yet clear how exactly the judgment stroke is to function, there can be no question that Frege thought that it was needed in logic. Much the same might be said of the identity sign in Frege's logic. Though Frege, at least at first, could not understand how identities can be significant, that they are essential in logic seemed to him to be obvious. Frege explains in §8 of the *Begriffsschrift* why logic needs the identity sign:

The same content can be fully determined in different ways; but, that the *same content*, in a particular case, is actually given by *two [different] modes of determination* is the content of a judgement. Before this [judgement] can be made, we must supply two different names, corresponding to the two [different] modes of determination, for the thing thus determined. But the judgement requires for its expression a symbol for identity of content to combine the two names.

Statements of identity are sometimes required in the course of a proof; but, Frege sees, if object names are merely referring expressions, representatives of objects, then, assuming that “a” and “b” are coreferring (and functioning normally), the sentence “ $a = b$ ” must have the same judgeable content as “ $a = a$ ”. We know, however, that these two sentences do not have the same judgeable content (since the former can require proof, while the latter is trivially true). So, Frege argues, “symbols . . . usually only representatives of their contents . . . at once appear *in propria persona* as soon as they are combined by the symbol for identity of content, for this signifies the circumstance that the two names have the same content” (BGS, §8). The strategy is hopeless. First, it requires an ambiguity in the meanings of symbols: “a bifurcation is necessarily introduced into the meaning of every symbol, the same symbols standing at times for their contents, at times for themselves” (BGS, §8), and that contravenes Frege’s requirement, set out in §1 of the *Begriffsschrift*, that symbols “have a completely fixed sense.” But even if such systematic ambiguity were admitted, the proposed solution could not work. Suppose that it is true that $a = b$, that is, that “a” and “b” have the same content. It follows that “ $f(a)$ ” and “ $f(b)$ ” must have the same content, since in this context the symbols “a” and “b” “[stand] for their contents,” not for themselves. But, we are told in §3, Frege’s *Begriffsschrift* does not “distinguish between propositions which have the same conceptual content.” Indeed, it is clear that even if $a = b$, nevertheless “ $f(a)$ ” and “ $f(b)$ ” do not have the same conceptual content; for not all consequences derivable from the one combined with certain other judgments can be derived from the other combined with those same judgments. (Consider, for example, the premise “if $f(a)$ then $g(c)$.”) Frege’s *Begriffsschrift* solution to the problem of the logical significance of identities is no solution at all.

In the early discussions, Frege explicitly claims that object names (in ordinary contexts) function solely as representatives of objects. Their logical function is to designate objects, and a name that fails to designate anything is an “empty sound” (PW, p. 60).²¹ Frege tries in the early logic to explain how identities are meaningful nonetheless. Wittgenstein, we have seen, argues that since the logical role of an object name is merely to be representative of the object, sentences that have the form of identities must be either contradictory (asserting the identity of two things) or useless (asserting the identity of one thing). In “Über Sinn und Bedeutung,” Frege argues contrapositively. Since sentences of the form “ $a = b$ ”, by contrast with those of the form “ $a = a$ ”, can “contain very valuable extensions of our knowledge and cannot always be established a priori” (CP, p. 157), the logical role of an object name cannot be merely to refer to an object in order that a characterization of it be given. Object names, Frege now thinks, do not only refer to objects; they also express Sinn. Indeed, though he does not explicitly draw the Sinn-Bedeutung distinction in the *Grundlagen*, already there the essential point is made: “Why is

it, after all, that we are able to make use of identities with such significant results in such divers fields? Surely it is . . . because we are able to recognize something as the same again even although it is given in a different way" (GL, §67). An identity such as "7 + 5 = 12" is true just in case it is "possible in every judgement to substitute without loss of truth the right-hand side of our putative identity for its left-hand side" (GL, §107). Nevertheless, the identity does not merely give the same object twice. It expresses what Frege calls a recognition judgment, the recognition that something is "the same again even although it is given in a different way."

In his letter of January 15, 1914, Jourdain quizzes Frege also about his notion of Sinn, asking "whether, in view of what seems to be a fact, namely that Russell has shown that propositions can be analyzed into a form which only assumes that a name has a '*Bedeutung*', & not a '*Sinn*', you would hold that '*Sinn*' was merely a psychological property of a name" (PMC, p. 78). Jourdain is referring, of course, to Russell's theory of descriptions (endorsed also by Wittgenstein), a theory that meets all the demands that are met by Frege's notion of Sinn—including accounting for the cognitive significance of identities—without having to appeal to that notion. On Russell's view, most names are really disguised definite descriptions. Frege's response begins by distinguishing between a thought, for example, a thought about Mount Edna, and the object the thought is about, Mount Edna itself. Mount Edna, Frege argues, cannot be a part of the thought, a constituent of it. So, he concludes,

both things seem to be necessary: (1) the meaning [*Bedeutung*] of a name, which is that about which something is being said, and (2) the sense [*Sinn*] of the name, which is part of the thought. Without meaning, we could indeed have a thought, but only a mythological or literary thought, not a thought that could further scientific knowledge. Without a sense, we would have no thought, and hence also nothing that we could recognize as true. (PMC, pp. 79–80)

This conclusion is then reinforced by consideration of cases such as that of Hesperus and Phosphorus in which the identity of an object known in two different contexts must be discovered: "An object can be determined in different ways, and every one of these ways of determining it can give rise to a special name, and these different names then have different senses; for it is not self-evident that it is the same object which is being determined in different ways" (PMC, p. 80). This response to Jourdain is remarkable—not so much for what it says, which is just what Frege has been saying about the sense of an object name since 1892, as for what it does not say. Nowhere does Frege even mention Russell's theory. Since that theory provides an alternative account of each of the phenomena Frege discusses, Frege's response seems completely to miss the point of Jourdain's question. Once again it is clear that Frege stands firm in his view but utterly mysterious why he does.

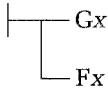
Wittgenstein argues in the *Tractatus* that the laws of logic are sinnlos, empty, that they are mere forms. Causal laws are rejected altogether. According to Frege, laws of logic are substantive truths, and there is no in principle difference between causal laws and laws of logic. The science of logic differs from other sciences only in being more general:

How must I think in order to reach the goal, truth? We expect logic to give us the answer to this question, but we do not demand of it that it

should go into what is peculiar to each branch of knowledge and its subject-matter. On the contrary, the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject-matter. We must assume that the rules for our thinking and our holding something to be true are prescribed by the laws of truth. . . . Logic is the science of the most general laws of truth. (PW, p. 128)

"Logic," Frege thinks, "has much the same relation to truth as physics has to weight or heat" (CP, p. 351).²²

Frege's *Begriffsschrift*, the logical language developed first in his 1879 monograph and brought to maturity (Frege says) in the *Grundgesetze* of 1893, is everywhere regarded as a notational variant of our quantificational logic, its concavity as a universal quantifier, and its indicating letters as variables. If *Begriffsschrift* were such a logic, then its generalities could not express laws. That is to say, if what is expressed by the *Begriffsschrift* judgment



is correctly expressed also by

$$(\forall x)(Fx \supset Gx)$$

then a generalized conditional of *Begriffsschrift* cannot be used in the expression of a law. For the sentence " $(\forall x)(Fx \supset Gx)$ " of quantificational logic is true just in case everything is either G or not F, whether or not that is necessarily the case. The generality may be true in virtue of a lawful relation (i.e., because it is necessary that any F be also G), but the sentence does not itself express this necessity. The necessity (or contingency) of the fact that everything F is G is no part of the content of the sentence. Frege, however, does seem to think that causal connections can be expressed in his logical language. Indeed, he suggests, *Begriffsschrift* was especially developed as a language within which to express laws, whether laws of logic or laws of the special sciences. It is worth a brief review of some familiar texts to remind ourselves of Frege's own conception of his logical language *Begriffsschrift*.

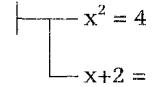
Frege developed his logical language as a special-purpose instrument—like a microscope, he tells us in the preface of *Begriffsschrift*—to further his logicist program. He wanted to show that arithmetic is "simply a development of logic, and every proposition of arithmetic a law of logic, albeit a derivative one" (GL, §87; cf. GG, p. vii), and to do so by proving even the most basic arithmetical truths by appeal only to the laws of logic and definitions. His reasoning, then, had to be absolutely rigorous; "so that something intuitive could not squeeze in unnoticed," everything on which a chain of reasoning depended was to be stated in advance (CN, p. 104). Frege at first tried to formulate his definitions and proofs in natural language. But, as he reports, "despite all the unwieldiness of the expressions, the more complex the relations became, the less precision—which my purpose required—could be obtained" (CN, p. 104). The problem, as Frege reiterates time and time again, is that in natural language forms of inference "are so varied, so loose and vague, that presuppositions can easily slip in unnoticed and then be overlooked when the necessary conditions on the conclusion are enumerated" (CN, p. 85). "The reason why verbal languages are ill suited to this purpose [of formu-

lating fully rigorous proofs] lies not just in the occasional ambiguity of expressions, but above all in the absence of fixed forms for inference" (CP, p. 235).²³ Because "in [ordinary] language, logical relations are almost always only hinted at—left to guessing, not actually expressed" (CN, p. 85), it is impossible "on the basis of linguistic form . . . [to] distinguish between a 'gapless' advance [in the argument] and an omission of connecting links" (p. 85).

Natural language, Frege thinks, is logically defective because it contains an "excessive variety" of modes of inference (GL, §91), "an almost unsurveyable multitude" of acceptable transitions from one sentence to another (CP, p. 235). Frege's *Begriffsschrift* is designed to overcome exactly this difficulty. Not only all the axioms on which proofs depend but also, as far as possible, all the rules to be employed—both rules of material inference (such as the rule that if angle a is smaller than angle b then a is not larger than b)²⁴ and rules of formally valid inference (e.g., that from a conjunction one can infer one or other conjunct)—are to be stated in advance. Furthermore, because rules of inference must be stated in natural language, Frege restricts himself to only one "giving as formulae what could otherwise have also been introduced as modes of inference" (PW, p. 37; cf. CN, p. 107; BGS, §6). The "fundamental idea" of Frege's conceptual notation is to "limit to the bare minimum the number of modes of inference, and to set these up as rules of this new language" (CP, p. 236). All other modes of inference are to be expressed as formulas and either laid down as self-evident basic laws, as axioms, or derived from those basic laws by the one rule of inference laid down in advance.²⁵ Frege's *Begriffsschrift* is in this way to be a language within which to reason with the utmost rigor, a language within which rules of formally and materially valid inference are expressed as judgments, as laws.

As expressed in the form of judgments, rules of inference, whether formal or material, have the form of generalized conditionals, or, as Frege thinks of them, "genuine hypotheticals" (PW, p. 52).²⁶ Indeed, Frege suggests, it is only by considering its role in the formulation of laws that we can adequately understand the conditional stroke. The conditional "looks strange at first sight": "people probably feel the lack of an inner connection between the thoughts: we find it hard to accept that it is only the truth or falsity of the thoughts that is to be taken into account, that their content doesn't come into it at all" (PW, pp. 186–87). Nevertheless, Frege claims, it is this connective that is needed in logic:

Now I understand by  A

the negation of the third case [i.e., of the case not-A and B]. This stipulation may appear very artificial at first. . . . The reason, however, will be immediately evident from an example:  $x^2 = 4$

$$x+2 = 4$$

denies the case that x^2 is not equal to 4 while nevertheless $x + 2$ is equal to 4. We can translate it: if $x + 2 = 4$, then $x^2 = 4$. This translation reveals the importance of the relation embedded in our symbol. Indeed, the hypothetical judgment is the form of all laws of nature and of all causal connections in general. (CN, p. 95)

Rules of inference, whether formal or material, are to be expressed in *Begriffsschrift* as generalized conditionals. For Frege, a lawful generality such as “if x is a whale then x is a mammal” differs from a law of logic, say, “if p then, if q then p ”, only in virtue of the fact that the laws of logic are more general, indeed the most general truths, those governing thinking no matter what its subject matter.

According to Wittgenstein, inference rules have “no sense and are superfluous” (5.132). Like assertion, inference (on his view) has merely psychological significance: “proof in logic is merely a mechanical expedient to facilitate the recognition of tautologies in complicated cases” (6.1262). Because “a proposition affirms every proposition that follows from it” (5.124), the conclusion of an inference merely repeats something affirmed already in the premises, merely makes explicit something implicit already in those premises. Frege offers us a very different account of the nature of inference.

We have already seen that Frege recognizes materially valid inference rules, rules such as the rule that if one angle is smaller than another, the former is not larger than the latter. He would not, then, consider the following inference enthymematic:

Angle A is smaller than angle B .

Therefore, angle A is not larger than angle B .

Given the lawful relation between one angle’s being smaller than another and the former’s not being larger than the latter, the argument is valid just as it stands. Rules of inference such as that licensing this inferential step are to be expressed as formulas and included in proofs as premises not because the proofs would otherwise be enthymematic but because they would otherwise be less perspicuously gap-free.²⁷ That is why Frege thinks that even strictly deductive proofs can yield fruitful extensions of our knowledge. In such cases the conclusion is contained in the premises only “as plants are contained in their seeds, not as beams are contained in a house” (GL, §88). The conclusion is contained in the premises not implicitly, as Wittgenstein thinks, but rather potentially. Actualizing that potential requires an inference. The step from premises to conclusion must be taken (PW, p. 258).

Frege holds that both the laws of logic and causal laws, that is, the laws of special sciences such as physics, are substantive truths expressible in his *Begriffsschrift*. Once again, however, he is at first unclear about how this can be. Frege’s thought is that laws, which have the form of generalized conditionals, express relations of subordination of concepts. But if, as he at first thinks, concepts have no independent existence, if they are merely expressions formed by imagining some symbol in a sentence as replaceable by others, it is impossible to see how they can be. A concept, Frege says in §9 of the *Begriffsschrift*, is “the part of the expression that shows itself invariant [under such replacement].” Like the symbol “R” in “ aRb ” as Wittgenstein understands it, a concept in Frege’s early account is an expression that does not designate anything. It is not a significant bit of language in its own right. But if a concept expression is not significant in its own right, if it has a use only in characterizing objects otherwise given, then there can be no sentences expressing relations among concepts. For there is in that case nothing to relate. Already in the *Grundlagen* (1884), Frege has modified his account: “The business of a general concept word,” he now thinks, “is precisely to signify a

concept" (GL, §51). In "Funktion und Begriff" (1891), Frege provides a geometrical analogy to illustrate his new conception of a concept, or function. We are to imagine a (dense) line of finite length divided into two by a point:

If we want to make a clean division, i.e. so as not to count anything twice over or leave anything out, then we may only count the dividing point along with one segment. This segment thus becomes fully complete in itself, and may be compared to the argument; whereas the other is lacking in something—viz. the dividing point, which one may call its endpoint, does not belong to it. Only by completing it with this endpoint, or with a line that has two endpoints, do we get from it something entire. (CP, p. 141)

As a line without an endpoint is lacking something, is not something entire or complete, but is nevertheless a perfectly objective entity, so, Frege now sees, a function (concept) is incomplete or unsaturated but nevertheless perfectly objective, something in its own right. A function is not an expression containing a variable (i.e., an expression with a fixed part and a variable part) as Frege had at first thought.²⁸ A function is what is designated by that part of the expression that is left over when the object name is removed.

Both in his early logic and in the mature logic of *Grundgesetze*, Frege holds that the judgment stroke is essential in logic, that identities can express substantial truths that extend our knowledge, and that both the laws of logic and laws of the special sciences are properly speaking truths expressible in his logical language. The two sorts of laws differ, he thinks, only in virtue of the greater generality of the former. As we have also seen, Frege is unable in his original *Begriffsschrift* account satisfactorily to explain any of these points. The reason he cannot, I have indicated, is that he has in the early logic essentially the conception of meaning that Wittgenstein endorses in the *Tractatus*. In the *Begriffsschrift*, Frege takes object names to function referringly in such a way that a name lacking a bearer is merely an empty sound, and concepts to be merely expressions enabling the characterization of objects otherwise given; a sentence, he thinks, expresses a state of affairs, the circumstance that things are thus and so. What is needed is an alternative conception of meaning, that provided by Frege's notions of Sinn and Bedeutung.

III. Frege's Mature Conception of Language

If one had only "Über Sinn und Bedeutung" to go on, one might naturally take Frege's introduction of the notion of Sinn as the introduction of a theory of the cognitive aspect of language use, one that is supplementary to the properly logical theory Frege develops in his *Begriffsschrift* of 1879. Following Dummett, Evans takes this view:

Frege's theory of meaning [*Bedeutung*] for the fragment of language he was concerned with after 1890 corresponds exactly to the theory that was implicit in the earlier works, the *Begriffsschrift* and the *Grundlagen*. The analysis of singular sentences did not alter with the discovery of sense; rather Frege saw more clearly what kind of analysis he had provided,

and saw the need for something more. In later works, Frege was grafting onto this enduring semantic theory his new conception of sense. (Evans, 1982, p. 8)

But this is not Frege's view. As Frege tells us in the introduction to *Grundgesetze*, his discovery of the Sinn-Bedeutung distinction is a "vital advance" in his logic; it is a logical distinction, one that brings his logic "to maturity" (GG, p. 7).

Frege did not discover a notion of sense to supplement his semantic theory. What he discovered was that his earlier view of concepts and objects involved a conflation of two different distinctions. As Frege explains (in his "Comments on Sense and Meaning"), after remarking that the distinction of sense and meaning first drawn in "Über Sinn und Bedeutung" for the case of object names "can also be drawn for concept words" (PW, p. 118), "it is easy to become unclear about this by confounding the division into concepts and objects with the distinction between sense and meaning, so that we run together sense and concept on the one hand and meaning and object on the other" (PW, p. 118). Frege claims that we tend to confuse the logical notion of the meaning of an expression, the notion of Bedeutung, with the logical function of referring, and correspondingly to confuse the logical notion of Sinn, of the cognitive content of an expression, with the characterizing role played by predicative expressions. Confounding the two distinctions, we think that concept words have only Sinn or cognitive significance. We take what is in fact the sense of a concept word (more exactly what pictures or exhibits sense) to be the concept itself. The notion of cognitive content is thus collapsed into that of descriptive or conceptual content. Object names, because they contrast with concept words, are then taken to be devoid of content. They merely refer, and where no object is designated, the object name is an empty sound or meaningless mark. Conversely, having collapsed the notion of the Bedeutung of an expression with that of reference to an object, we come to think that semantic content, objective meaning, lies in the relation of a representation to an object. Having collapsed the Sinn-Bedeutung distinction into the concept-object distinction, one takes object names to function solely to refer to objects, to have no cognitive content. Object names on this view are, in effect, blind. Concept words, correlatively, are taken to be empty independent of reference to any object. They have no meaning in isolation. They are mere forms.

Frege claims that we tend to collapse the distinction between Sinn and Bedeutung into the distinction between concept and object, and, as a result, understand objective content in terms of relation to an object and cognitive content descriptively. (So, for example, if object names are to have cognitive content as well as reference, on this view, that can only be by virtue of their being disguised descriptions.) Frege at first assumes just such a view. That is to say, he does not, at first, distinguish between the notion of a concept and the notion of sense, between conceptual content and inferential content. He thinks (at first) that inferential content—which is what *Begriffsschrift* sentences express, "everything necessary for a correct inference" (BGS, §3)—just is given by truth conditions.²⁹ In fact, as he comes explicitly to hold, it is the sense of an expression understood in terms of inference that is directly expressed in Frege's logical language. Only relative to

an analysis can a Begriffsschrift sentence be understood to express a possible state of affairs, the circumstance that things are thus and so.³⁰

In the early 1890s Frege comes to think that his early view of the way concept words and object names function—which, we have seen, is also Wittgenstein's view—rests on a conflation of two different distinctions. This development, furthermore, seems to have been motivated by what Frege had from the beginning taken to be manifest inadequacies in just the sorts of conclusions that Wittgenstein would later draw in the *Tractatus*. Object names, Frege argues, cannot be merely names for objects; for if they were, there could be no accounting for the cognitive significance of some identities. Concept words correlatively cannot be merely cognitively significant, ways we can think of objects being; for then there could be no accounting for the objective content of laws. What is needed, he comes to think, is a distinction between distinctions, between the distinction of concept and object and the distinction of Sinn and Bedeutung. Having drawn that distinction of distinctions, we are to see that object names have not only reference (Bedeutung) but also sense (Sinn) and so, in a general way at least, how identities might constitute “very valuable extensions of our knowledge” (CP, p. 157), and correlatively that concept words have not only sense but also reference and hence, again at least in general, how laws expressing relations among concepts might be thought to be properly speaking true or false.³¹

But how exactly are we to understand Frege's notions of Sinn and Bedeutung? An answer is suggested by the formula language of arithmetic on which Frege's Begriffsschrift notation is modeled.³² Just as the formula language of arithmetic is, Frege's logical language was to be at once a Leibnizian lingua characterica and a calculus ratiocinator, “both, with equal emphasis” (CP, p. 242). Understanding the sense in which the formula language of arithmetic is at once a lingua characterica and a calculus ratiocinator will help us understand Frege's mature conception of language in terms of Sinn and Bedeutung.

A lingua characterica, Frege tells us, is a language that “directly expresses the facts without the intervention of speech” (CN, p. 88); it expresses “*non pas les paroles, mais les pensées*” (PW, p. 13). It achieves this expression by virtue of an isomorphism between the representation and that which is represented. Leibniz, in his “Dialogue on the Connexion between Words and Things” (1677), explains: “although the signs themselves may be arbitrary, their use and connexion has something which is not arbitrary, namely a certain proportion between symbols and things and the mutual relations of different symbols which express the same things. And this proportion or relation is the ground of truth [*fundamentum veritatis*].”³³ In the case of arithmetic, one might think, an adequate notation would express relations of relative magnitude. Two is twice as many as one, so supposing we depict one as “i”, we should express two as “ii”. Three is three times as many as one and one more than two, so it is symbolized “iii”. For convenience, abbreviations are introduced, five for instance as “v” to abbreviate “iiii”. Then we can express four as “iv” on the grounds that four is one less than five, and six as “vi” because it is five and one more.³⁴ In such a numeration system, “bigger” numbers are represented by symbols that are bigger (composed of a greater number of tokenings of “i”), and bigger by as much as the numbers themselves are bigger. Three as represented in a roman numeral is literally and manifestly two

and one; one can see in the symbols themselves that two and one make three. The roman numeration system is in this way a lingua characterica of relative magnitudes and thereby, so it might seem, of the numbers themselves.

If the roman numeration system were a perspicuous notation of arithmetic, a lingua characterica, then the arabic numeration system would be a remarkably unperspicuous symbol system of arithmetic. It would be unperspicuous precisely because the symbols—"1", "2", "3", and so on—do not themselves exhibit relative magnitudes. It is inscribed in the symbols themselves that iii is greater than ii, and by how much; it is not inscribed in the symbols themselves that 3 is greater than 2. To know which of the two arabic signs represents the larger number requires knowing what they mean. Although 222 is a smaller number than 999, it does not look any smaller in the arabic numeration system. From the perspective according to which roman numerals perspicuously represent the numbers, the ten digits of the arabic notation seem quite like the convenient abbreviations "v," "x," and so on, that we find already in the roman system. Larger numbers, which in the arabic system are given by the relative positions of digits in combinations of them, give the numbers through a kind of a code. Knowing the code is being able to get back to the numbers themselves as needed.

The arabic system, we are assuming, is an unperspicuous notation of arithmetic. Because it does not map relative magnitudes, it seems not to be a lingua characterica of arithmetic. Nevertheless, that symbol system is enormously powerful. Because algorithms are formulable in the arabic numeration system for all the basic arithmetical operations, one can formalize (i.e., rigorize) arithmetic using arabic numerals. Furthermore, this is impossible in the roman numeration system. The results of calculations can be recorded in roman numerals, but one cannot calculate in the language. Only the arabic numeration system is a language within which to calculate; only that system is a calculus ratiocinator.

It is the arabic numeration system rather than the roman that enables the rigorization of basic arithmetic. But it is the roman numeration system and not the arabic that is (we are supposing) a perspicuous language of arithmetic, a lingua characterica of the numbers. What, then, is the relationship between the rules governing our calculations using arabic numerals and the arithmetical truths thereby proved? What is the relationship between calculability in the arabic notation and arithmetical truth perspicuously representable only in the roman system, between, as it were, the proof theory of arithmetic and its semantic notion of truth in terms of the picturing of relative magnitudes? The question has no answer.

We have been assuming that the roman numeration system is a lingua characterica of arithmetic in virtue of the fact that it displays in its symbols the relative magnitudes of numbers. This system codifies a conception of number, that is to say, according to which number is a collection or multiplicity of units. But, Frege argues in the *Grundlagen*, that conception of number is fundamentally flawed. Numbers are not formed by the putting together of units; "+" does not mean the same as "and". It follows, assuming he is right, that numbers are not perspicuously represented as series of strokes, or of tokenings of "i." Is it perhaps, then, the arabic system that is a perspicuous notation, a lingua characterica, of arithmetic? Is it perhaps the arabic numeration system that is at once a lingua characterica and a calculus ratiocinator "both, with equal emphasis"?³⁵

To learn the arabic numeration system is to learn the rules governing the manipulation of its symbols, both the rules governing the formation of the numerals (that in “673,” say, the “6” is in the hundreds place, and so on), and the algorithms of addition, subtraction, and so on. What is expressed by an arabic numeral as determined by those rules is, we might say, its computational content. Given the algorithms for the formation of the canonical names of the numbers and for the basic arithmetical operations, the language expresses everything necessary for a correct computation; nothing is left to guessing. The computational content of, for instance, “375” or “58 + 62” is fixed by the rules governing the use of the symbols involved in these expressions.³⁶ To know those rules and so how correctly to manipulate the symbols according to the rules is to grasp the computational content of the expression. An expression such as “58 + 62”, we can say, *expresses* its computational content; and where the signs are different, as they are in, say, “80 + 40”, the computational content is also different. That the computational content is different is evident from the fact that different rules would need to be applied, or to be applied to different cases, or in a different order. What the arabic system directly expresses, then, what it maps or pictures, what it traces in its symbolism, is not the numbers themselves, but rather computational content. That is why algorithms are formulable for all the basic arithmetical operations in the arabic system.

But arabic numerals such as “37” and “58 + 62” do not only express computational contents. They also designate numbers. What does “58 + 62” designate? The rules governing the use of the signs enable an answer: the positions of the “8” and the “2” determine (in light of the “+”) that we need to write a “0” in the ones column and carry a “1” over to the tens, where it is added to “5 + 6” to give “12”, and so we write a “1” in the hundreds column and a “2” in the tens and there is our answer, 120. Computing, guided by the computational content pictured in the notation of arabic numerals, enables the advance from that content to a canonical expression of the number designated. The expression “58 + 62” designates precisely the same number as “120”, but it gives it in a different way. It gives it by way of a computational content that is different from that exhibited in “120.” Furthermore, one can just see that the computational contents are different—for the signs are different.

The arabic numeration system takes computation, rather than representations of relative magnitudes, to reveal the essence of number. In much the same way, Frege’s logical language takes reasoning, rather than the representation of states of affairs, to reveal the essence of truth. What Begriffsschrift sentences exhibit, what they express (directly map, picture, or trace) are inferential contents determined by the rules that govern the manipulation of the symbols involved. To know those rules is to grasp the inferential content of the expression. A Begriffsschrift sentence does not, in other words, directly map or picture a state of affairs, things as thus and so, in the way that a Tractarian sentence does.³⁷ In a Tractarian language, a state of affairs is depicted directly, by way of a picture of things as thus and so, names standing for objects and their manner of presentation indicating their properties and relations. In Begriffsschrift what is directly depicted is inferential content. In Begriffsschrift “everything necessary for a correct inference is fully expressed; but what is not necessary usually is not indicated; *nothing is left to guessing*. In this I strictly follow the example of the formula language of mathematics, in which also,

one can distinguish subject and predicate only by doing violence [to the language]" (BGS, §3). Frege's *Begriffsschrift* is a lingua characterica of inferential content.

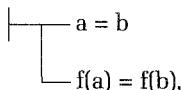
In a roman numeration system, one pictures collections of objects by way of strokes or marks such as "i," one for each object. That certain collections are given special signs, for instance, "v" for "iiii," is merely a convenient abbreviation. Similarly, in Wittgenstein's picture language, one pictures states of affairs by means of arrangements and forms of names for objects. That certain truth possibilities are given special signs, for instance, " $(\forall x)Fx$ " for the conjunction of pictures of objects as having the property F, is likewise merely a useful abbreviation. Just as one cannot calculate in the language of roman numerals, so one cannot reason in Wittgenstein's picture language. Because a picture of a state of affairs must contain already everything that follows from it, reasoning, like asserting, is only of psychological significance in a Tractarian language. Just as a representation of five objects as iiuii and of three objects as iii is already thereby a representation of eight objects, whether or not anyone notices that it is, so a Tractarian picture of Socrates as human together with such a picture of all humans as animals is already thereby a picture of Socrates as an animal, whether or not anyone actually notices that it is. A Tractarian language pictures states of affairs conceived in terms of objects with their properties and relations. Because it does, a system of logic within which to reason is, relative to it, a mere calculus ratiocinator, and proof is "merely a mechanical expedient" (6.1262).³⁸

By his own account, Frege's logic is at once a lingua characterica and a calculus ratiocinator, "both, with equal emphasis." It is both in much the way the arabic numeration system is. A sentence of *Begriffsschrift* expresses an inferential content much as an expression of arithmetic such as "23² – 17" expresses a computational content. Hence, much as "5 × 5" and "13 + 12" differ in computational content (the one being a product and the other a sum) while designating the same number, so the names "Hesperus" and "Phosphorus" differ in inferential content while designating the same planet. We have, then, an account of why assertion and inference are of logical, and not merely psychological, significance. To assert that a sentence is true is analogous to giving the solution to a problem in arithmetic such as "2³ – 1". A thought, an inferential content, poses in this way a question; it does not already say how things stand if it is true. (Of course, in the logical case, by contrast with the arithmetical case, only two answers are possible: "yes" and "no".) To judge is to acknowledge the truth of a thought, or of its negation.³⁹ One marks this advance to a truth-value, this acknowledgment of truth, using the judgment stroke. And here one can be mistaken.

Just as some combinations of symbols in arithmetic, "2/0", for instance, do not designate numbers even though they are syntactically correct (i.e., of the same form as other combinations that do designate numbers), so some combinations of symbols in a logical language do not designate objects, though they are syntactically correct. The sentence "the largest proper fraction is less than one," for example, fails to have a truth-value because the expression "the largest proper fraction" does not designate any object. The sentence expresses a Fregean thought—one knows what would follow from it were it true (or were it false), what would be logically incompatible with it were it true (or false), and so on—but because there is no object about which to judge in this case, the question of truth, whether the object is or is not less than one, cannot be asked. The sentence has no truth-value. The step from the level

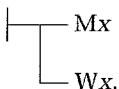
of thoughts to the level of meaning cannot be taken in such a case. The sentence expresses a perfectly determinate thought but fails to have a truth-value.⁴⁰

I have sketched a conception of meaning according to which a sentence directly exhibits not a state of affairs, the circumstance that things are thus and so, but instead an inferential content as determined by the rules governing the manipulation of the symbols involved.⁴¹ There is, then, an objective difference between a sentence of the form “ $a = a$ ”, which merely presents the same thing twice in the same way, and a sentence “ $a = b$ ”, which can be a valuable extension of knowledge, a recognition judgment properly speaking, because it reveals (assuming it is true) that two signs with quite different inferential contents nonetheless designate a single object and so are substitutable *salva veritate* in all extensional contexts. There is also an objective difference between the thought “— $a = b$ ” which (assuming that “ a ” and “ b ” designate objects) can be judged to be true on the condition that a and b share all their properties, that is,

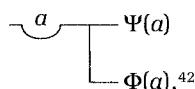


and the judgment “ $\vdash a = b$ ”, in which the thought expressed by “— $a = b$ ” is acknowledged to be true unconditionally. Where either “ a ” or “ b ” fails to refer, then while it may seem to a deluded user of the name that such judgments as these are possible, in fact they are not. It is, again, an objective matter whether or not it is correct to attach the judgment stroke in any given case, whether conditionally or unconditionally.

Laws, Frege thinks, express relations of subordination between concepts. The law that all whales are mammals, for instance, is expressed in the form of a judgment as a generalized conditional:



We can render it in English as “being a mammal follows being a whale.” Such a sentence, in other words, expresses a rule of inference, an inference license, as a claim, one that is true, if it is, in virtue of an objective relation among concepts. As we can think of it, to say of any particular whale that it must thereby be a mammal is to say that as a matter of fact the concepts *whale* and *mammal* are related in a certain way. That relation, the logical relation of subordination, is a second-level (two-place) concept in *Begriffsschrift*:



This concept takes two first-level concepts as arguments to yield a truth-value as value; and it can do so because having distinguished between the Sinn-Bedeutung distinction and the distinction between concept and object, we can understand concept words as more than mere forms enabling the characterization of objects otherwise given. Concept words designate concepts in Frege's mature logic, and

the objective laws of the special sciences express logical relations among such designated concepts.

IV. Conclusion

Frege's logic, first formulated in his *Begriffsschrift* of 1879 and brought to maturity with the introduction of the Sinn-Bedeutung distinction in the early 1890s, is universally regarded as a notational variant of the logic Russell developed at the turn of the century. Were it such a logic, Wittgenstein's criticisms of it in the *Tractatus* would be entirely justified. Wittgenstein had an unusually penetrating and philosophically rich understanding of the logic he inherited from Russell. But things are at once more complex and more interesting. Frege's logical language, *Begriffsschrift*, was designed to express everything necessary for a correct inference, inferential content, or, as Frege called it, *begrifflicher Inhalt*. At first Frege thought that to express inferential content just is to express judgeable content, that is, *beurtheilbare Inhalt*, the circumstance that obtains if a sentence is true. His early conception of content, that is, is in all essential respects Wittgenstein's Tractarian conception. On Frege's early view, object names function to refer to objects, and concept words serve to present those objects as thus and so. It follows, as Wittgenstein saw, that identities are logically useless, that laws, which abstract from all relation to any object, are sinnlos, that is, without objective significance, and that both judgment and inference are of merely psychological significance. But Frege also came to see this. His response, however, was not to embrace those conclusions (as Wittgenstein does), but instead to reject the conception of meaning on which those conclusions are based. The conception of meaning we find in the *Tractatus*, which is grounded in the fundamental logical distinction between referring and predicative expressions, rests on a conflation. It confuses two different logical distinctions, that between concept and object, and that between sense and meaning. None of Wittgenstein's conclusions follow on the basis of the conception of language Frege endorses after 1890. The question, then, of the adequacy either of Frege's views or of Wittgenstein's criticisms of those views essentially depends on the prior question of the adequacy of their respective conceptions of meaning. Until this latter question is resolved, there can be no definitive assessment of the cogency of Wittgenstein's Tractarian criticisms of Frege's views.

Notes

I am grateful to Erich Reck for comments on an earlier draft.

1. Wittgenstein, 1921, p. 3. Where it is obvious, citations of passages in the *Tractatus* will be given by the relevant section number alone.

2. Geach (1976) argues that "a great deal of the *Tractatus* is best understood as a refashioning of Frege's function-argument analysis in order to remove his mistaken treatment of sentences as complex names" (p. 64). Geach, 1982, and Diamond, 1991, chap. 6, develop related accounts. Ricketts (1985), focusing instead on Frege's "underlying view of judgment and the role for logic it dictates," takes that view to be incom-

pative with Frege's conception of the propositions of logic as truths, and aims to show that "Wittgenstein's *Tractatus* is in large measure a response to these tensions in Frege's thought" (p. 3)—though see also his essay in this volume, Conant, 1991, p. 139, seconds the view. In his essay in this volume, Conant argues further that Frege's account of the workings of language is vitiated by a tension between an austere and a substantial conception of nonsense, one that is satisfactorily resolved only in the *Tractatus*.

3. As will already be evident, my reading of Frege (which I develop and defend in *Frege's Logic*, in progress) is fundamentally opposed to both the Dummettian reading and the reading developed by Ricketts and others.

4. This correspondence is included as Appendix III in Wittgenstein, 1979, pp. 121–22.

5. The 1913 "Notes on Logic" are included as Appendix I in Wittgenstein, 1979, and are cited here as NL followed by the page number in that appendix.

6. See Sellars, 1962; also Sellars, 1981.

7. As will become clearer below, the sign for negation does not represent anything; "the 'logical constants' are not representatives" (4.0312). There are only objects and in a different way their properties and relations. That is what is pictured: objects as thus and so. And that is all that can be pictured. "p" and "not-p" could not possibly have different meanings.

8. Although the context principle makes its first appearance in the *Prototractatus*, it becomes an important organizing principle for Wittgenstein only in the *Tractatus*. See Kremer, 1997.

9. As Winch (1987) says, "the discussion of what it is for a name to function in a certain way in the symbolism *is* a discussion of what it is for it to have reference" (p. 10).

10. The logical notion of an object should not be confused with the notion of a logical object. Wittgenstein explicitly rejects the notion of a logical object (4.441, 5.4).

11. Sellars (1952) puts the point this way: "we can, indeed, say that the fact that α is φ consists of a '*this*-factor' and a '*such*-factor,' but the '*this*-factor,' instead of being a bare particular, is nothing more nor less than an instance of φ , and the '*thing*' which consists of these factors [namely, the fact] is so far from exemplifying φ that it cannot be meaningfully be said to do so. To say that a blue particular consists of Blue and a particular is indeed to talk nonsense; but it is nonsense which arises not out of a dualism of particulars and universals, but out of a confusion between particulars and facts" (p. 287).

12. Indeed, Wittgenstein claims, "it is self-evident that \vee , \supset , etc. are not relations in the sense in which right and left etc. are relations" (5.42).

13. This, as Wittgenstein sees, also follows on the picture theory. Because there can be no laws of logic in Wittgenstein's conception of logic and language, there are no logical moves one can make as licensed by such laws. All that a proof does is to make explicit something already implicit in the premises. Inference, then, is of no logical significance. For Wittgenstein, inference has essentially the same status as assertion; both are of merely psychological significance (see the 5.12s).

14. See Ricketts, 1996, and Kremer, 1992, for a more detailed discussion of Wittgenstein's conception of generality.

15. See note 13.

16. References to translations of Frege's writings will be given in the body of the text using the following abbreviations: *Begriffsschrift*, in Frege, 1972, as BGS; *Grundlagen*, in Frege, 1968, as GL; *Grundgesetze*, in Frege, 1967, as GG; other published writings, in Frege, 1984, as CP, except those that appear in Frege, 1972, cited as CN; posthumous writings, in Frege, 1979, as PW; and Frege's correspondence, in Frege, 1980, as PMC.

17. Frege here intends, I think, to use the word "*Vorstellung*" in what he describes in *Grundlagen* (§27 fn. 1) as the objective sense, to mean objects and concepts. That is why he goes on (in §2 of *Begriffsschrift*) to say that we can paraphrase an unasserted

content “by means of the words ‘*the circumstance that*’ or ‘*the proposition that*.’” A combination of ideas in the subjective sense cannot amount to a proposition or possible state of affairs; a combination of objects and concepts can (cf. PW, p. 174). By the time he is writing *Grundlagen* (which appeared in 1884), Frege has resolved to use the word “*Vorstellung*” only in the subjective sense, never to mean objects and concepts.

18. In the *Begriffsschrift* §2, Frege suggests that one might derive conclusions from a sentence lacking a judgment stroke in order to “test the correctness of the thought.” He similarly at first characterizes the task of logic as “to set up laws according to which a judgment is justified by others, irrespective of whether they are themselves true” (PW, p. 175).

19. In the version of the letter that was sent, Frege’s remarks on *Principia* are greatly expanded; he never gets to the discussion of the judgment stroke.

20. Frege remarks in the earliest logic manuscript: “what is true is true independent of our recognizing it as such. We can make mistakes. The grounds on which we make a judgment may justify out recognizing it as true; they may, however, merely give rise to our making a judgment, or make up our minds for us, without containing a justification for our judgment. Although each judgment we make is causally conditioned, it is nevertheless not the case that all these causes are grounds that afford a justification” (PW, p. 2).

21. “The sentence ‘*Leo Sachse is a man*’ is the expression of a thought only if ‘*Leo Sachse*’ designates [bezeichnet] something” (PW, p. 174).

22. At first Frege takes logic to contrast with a science like physics because, he thinks, it is a normative discipline like ethics which does not describe what is but instead prescribes what ought to be. He later comes to hold that logic is first and foremost descriptive. Frege writes in the late essay “Thoughts”: “laws of nature are general features of what happens in nature, and occurrences in nature are always in accordance with them. It is rather in this sense that I speak of laws of truth. Here of course it is not a matter of what happens but of what is. From the laws of truth there follow prescriptions about asserting, thinking, judging, inferring” (CP, p. 351).

23. Even Euclid draws inferences in his proofs that are legitimated only by unstated rules of natural language:

Thus in the proof of the nineteenth theorem of the first book of *The Elements* (in every triangle, the largest angle lies opposite the largest side), [Euclid] tacitly uses the statements:

- (1) If a line segment is not larger than a second one, the former is equal to or smaller than the latter.
- (2) If an angle is the same size as a second one, the former is not larger than the latter.
- (3) If an angle is smaller than a second one, the former is not larger than the latter.

Only by paying particular attention, however, can the reader become aware of the omission of these sentences, especially since they seem so close to being as fundamental as the laws of thought that they are used just like those laws themselves. (CN, p. 85)

24. The example is Frege’s. See note 23.

25. After one has shown that everything can be done with one’s single rule of inference and one’s axioms, it is unobjectionable, indeed desirable on grounds of brevity, to transform some of one’s theorems back into rules of inference (see, e.g., PW, p. 29).

26. Only a generalized conditional is a genuine hypothetical: mere conditionals “do not count as laws, since they lack the generality which distinguishes laws from particular facts” (PW, p. 258). “[Indefiniteness] is always present when we express a general law” (PW, p. 199).

27. Once again, because “in [ordinary] language, logical relations are almost always only hinted at—left to guessing, not actually expressed,” it is impossible “on the basis of linguistic form . . . [to] distinguish between a ‘gapless’ advance [in the argument] and an omission of connecting links” (CN, p. 85).

28. Frege does employ the notion of a variable in the discussion of the 1879 logic, for instance, in his geometrical example to illustrate the need for the identity sign. (The comparable example in “Über Sinn und Bedeutung,” it should be noted, does not employ any variables.) Although I cannot argue the point here, Frege does not employ any variables at all in his mature logic. He has no need of them once he has achieved his mature understanding of the sense and meaning of a concept word.

29. That is why he calls his logical language—which, as a language of pure thought, might more naturally be called a *Sinnsschrift*, a sense notation or sense script—a *Begriffsschrift*, a conceptual notation or concept script.

30. That is to say, one can recover truth conditions, a conception of the state of affairs presented in a sentence in *Begriffsschrift*, but only by analyzing the sentence into function and argument. Relative to an analysis, we can say that the sentence is about the entity designated by the argument and says of it whatever is the relevant concept designated by the function expression. Frege makes the point for the case of arithmetic. He argues (PW, pp. 16–17) that depending on how it is analyzed the sentence in arithmetic “ $2^4 = 16$ ” is about two, that it is a fourth root of sixteen, or about four, that it is a logarithm of sixteen to the base two, and so on. Independent of an analysis the content of the sentence is not formulable this way, in terms of truth conditions. Similarly, truth conditions attach to a sentence of *Begriffsschrift* only relative to an analysis. Though it is impossible to explain the point here, this feature of Frege’s logic, that truth conditions can be assigned to a sentence only relative to a particular analysis, helps explain why *Begriffsschrift* is a two-dimensional script rather than a one-dimensional script like Peano’s.

31. It is worth noting that the third of Jourdain’s questions (first in the order of asking) in his letter of January 15, 1914, is whether Frege’s “theory of functions ‘erster bzw. zweiter Stufe’ is not the same as Russell’s theory of orders (cf. *Principia Mathematica*, I. 54ff)” (PMC, p. 78). In the draft Frege suggests that perhaps it is, though he worries that his understanding of English is inadequate to the task of determining the matter one way or another. The final version makes no reference to difficulties with English. The difficulties, Frege now thinks, are with *Principia* itself, and in particular with Russell’s notion of a variable. Though I cannot adequately clarify the point here, the issue raised by Jourdain’s third question is essentially that raised by the question of the nature and status of laws, which raises in turn the question of an adequate conception of a concept. Jourdain’s three questions thus direct us to exactly the concerns that drive the discussion here: that of the logical status of the judgment stroke and of the sign for identity, and that of the nature and possibility of laws.

32. Frege subtitles his 1879 monograph “A Formula Language of Pure Thought Modeled on the Formula Language of Arithmetic.”

33. Quoted in Kneale, 1966, p. 208.

34. These abbreviations, in particular the notation for four, make even the roman numeration system slightly more complex than a pure lingua characterica of relative magnitude.

35. Whether, in the end, the arabic numeration system adequately symbolizes “in its own right the numerical element” (GL, §28) is not at issue here. What matters is only a particular understanding of the symbolism as it provides a model for thinking about how Frege’s logical language functions, as a model of the sort of symbol system Frege takes *Begriffsschrift* to be.

36. Elucidations in Frege’s sense are also needed fully to understand the language. Otherwise one would know how mechanically to manipulate the symbols but would not be able to apply them. The application is no part of the computational content of

expressions in an arabic notation; but it is nonetheless understood by anyone who understands the language.

37. Again, truth conditions are generally specifiable for a *Begriffsschrift* sentence, though only relative to an analysis. They are not, as they are in a Tractarian language, directly expressed in the symbolism. See note 30.

38. As in the arithmetical case, and for analogous reasons, a gap is thereby opened up between one's proof theory and one's semantic notion of truth. The question of the relationship between them comes to seem fundamental and urgently in need of resolution.

39. Frege from the beginning thought of judgment this way (see PW, p. 8, for the earliest formulation). But it is only his later understanding of meaning in terms of the Sinn-Bedeutung distinction that enables a coherent account of how a content of possible judgment can be posed as such a question, as a question of *truth* rather than as a question of whether the speaker takes it to be true.

40. In *The Varieties of Reference*, Evans describes Frege's claim that a sentence can express a thought yet fail to have a truth-value as "the great fault-line in Frege's mature philosophy of language" (p. 24). Evans's problem, of course, is that given his conception of meaning in terms of truth—in terms of the idea that the meaning of a sentence can be given by its truth conditions—it is utterly incomprehensible that a sentence should be meaningful without being true or false. (Dummett tries to get around this on Frege's behalf by positing a third truth-value for these sentences to have.) Indeed, so obvious is it that on the truth-conditional conception of meaning a sentence can have meaning if and only if it has a truth-value, Evans might have more naturally argued that, since Frege did take the idea of a contentful but truth-valueless sentence to be intelligible he must have had a different conception of meaning than that Evans finds most natural. Evans never so much as considers this possibility.

41. Again, it is worth emphasizing that relative to an analysis, a state of affairs can generally, thought not invariably, be specified as something indirectly expressed by the sentence.

42. Because we have here a concept word (for a second-level concept), Frege's gothic letters and concavity must be employed instead of his italic roman letters. Italic roman letters are suitable only to indicate generality of content and only in the case of a sentence whose truth is acknowledged (see BGS, §11, and GG, §17 fn. 22). The reasons for this are explained in my *Frege's Logic*.

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Wittgenstein against Frege and Russell

Thomas Ricketts

Wittgenstein's explicit discussions of Frege in the *Tractatus* and pre-*Tractatus* writings are problematic. They appear to reflect at best a superficial understanding of Frege's views, and in places to contain misunderstandings. As a result, Wittgenstein's criticisms of Frege seem variously to miss their target, to beg the question, or to be easily answerable.¹ Wittgenstein's explicit discussions of Frege are also relevant to an interpretive issue that has recently arisen between Warren Goldfarb on the one hand and Cora Diamond and myself on the other.² Against Diamond and me, Goldfarb urges that the *Tractatus* is not a product of Wittgenstein's deep engagement with Frege's philosophy, a product that is itself a response to deep tensions within that philosophy.

I shall not here address the general issue Goldfarb has raised.³ I want to explore a prior question: What do Wittgenstein's explicit criticisms of Frege in the *Tractatus* tell us about how Wittgenstein understood Frege? I will argue that Wittgenstein's critique is mounted from within Wittgenstein's own distinctive approach to language and logic, an approach that receives its first sustained articulation in his 1913 "Notes on Logic."⁴ In contrast to Frege and Russell, Wittgenstein places sentences at the center of his philosophy of logic. Although the terminology does not enter until 1914, I hold that even in 1913 Wittgenstein conceives of sentences as models of reality. His leading idea is that a proper understanding of the relation of sentences to reality that makes them correct or incorrect models of reality will encompass a proper understanding of the logical relationships among sentences, above all the relationship of logical consequence. In particular, Wittgenstein, sharply distinguishes the role in sentences of genuine predicates from the role of logical connectives like " \sim " and " \supset ". This distinction in roles lies at the heart of Wittgenstein's critique of Frege and Russell, a critique that is basi-

cally the application to these thinkers of Wittgenstein's *Grundgedanke* from 4.0312:

The possibility of sentences is based upon the principle of the representation (*Vertretung*) of objects by signs.

My fundamental thought is that the "logical constants" do not represent. That the *logic* of facts cannot be represented.⁵

As I read him, Wittgenstein then relies on his own emerging view of logic to diagnose where Frege and Russell have gone wrong. Thinking through Wittgenstein's critique of Frege and Russell will help us to understand the content of Wittgenstein's *Grundgedanke* and its connection to the conception of sentences as models of reality. It will also lead us to a fuller appreciation of the gulf separating Wittgenstein's conception of logic from Frege's and Russell's universalist conceptions. It will turn out that, while Wittgenstein does not meet Frege on Frege's own terms, neither does he straightforwardly misunderstand Frege, nor conflate Frege's pre- and post-1891 views, nor run Frege and Russell together. Moreover, once we recover Wittgenstein's perspective, we see his criticisms of Frege to be informed, serious, and deep.

I shall focus on the mixture of commendation and criticism Wittgenstein directs at Frege in 4.431. Discussion of this matter will lead us to other mentions of Frege, most notably 4.063. Along the way, there will be a lengthy detour into Russell in order to motivate and present Wittgenstein's 1913 viewpoint by considering the morass in Russell's philosophy of logic awaiting Wittgenstein, when he arrived in Cambridge in 1911.

I. An Exegetical Puzzle

The 4.4s introduce Wittgenstein's conception of sentences as truth-functions of elementary sentences. Remark 4.431 reads:

The expression of agreement and disagreement with the truth-possibilities of elementary sentences expresses the truth-conditions of a sentence.

A sentence is the expression of its truth-conditions.

(Thus Frege was quite right to use them as a starting point when he explained the signs of his conceptual notation [*Begriffsschrift*]. But the explanation of the concept of truth that Frege gives is mistaken: if "the True" and "the False" were really objects, and were the arguments in $\sim p$ etc., then Frege's determination would not at all determine the sense of " $\sim p$ ".)⁶

What is Wittgenstein praising in Frege, and why does he mention only Frege and not Russell here?

In Wittgenstein's eyes, Frege's logical notation and his explanations of it are seriously flawed. Wittgenstein finds the theory of classes superfluous in mathematics (6.031), and so has no sympathy with Frege's inclusion in logic of a notation for value-ranges of functions. In sharp contrast to Frege, Wittgenstein holds that identity is not a relation between objects, that the identity sign is not an essential component of a logical notation (5.53s). Finally, Wittgenstein criticizes Frege's

introduction of the universal quantifier (5.521). By process of elimination, I conclude that Wittgenstein is commanding Frege's explanations of the truth-functional connectives. Furthermore, I suggest that Wittgenstein has in mind more Frege's introduction of these connectives in the 1879 *Begriffsschrift* than in the 1893 *Grundgesetze der Arithmetik*; for Wittgenstein is criticizing the 1893 explanation in 4.431. In *Begriffsschrift*, Frege lists the four possibilities of truth and falsity for two judgeable contents A and B, and says that " $\vdash B \supset A$ " signifies (*bedeuten*) the judgment that excludes the case that, as Frege puts it, A is denied and B is affirmed so that one of the other three possibilities obtains.⁷ Similarly, Frege explains that " $\vdash \sim A$ " means that the content A does not obtain.⁸

In contrast to Frege's 1879 explanation, Russell in *Principles of Mathematics* explains his sign for the material conditional as a predicate that designates a dyadic relation over propositions.⁹ Somewhat reluctantly, in later writings Russell treats negation as the predication of falsity. So, *Principia Mathematica* *1 introduces the tilde notation as follows: "If p is any proposition, the proposition 'not-p' or 'p is false' will be represented by ' $\sim p$ '."¹⁰ In the *Tractatus*, Wittgenstein forcefully maintains that the logical connectives are not predicates, that material implication, disjunction, and so on are not relations (see especially 5.4–5.44). His opposition to Russell here is already fully present in and central to the 1913 "Notes on Logic."¹¹

Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms. (p. 98[10])

All that is essential about molecular functions is their T-F schema (i.e. the statement of the cases when they are true and the cases when they are false). (p. 98[11])

Wittgenstein must see in Frege's 1879 explanations an instinctive appreciation of the point made in the second quoted remark from NL. In 4.431, he alleges that this healthy insight is throttled by Frege's post-1891 logical grammar that classifies sentences as proper names of truth-values, and takes the sentential connectives to mean functions that map truth-values to truth-values.¹² Wittgenstein's objection to Frege's post-1891 explanations of the logical connectives invokes the notion of sense. We need, then, to consider Wittgenstein's conception of sense. This is a larger task than it might first appear; and the place to begin is not with Frege, but with Russell. It is difficulties in Russell that in the first instance motivate and shape Wittgenstein's conception of sense.

II. Russell's Multiple Relation Analysis of Judgment

I take Wittgenstein's work on the *Tractatus* to begin in 1913. It is preceded by a year and a half of intense philosophical engagement with Russell. Russell had developed what I call a metaphysics of propositions to serve as a foundation for logic, for the logic of *Principia Mathematica*. In 1910, Russell rejected this metaphysics of propositions in favor of a metaphysics of facts. This new metaphysics brings with it its own problems in connection with Russell's multiple relation

analysis of judgment. Furthermore, it requires the wholesale replacement of the foundations for the logic of *Principia* that the metaphysics of propositions was to provide.

In thinking through the problems facing Russell, Wittgenstein acquires an approach to logic very different from Russell's old propositions approach—very different from Frege's approach as well. To understand Wittgenstein's perspective better, I propose first to survey some central features of Russell's metaphysics of propositions. Then I shall describe the shift to the metaphysics of facts and the problems this shift causes Russell. After that, we will be in a position to appreciate the notion of sense Wittgenstein introduces in 1913.

Russell acquires the notion of a proposition from G. E. Moore, when he joins Moore in rejecting Bradley's monistic idealism in favor of pluralist realism. This pluralism does not lead Moore and Russell back to a Lockean view with ideas representing mind-independent things. They appear to think that any view of cognition that erects a veil of ideas between our minds and mind-independent objects leads ineluctably back into the idealist swamp. Instead, cognition for Moore and Russell is direct, unmediated by mental representations; and propositions are what we cognize. Propositions are nonmental, nonlinguistic complex entities whose existence is independent of minds and cognition. Judgment is a dyadic relation between minds and propositions. Russell's logic is the general theory of propositions and propositional functions.

Peter Hylton has observed that in breaking with Bradley's monism and embracing a pluralism of independently subsisting items, Russell takes as the fundamental form of ontological complexity that of a relation to its relata.¹³ The simplest propositions, atomic propositions, are complexes in which, for example, two individuals are joined together by a relation to form a whole. It is intrinsic to propositions that they are true or false. So, in the proposition that Desdemona loves Othello, the individual Desdemona is joined by the relation of loving to Othello to form a true proposition. In the proposition that Desdemona loves Cassio, Desdemona is joined by the relation of loving to Cassio to form a false proposition. These simplest propositions do not represent anything; nothing makes the first proposition true and the second false. Indeed, on this view, for Desdemona to love Othello is for the proposition that Desdemona loves Othello to be true. In general, for something to have a property, or for one thing to bear a relation to another, is for propositions containing the individuals, properties, and relations to be true. Moore and Russell thus in effect identify facts with propositions that are true. False propositions, however, subsist on an ontological par with true ones. Truth and falsity themselves are indefinably simple, mutually exclusive properties of propositions.

I have used atomic propositions to illustrate Russell's general conception of a proposition as a complex consisting of items joined by a relation. In *Principles of Mathematics* and beyond, Russell works to accommodate other kinds of propositions to this model. I want to consider two issues especially important for understanding Wittgenstein's criticisms of Frege and Russell.

The first issue has already been mentioned: Russell's treatment of the logical connectives. Russell, consonant with his relation-relata view of propositional complexity, follows out the idea implicit in Moore's analysis of judgment as a relation between minds and propositions. On this analysis, the proposition expressed by the sentence "Cassio judges that Desdemona loves Othello" is a complex

in which the relation of judging joins Cassio to the proposition that Desdemona loves Othello. Here, then, a proposition itself occurs as the relatum of a relation in another, larger proposition. In the same way, in the proposition signified by the conditional “Socrates is human implies Socrates is mortal”, the relation of material implication joins the proposition signified by the antecedent to that signified by the consequent to form a molecular proposition. When Russell comes to explaining his negation sign, “~”, it is not enough for him to tell us that “~p” signifies the negation of the proposition p. Russell has to tell us what proposition this is, what combination of constituents. He is thus backed into taking the negation sign to be a predicate signifying falsity.

This treatment of sentential connectives as signifying properties and relations of the propositions signified by sentences fits well with Russell's universalist conception of logic, for it enables him to quantify into sentential positions in truth-functionally compound sentences. The treatment also explains one jarring feature of Russell's metaphysics of propositions: the admission of false propositions in which relations join things together. In some places, most notably his 1904 paper “On Meinong's Theory of Complexes and Assumptions,” Russell shows himself reluctant to admit false complexes on an ontological par with true ones. But he can find no other way to accommodate what is signified by true compound sentences with a false compound: false complexes are required to serve here as the significata of false sentences. Russell's example is a material conditional with a false antecedent. After the shift to the metaphysics of facts, Russell presents this point as the strongest reason for retaining the metaphysics of propositions.¹⁴

The second issue is one Peter Hylton has emphasized.¹⁵ Russell's relation-relata conception of propositional complexity does not easily accommodate those expressed by sentences containing complex singular terms. Consider, for example, the proposition expressed by “Xantippe's spouse taught Plato.” This proposition, like the proposition expressed by “Socrates taught Plato,” should consist of something joined by the teacher-of relation to Plato. But what? Not the man Socrates. For then the two sentences would signify the same proposition, and so could be used to communicate the same judgment, which they manifestly cannot. Might the phrase “Xantippe's spouse” signify some non-propositional complex composed of the spousehood and Xantippe? But no such complex is identical to Socrates, although Xantippe's spouse is identical to Socrates, that is, the proposition that Xantippe's spouse is identical to Socrates is true. It looks, then, as if we must admit another kind of complexity into some propositions—something that serves some of the same purposes as Frege's sense-meaning distinction.

This is exactly what Russell does in *Principles* with the apparatus of denoting concepts. In 1905, in “On Denoting,” he abandons denoting concepts, replacing them with the theory of descriptions. By means of this theory, Russell avoids the introduction of special nonrelational complexity into propositions in order to handle compound singular terms. Sentences containing complex singular terms are analyzed into a kind of existential generalization. In the proposition signified by this generalization, there is no constituent corresponding to the original complex singular term. The original complex term, thus lacking independent significance, is an incomplete symbol. The only sort of complexity the theory of descriptions requires besides relation-relata complexity is the complexity of propositional functions and variables that Russell uses after 1905 in his account of general propositions.

In these last remarks, I have several times spoken about language, asking what propositions are signified by what sentences. Russell's primary interest is with propositions, not language.¹⁶ His basic conception of the way in which language symbolizes propositions is crude and remains undeveloped. In reaction to Bradley's monism, Russell in *Principles* posits a rough-and-ready correspondence between the words and phrases of sentences and the constituents of propositions expressed by those sentences.¹⁷ Individual words signify propositional constituents so that sentences built up from those words signify propositions composed of the signified constituents. There is no account of *how* a sentence, an arrangement of designations of propositional constituents, signifies a complex of those constituents. This basic view of language remains intact after the introduction of incomplete symbols in "On Denoting." Russell gives up the view that the grammar of colloquial sentences is a guide to the constituents of the proposition it signifies, for colloquial sentences contain incomplete symbols. He, however, believes that analysis leads us to sentences from which incomplete symbols have been eliminated. Here at the bottom level of analysis, the correspondence between linguistic expressions and propositional constituents is restored. Russell still provides no account of how the sentences at the bottom level of analysis signify propositions, and in effect operates with a notion of signification that covers both words and sentences.

In 1910, Russell embraces a multiple relation analysis of judgment; and, in so doing, he replaces propositions with facts as the complexes of his metaphysics. On the 1910 version of the theory, when Cassio judges that Desdemona loves Othello, a four-place relation of judging relates Cassio, Desdemona, loving, and Othello, respectively. On the old analysis, this judgment was ascribed by a sentence of the form " $J(x,p)$." Now it is ascribed by a sentence of the form " $J(x,a,R,b)$." Cassio's judgment is true, for there exists a corresponding fact, Desdemona's loving Othello. Russell conceives of this fact as he earlier thought of propositions, as a complex in which the relation of loving unites Desdemona to Othello. Cassio's judgment is similarly a complex that consists of a four-place judging relation holding among Cassio, Desdemona, loving, and Othello, respectively. On the basis of this analysis of judgment, Russell eliminates false complexes from his ontology. There is, for example, no longer a false complex in which the relation of loving joins Desdemona to Cassio. Truth thus consists in the existence of a fact corresponding to a judgment-fact; the absence of a corresponding fact falsifies Othello's judgment that Desdemona loves Cassio.

Russell rejects his metaphysics of propositions in favor of a metaphysics of facts, as he and Whitehead are completing the first volume of *Principia Mathematica*.¹⁸ Russell tries to accommodate the logic of *Principia* to his new metaphysics by claiming that propositions are, so to speak, incomplete symbols. His strategy is to analyze the kind of judgment correlated with (or expressed by) each kind of sentence without reference to any complexes except facts. In this way, the clauses of compound sentences usable to express judgments become inert, incomplete symbols that do not signify anything. However, the multiple relation analysis, as far as Russell develops it, at best handles only judgments expressed by atomic sentences. In particular, Russell does not explain how to accommodate the quantification over propositions and propositional functions that lies at the heart of *Principia*.

Russell planned to address these problems in the third, never-to-be-drafted section of his 1913 *Theory of Knowledge* manuscript. In the meantime, Russell's shift from a metaphysics of propositions to a metaphysics of facts robbed him of a coherent construal of the logic of *Principia*.

Despite its manifold problems, Russell persists with the metaphysics of facts.¹⁹ In the 1910 paper defending the shift from propositions to facts, Russell says: "[I]f we adhere to the opinion that there are true and false objectives [= propositions], we shall be compelled to regard it as an ultimate and not further explicable fact that objectives are of two sorts, the true and the false. This view, though not logically impossible, is unsatisfactory and we shall do better, if we can, to find some view which leaves the difference between truth and falsehood less of a mystery."²⁰ Wittgenstein picks up this theme in 6.111 and links it to his dissatisfaction with Frege's and Russell's universalist conception of logic as the maximally general science:

Theories which make a sentence of logic appear substantial [*gehaltvoll*] are always false. One could e.g. believe that the words "true" and "false" signify two properties among other properties, and then it would appear as a remarkable fact that every sentence possesses one of these properties. This now by no means appears self-evident, no more so than the sentence "All roses are either yellow or red" would sound even if it were true. Indeed our sentence now gets quite the character of a sentence of natural science and this is a certain symptom of its being falsely understood.

(Ramsey-Ogden translation)

As early as 1912, Wittgenstein had rejected a universalist conception of logic: "Logic must turn out to be a TOTALLY different kind than any other science."²¹ Remark 6.111 takes us to the core of Wittgenstein's antipathy to the metaphysics of propositions as a foundation for logic: it ignores the intrinsic logical connectedness of sentences. This indifference is most salient in its treatment of contradiction and negation. In the metaphysics of propositions, negation is the predication of falsity; and truth and falsity are two properties among others. This metaphysics thus provides no articulation of the direct disagreement, the complete opposition between a sentence and its negation. On its telling, the conjunction of a sentence with its negation is just one more falsehood, parallel to other false conjunctions.

This feature of the metaphysics of propositions is deeply embedded in it. It is the product of taking truth and falsity to be properties of items signified by sentences combined with taking the logical connectives to signify properties and relations of what sentences signify. The shift to a metaphysics of facts, by breaking this logjam, holds forth the promise of an alternative understanding of negation and contradiction. For with the elimination of propositions, sentences can no longer be taken to signify complexes, complexes that have properties called "true" and "false". Wittgenstein's leading idea is that a sentence agrees or disagrees with the facts; the negation of a sentence agrees with the facts just in case the negated sentence disagrees. In this way, a sentence and its negation are themselves opposed: they disagree with each other. Here is where Wittgenstein's notion of sense makes its entrance. To see how it enters, we need to consider more closely Russell's multiple relation analysis of judgment.

III. Wittgenstein against Russell

Recall that according to the multiple relation analysis, for Cassio to judge that Desdemona loves Othello is for a four-place relation of judging to relate Cassio, Desdemona, loving, and Othello. This judgment is true if there is a corresponding complex formed from Desdemona, loving, and Othello; it is false if there is no corresponding complex. On this approach, then, the “content” of the judgment is in effect represented by the identities of the relata of the judging relation over and above the judging mind. The approach then leaves out *the way* these relata can combine to form a complex. This lacuna is conspicuous in the case of asymmetrical relations like loving. There are two ways in which the relation of loving can combine Desdemona and Othello to form a fact: Desdemona can love Othello; or Othello can love Desdemona. Russell struggles without success to overcome this particular deficiency in the three versions of the multiple relation analysis he produces from 1910 to 1913.²²

Wittgenstein, in his explicit criticism of the multiple relation analysis, brings out the significance of the general point from another direction. In NL, he says: “Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell’s theory does not satisfy this requirement” (NL, p. 103[3]; cf. 5.5422). To bring out the force of Wittgenstein’s objection, let us consider the view of language that must implicitly accompany Russell’s multiple relation analysis. The words in sentences—at least the atomic sentences at the bottom level of analysis—must signify constituents of complexes; sentences express judgments made true or false by the existence or nonexistence of complexes formed from those constituents. If the “content” of the judgment is represented by the identity of relata of the judging relation, then the only feature of atomic sentences relevant to the expression of a judgment is the identity of the items signified by the words. Sentences are thus essentially collections of names. What about a nonsensical array of names like “Desdemona Othello”? On Russell’s approach it should be false: after all, there is no complex formed from just two individuals. Russell thus fails to distinguish falsehoods from nonsense.

Russell has a ready reply to this objection. “Othello Desdemona” cannot express a judgment. It cannot express a judgment because there is no judgment-relation of the right multiplicity—no judgment-relation that takes as arguments, in addition to a judging mind, just two individuals. The reason that there is no such judging-relation is that it is not *possible* for two individuals to combine to form a fact. However, in Russell’s metaphysics of facts, there are no facts about possible combinations of items. Russell cannot then rest content with a brute appeal to logical possibility: “The notion of what is ‘logically possible’ is not an ultimate one, and must be reduced to something that is *actual* before our analysis can be complete.”²³ He accordingly introduces forms of complexes.²⁴ Russell now adds to judgment-relations an argument place for forms. So, on the 1913 version of the multiple relation analysis, a judgment-relation has, besides an argument place for the judging mind, an argument place for the form of a complex in addition to argument places for items of the right logical types to combine into a complex of that form. A judgment-fact formed by that relation is true if there is a complex of the contained form with only the contained items as constituents. It is false if there is no such corresponding complex.²⁵

I suspect that this 1913 revision in the multiple relation analysis was prompted by earlier conversations with Wittgenstein. In conversations with Russell in May of 1913, Wittgenstein criticized this revision. On the interpretation of Wittgenstein's worry I am presenting, Russell's modification does not adequately meet Wittgenstein's point. Again let us consider matters from a linguistic perspective. Sentences that express judgments should still be collections of names, only now a fully analyzed atomic sentence contains a name of a form and names of items of the right logical types to form a fact of that form. Let " $(\exists x,y,\phi)x\phi y$ " designate the form of the fact of Desdemona's loving Othello. The collection of names "Desdemona Othello $(\exists x,y,\phi)x\phi y$ " will be nonsensical, for there is no complex of the designated form containing only two individuals. However, this version of the multiple relation analysis, while nominally distinguishing falsehood and nonsense, still assimilates them. One kind of nonexistence of a complex inflicts falsity on an array of meaningful names; another kind of nonexistence of a complex makes an array of meaningful names nonsensical. Falsehood and nonsense thus appear on a level as two species of an unnamed genus. Wittgenstein, accordingly, finds Russell's 1913 revision no improvement.²⁶

The shift to a metaphysics of facts requires some notion of representation in order to characterize the correspondence that makes some judgments true. Russell's difficulties arise from the attenuated and crude conception of representation built into the multiple relation analysis. This view of representation is in turn abetted by Russell's direct realism combined with a view of language in which the atomic sentences at the bottom level of analysis are true or false by virtue of an association of words with things—a view of language that thinks of meaningful words as names, and names as mere labels. Here we have an adaptation of the classic empiricist view of language, with an association of words and things replacing the older association of words and ideas.

Wittgenstein breaks decisively with this view of language in NL, and so introduces a view of representation adequate to a conception of truth as correspondence. On Wittgenstein's new view, "In ' aRb ' it is not the complex that symbolizes but the fact that the symbol ' a ' stands in a certain relation to the symbol ' b '. Thus facts are symbolized by facts, or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world" (NL, p. 96[4]). Atomic sentences are facts in which simple symbols are related to each other. Sentential facts are coordinated with facts in the world so that a sentence either agrees or disagrees with the facts. This is the sentence's fundamental representational relationship to reality that Wittgenstein calls sense. In NL he describes it in the following terms:

The form of a proposition [= sentence] has meaning in the following way. Consider a symbol " xRy ". To symbols of this form correspond couples of things whose names are respectively " x " and " y ". The things xy stand to one another in all sorts of relations, amongst others some stand in the relation R , and some not; just as I single out a particular thing by a particular name I single out all behaviours of the points x and y with respect to the relation R . I say that if an x stands in the relation R to a y the sign " xRy " is to be called true to the fact and otherwise false. This is a definition of sense. (NL, p. 95[3])

An atomic sentence with sense is thus “a standard to which facts behave” (NL, p. 95[2]), a measuring rod laid against the facts. So conceived, the sentence is essentially true-false; it is bipolar, with true-false poles (NL, p. 99[1]).

Let us explore this conception further. Simple symbols are names. They symbolize via a rule that associates them with objects. Forms are the ways that names are related in sentences. Forms symbolize via a rule for comparing sentences of that form with the facts. So, a sentence of the form “*x* envies *y*” is true if the individual designated by the name in the *x*-position envies the individual designated by a name in the *y*-position, and is otherwise false. Remarks in the 1914 “Notes Dictated to Moore”²⁷ indicate that Wittgenstein conceives of the application of rules of designation for names and rules of comparison for a form to an individual atomic sentence in the following terms: that a name “*a*” is related thus and so to a name “*b*”—let us say, that “*a*” envy-leftstands “*b*”—says that the individual *a* envies individual *b*. An ascription of truth to the sentence

“*a* envies *b*” is true

is then analyzed as the conjunction: “*a*” envy-leftstands “*b*”, and that “*a*” envy-leftstands “*b*” says that *a* envies *b*, and *a* envies *b*.²⁸ In this way, our atomic sentence is a model, a picture of reality that represents how things might be related. It is true if the things are so related; and it is false otherwise. Here, then, we have the makings of a replacement for Russell’s crude view of linguistic representation.

This view of sentences as models of reality is Wittgenstein’s overpowering insight of 1913 that sets him on the path to the philosophy of the *Tractatus*. Reflecting on it in September 1914, he draws two stick figures that might be used to say that two persons are fencing and comments: “The sentence in picture-writing can be true and false. It has a sense independent of its truth or falsehood. It must be possible to demonstrate everything essential by considering this case” (NB 29.9.14 [5]). Once acquired, he finds the approach deeply intuitive. Thus, after the abstract discussion of picturing in the 2s, Wittgenstein asserts in 4.01 that sentences are pictures of reality. And while he notes in 4.011 that sentences in general do not seem to be pictures of reality, in 4.012 he points to a place where the essence of linguistic representation shines through: “It is obvious that a sentence of the form ‘*aRb*’ strikes us as a picture. In this case the sign is obviously a likeness of what is signified.”

Most of the first two surviving wartime notebooks wrestle with various problems in giving this approach to linguistic representation general application. The *Tractatus* presents itself in very large measure as an expression of the worked-out view Wittgenstein arrives at.

The first fruit of Wittgenstein’s 1913 insight is his treatment of the logical connectives, as they figure in singular molecular sentences. I shall first outline Wittgenstein’s treatment, emphasizing the link he makes between his conception of sentences as models of reality and the iterability of the logical connectives. Here I follow Cora Diamond, who has argued that this link between sense and iterability is the basis for Wittgenstein’s critique of Frege’s and Russell’s account of these connectives.²⁹ In the rest of the section, I consider the critique of Russell. In the next section, I return to the critique of Frege.

As already noted, Wittgenstein says in NL: “All that is essential about molecular functions is their T-F schema (i.e., the statement of the cases when they are

true and when they are false)." Let "p" and "q" stand in for atomic sentences. The sentence " $p \supset q$ " is a truth-function of "p" and "q". It is false if "p" is true and "q" is false; otherwise it is true. As atomic sentences, "p" and "q" each has a sense—each models a state of things. Our molecular sentence is, then, as 4.431 has it, an expression of agreement and disagreement with the truth-possibilities of "p" and "q". In thus giving the T-F schema associated with " \supset ", the projection of "p" and "q" onto reality is exploited to project " $p \supset q$ " onto reality—we stipulate how " $p \supset q$ " is to be compared with reality in terms of the comparisons of "p" and "q" with reality. In this way, the sense of " $p \supset q$ " is a function of the sense of "p" and the sense of "q". All that matters for the application of the T-F schema associated with " \supset "—what the *Tractatus* calls a truth-operation (5.234–5.2341)—is the bipolarity, the possession of sense, by the sentences to which it is applied. So, in the same way that the T-F schema is applied to atomic sentences, it may also be applied to truth-functions of atomic sentences to form further truth-functions of those atomic sentences (NL, p. 102[8]). Thus, it is intrinsic to logical connectives to be iterable.

Russell, relying on a typographical similarity between " $p \supset q$ " and "aRb", takes the first sentence to assert that the item designated by "p" materially implies that designated by "q", just as "aRb" says that aRb. However, the conception of sentences as models blocks the construal of " $p \supset q$ " as a relational sentence. To understand this, we must bear in mind two features of this conception as it applies to relational sentences. First, "R" is not a meaningful part of "aRb". "R" does not symbolize anything; what represents in this sentence is one name's R-leftstanding another (see MN, p. 109[9]–p. 111). Second, no complexity in a name can be relevant to its use in a sentence to model how things stand vis-à-vis the named object.³⁰ Consider now a conditional formed from two atomic sentences: " $cRd \supset dSb$ ". Suppose " $x \supset y$ " were a form so that it is hook-leftstanding that represents a relation in our sample conditional. Then " cRd " and " dSb " would function in this sentence as names whose apparent complexity is irrelevant to their role in our sample conditional. The typographical resemblance between " cRd " as used as a name in this sentence and the sentence " cRd " would be an accidental, insignificant feature of the notation. The conditional would not be a genuinely compound sentence. Of course, it is a compound sentence, and the sentence " cRd " is one of its compounds. This much is evident in that " dSb " follows from " cRd " and " $cRd \supset dSb$ ".

Furthermore, on Russell's analysis, what holds for the antecedent and consequent of our sample conditional holds for the conditional itself, when it occurs, for example, as the antecedent of another conditional. Our sample conditional can no more appear as a sentence in our longer conditional than the " cRd " could appear as a sentence in it. For suppose, as regards the principal occurrence of " \supset " in this second conditional, that hook-leftstanding represents a relation. Then, embedded in this longer sentence on one side of " \supset ", our original conditional must function as a name—its typographical complexity must be irrelevant to how it symbolizes here. So, if hook-leftstanding represents a relation, then it cannot be iterated to form genuinely compound sentences.

Thus on Wittgenstein's conception of sentences as models, the logical connectives do not function as predicates, do not represent properties and relations of items designated by sentences. This, I believe, is the point Wittgenstein is making when he says in NL, "Logical indefinables cannot be predicates or rela-

tions, because propositions, owing to sense, cannot have predicates or relations" (p. 99[3]).

Wittgenstein approaches this last point from a different direction in MN, p. 116[7]: "It is very important that the apparent logical relations \vee , \supset , etc. need brackets, dots, etc., i.e. have 'ranges'; which by itself shows they are not relations" (cf. 5.461). In Peano-Russell notation, when logical connectives are iterated to form molecular sentences, it is essential to have some device to indicate the scope of each occurrence of a connective. The demarcation of scope is needed to fix the agreement and disagreement of the molecular sentence with the truth-possibilities of its component atomic sentences. No complexity in a name can be essential to their use in a sentence to model how things stand; and in a sentence names are related in a determinate way—this is what represents how things are related in reality (see 2.14, 2.15; and the 3.14s). Scope demarcation thus has no application within models. The need for scope demarcation in the iterated use of the logical connectives shows that these do not represent properties and relations.

We can now see Wittgenstein's critique of Russell's view of sentential connectives to be an application of his *Grundgedanke* that the so-called logical constants do not represent. The sentential connectives do not function in molecular sentences as either names or forms. They are not proxies for anything; they are not functioning parts of models. What does the representing in a molecular sentence are the component atomic sentences. The rest is an indication of a T-F schema that fixes the agreement and disagreement of the molecular sentence with the truth-possibilities of its atomic components. It is the sense of the atomic sentences, their bipolarity, that fits them for the iterated application of T-F schemata, that makes the iterated applicability of T-F schemata intrinsic to them.

IV. Wittgenstein against Frege

Let us now return to Wittgenstein's criticism of Frege in the *Tractatus*. This criticism is in very large measure the application to Frege of the *Grundgedanke* and of the conception of sentences as models that underlies it. The criticisms that I shall discuss all have antecedents in NL and MN. I continue to draw on these documents to interpret the *Tractatus* itself. There are important changes and development from NL and MN to the *Tractatus*, but on the points relevant to the criticism of Frege, I see a fundamental continuity.³¹

In Frege's post-1891 explanations of his begriffsschrift, the structure of complex arithmetical terms provides the template for the logical structure of sentences. In §1 of *Grundgesetze*, Frege introduces his notion of a function. Consider a complex term like " $(4 + (3 \times 4))$ ". If we remove both occurrences of the numeral "4", we obtain an expression with a blank: " $(\xi + (3 \times \xi))$ ". Completion of the blank by any designation of a number yields a designation of the result of adding the number to its triple. Our complex term is thus analyzable into a 'complete' part, a proper name, that designates a number, and an 'incomplete' part with a blank that designates a function.

In §2, Frege assimilates arithmetical sentences to arithmetical terms, saying:

" $0^2 = 4$," " $1^2 = 4$," " $2^2 = 4$," " $3^2 = 4$ "

are expressions of thoughts, some true and some false. I express this as follows: the value of the function $\xi = 4$ is either the truth-value of what is

true or that of what is false. From this it is seen that I do not intend to assert anything when I merely write down an equation. Instead, I just designate a truth-value, just as I don't assert anything when I write down "2²" but only designate a number.³²

At the beginning of the passage Frege invokes his notion of a thought as that concerning which the question of truth arises.³³ Thoughts are what are recognized to be true or rejected as false in acts of judging that are publicly manifested by the assertion of sentences that express the thought. A sentence, a series of marks or sounds, is thus true or false in virtue of the thought it expresses.³⁴ When we substitute for a numeral in an equation other designations of numbers, we obtain other equations, sentences that express thoughts, typically some true and some false. Frege goes on to redescribe matters. Paralleling the treatment of incomplete expressions like " $(\xi + (3 \times \xi))$ " in §1, he states that " $\xi^2 = 4$ " designates a function whose values are the two truth-values, objects that Frege calls the True and the False. The sentences that result from completing this second incomplete expression thus designate truth-values, just as the completions of the first incomplete expressions designate numbers. The object designated by a complete expression is its meaning. Distinguishing the sense of a name from its meaning, Frege now tells us that a thought is the sense of a name of a truth-value. He refers his readers to his paper "On Sense and Meaning" for the detailed substantiation of these points.

On Frege's view, to write down a sentence is then no more to say something than to write down a complex arithmetical term. In "Function and Concept," he puts the point like this:

When we write down an equation or inequality, say $5 > 4$, we ordinarily intend thereby to express a judgment; in our case, we intend to assert that 5 is greater than 4. According to the view set forth here, we have in " $5 > 4$ " or " $1+3 = 5$ " only expressions for truth-values in which no assertion is made. This separation of judging from the subject-matter of judgment appears unavoidable, because without it a mere assumption [*bloße Annahme*], the stipulation of a case, could not be expressed without simultaneously judging whether the case held. We, therefore, need a special sign in order to assert something.³⁵

To this end Frege introduces the sign " \vdash " to transform names of truth-values into assertions. This sign, Frege explains, is a combination of two signs. " $\neg\neg$ " " ξ " is the name of a function that maps the True to itself and everything else to the False. " \vdash ", the judgment stroke, is not a name at all. It is an expression of asserting force. By prefixing " \vdash " to a proper name, we declare that the named object, the object presented by the sense of the proper name, is the True.³⁶ Indeed, Frege officially restricts the term "(begriffsschrift) sentence" to proper names prefixed by " \vdash ".³⁷ This use of the term "sentence," the term *Satz*, must surely have leapt off the page at Wittgenstein.

Wittgenstein criticizes this nest of views in 4.063, a long passage taken from NL, p. 99[9].

An analogy to explain the concept of truth: a black spot on white paper; you can describe the shape of the spot by saying, for each point on the

sheet, whether it is black or white. To the fact that a point is black there corresponds a positive fact, and to the fact that a point is white (not black), a negative fact. If I designate a point on the sheet (a Fregean truth-value), then this corresponds to the assumption [*Annahme*] that is put forward for judgment, etc. etc.

But in order to be able to say that a point is black or white, I must first know when a point is called black, and when white; in order to be able to say, “p” is true (or false), I must have determined in what circumstances I call “p” true, and in so doing I determine the sense of the sentence.

Now the point where the simile breaks down is this: we can indicate a point on the paper even if we do not know what black and white are, but if a sentence has no sense, nothing corresponds to it, for it does not designate a thing (a truth-value) with properties perhaps dubbed “false” or “true”. The verb of a sentence is not “is true” or “is false”, as Frege thought: rather, that which “is true” must already contain the verb.³⁸

To understand Wittgenstein’s criticism, we need first to reconstruct how Wittgenstein viewed Frege’s explanation of his notation, especially the judgment stroke.³⁹ As Frege explains matters, names of truth-values, like names of numbers, do not assert anything. To use Wittgenstein’s terminology, these names, like other names, say nothing: they are not models of reality. As names, they have no representationally relevant structure—they could be replaced by simple signs. Thus, Frege’s names of truth-values, as names, lack sense, as Wittgenstein understands this term.

As Wittgenstein reads him, Frege attempts to bring Wittgenstein’s sense into his view with the judgment stroke plus horizontal.⁴⁰ For the addition of the judgment stroke to the designation of a truth-value transforms that name into a sentence that declares the designated truth-value to be (the) true. Presumably, there is another linguistic device for declaring a designated truth-value to be (the) false. (I shall return to this point below in connection with 4.431.) Frege thus makes “ \vdash ” function as Wittgenstein says predicates function.⁴¹ Indeed, on the view Wittgenstein finds in Frege, “is true” and “is false” are the *only* genuine predicates.

On Wittgenstein’s view, what is true or false are sentences; and, as 4.06 says, “A sentence can be true or false only by being a picture of reality.” Hence, truth can be attributed only to something that models reality, and so only to something that has representationally relevant structure—to something that “already contains the verb.” Consequently, declarations of truth and falsity cannot supplant other predictions, those that figure in the sentences to which truth and falsity are attributed. Indeed, on Wittgenstein’s view of truth, not only are “is true” and “is false” not the only predicates; they are not really predicates at all. This point is evident in the disappearance of the predicate “true” in the previously discussed schematic analysis of truth: “aRb” is true =_{df} “a” R-leftstands “b”; and that “a” R-leftstands “b” says that aRb; and aRb. The facts used as models of reality can, of course, be described; but there are no models of how models model reality—no sentences that contain names that go proxy for models. This point is an aspect of Wittgenstein’s *Grundgedanke*, one salient in its second phrasing in 4.0312, “the logic of facts cannot be represented [vertreten]”—nothing can go proxy for the logic of facts.⁴²

Let us now return to the exegesis of 4.063. So consider a black spot on a white sheet of paper. Each point on the paper is either black or white, depending on whether it lies inside or outside of the spot. We can describe the spot by saying of each point whether it is black or white, using, for example, Cartesian coordinates to designate the points. There is an obvious analogy here that Wittgenstein accepts: "The specification of all true elementary sentences describes the world completely. The world is completely described by the specification of all elementary sentences plus the specification, which of them are true and which false" (4.26, Ogden-Ramsey translation). The designations of points correspond to elementary sentences. Whether a designated point is black or white corresponds to the existence or nonexistence of the state of affairs presented by an elementary sentence.

Wittgenstein argues that the analogy cannot be pressed further, as the Fregean position does. In order to be able to say that a point I have identified is black or white, I have to know when points are to be called black, and when white. A parallel point holds for truth. Now it is sentences, at least in the first instance, that are called true or false, a point insinuated by the use of quotation marks in the second paragraph of 4.063. So, in order to say that a sentence is true, I have first to know the circumstances under which sentences are to be called true. Here the analogy does not quite fit: it is, after all, the points, not their designations, that are black or white. The Fregean maintains that a linguistic expression is by extension called true or false in virtue of designating something that has properties called true or false. So, to reinstate the analogy we have to imagine the designations of the points being called black or white, depending on whether the points designated are black or white. Since the way I designate the points by means of a coordinate system has nothing to do with the color of the surface, there is no motivation to extend the application of "black" and "white" from the points to the designations. The transparent artificiality of this extension thus highlights that knowledge of what black and white are is independent of the ability to designate the points. There is no such independence with respect to the use of the sentences that are compared to the designations of the points. Here is where the analogy breaks down. To have determined the circumstances under which a sentence is true is to determine its sense. Without sense, apart from its sense, a sentence has no relationship to anything outside of it, and a fortiori no relationship to anything with properties that might be dubbed "true" and "false".

Wittgenstein, drawing on his view of sentences as models, concludes 4.063 with my opening point. Sentences are true or false in virtue of modeling reality, and this relationship to reality requires that they have representationally relevant structure. Hence, Frege's judgment stroke cannot do for Frege what Wittgenstein thinks it must do, namely, convert a senseless designation of a truth-value into a sentence with sense. Remark 4.064 underscores this conclusion: "Every sentence must already have a sense: it cannot be given a sense by affirmation [*Bejahung*]. Indeed its sense is just what is affirmed. And the same applies to negation, etc."⁴³

Some may protest that Wittgenstein's objection to Frege in 4.063 underestimates the resources of Frege's position, the way that Frege's function-argument conception of logical structure, his sense-meaning distinction, and his understanding of judgment and the judgment stroke mesh. In particular, Frege is emphatic that the judgment stroke is not a predicate. Claiming that the sentence "The thought that 5 is a prime number is true" expresses the same thought as "5 is a prime number",

he argues that the expression of judgment in an assertion cannot be accomplished by any predication. He says that the relationship (*Verhältnis*) of a thought to the True is not that of subject to predicate, not that of an object to a property, as the use of the word “true” as a grammatical predicate suggests.⁴⁴ The relationship of the thought that 5 is a prime number to the True is rather that of the sense of a name to the meaning of the name. It is this relationship that we recognize in an act of judging, an advance (*Fortschreiten*) from a thought to its truth-value. We give linguistic expression to this relationship when we utter a name with this sense with asserting force. Judging itself cannot be explained in any more basic terms: “Judging is indeed something entirely singular and incomparable.”⁴⁵

While I do not attribute to Wittgenstein a full appreciation of the subtleties of Frege’s position, I do not think his objection underestimates Frege. To see how Wittgenstein in effect meets Frege’s sharp distinction between predication and asserting force, we need to consider Wittgenstein’s criticism of Frege on negation.

Wittgenstein finds another drawback to the Fregean view that names of truth-values are given sense by affirmation or denial. If negation is a predicate, and predication is modeling, then it is impossible to iterate negation. We encountered this point in the previous section in considering Russell’s account of logically compound sentences. The Fregean approach, applied generally, thus presents a distorted understanding of logically compound sentences. Wittgenstein makes this point against Frege in the present context at the conclusion of 4.0641: “The negated sentence can be negated again, and this in itself shows that what is negated is already a sentence, and not merely something that is preliminary to a sentence.”⁴⁶

Frege’s introduction of negation in *Grundgesetze* §6 is, after a fashion, responsive to this last point:

We don’t need a special sign to declare a truth-value to be the False, if we have a sign by means of which each truth-value is transformed into the opposite, which is in any case indispensable. I now stipulate:

The value of the function

— ~x

should be the False for each argument for which the value of the function

— x

is the True, and should be the True for all other arguments.

We have therefore in

— ~x

a function whose value is always a truth-value; it is a concept under which all objects, with the exception of the True, fall. . . .

According to our stipulation — ~2² = 5 is the True; hence

! ~2² = 5;

in words, 2² = 5 is not the True; or: the square of two is not five.

In the previous section, Frege had introduced the horizontal and the judgment stroke. Alluding to the line of argument spelled out in detail in the late paper “Negation”, Frege maintains that if there is a sign that converts a name of a truth-

value into a name of the opposite truth-value, then there is no need to supplement “ \vdash ”—the sign for declaring the truth-value designated by a proper name to be the True—with another sign to declare a named truth-value to be the False. Instead of declaring the truth-value designated by “ $2^2 = 5$ ” to be the False, we declare the opposite of this truth-value to be the True. Frege notes that we need such a sign anyway in order to express, for example, a conditional whose antecedent is the negation of a sentence. Of course, Frege’s view of the negation-sign as a function-sign makes the iterability of negation, as well its combination with the other logical connectives, unproblematic.

We are now ready to return to the criticism of Frege in 4.431. Here, once again, is Wittgenstein’s criticism from the end of the passage: “If ‘the True’ and ‘the False’ were really objects, and were the arguments in $\neg p$ etc., then Frege’s determination would not at all determine the sense of $\neg p$.⁴⁷ ” Wittgenstein’s point is straightforward.⁴⁷ According to Frege, the expression formed by completion of the argument place in the negation sign is a name of a truth-value, of the truth-value that is other than the truth-value designated by the name that completes the argument place. So names of the form “ p ” and “ $\neg p$ ” are names of different truth-values. One names the True, and the other names the False. As names, they have no representational structure; they do not say anything. Not agreeing or disagreeing with reality, they do not oppose, do not contradict each other.⁴⁸ Frege’s introduction of the negation sign thus gives no sense to expressions of the form “ $\neg p$ ”.

Indeed, this point against Frege is an adaptation of a point Wittgenstein makes in MN: “The reason why $\neg x$ is meaningless, is simply that we have given no meaning to the symbol $\neg \xi$. I.e., whereas σx and σp look as if they were of the same type, they are not so because in order to give a meaning to $\neg x$ you would have to have some *property* $\neg \xi$. What symbolizes in $\neg \xi$ is that σ stands to the left of a proper name and obviously this is not so in $\neg p$ ” (MN, p. 116[2]). If Frege is going to treat sentences as names of truth-values, the only way to give sense to an expression of the form “ $\neg \xi$ ” is to treat it as a (Wittgensteinian) predicate of objects. Frege does not do this. He cannot for, as we have seen, this treatment is incompatible with the iteration of negation. Instead Frege treats negation as a mathematical function. But this treatment robs those complex names that result from completing “ $\neg \xi$ ” of representationally relevant structure. As names, they have no sense.

Here, once again, we arrive at a point where Wittgenstein may appear to underestimate Frege’s position. But does he? Frege’s official stipulation of meaning for the negation sign in *Grundgesetze* §6 leaves out the opposition between a sentence and its negation, that the negation denies what the negated sentence says. Frege tries to slip this in, when he surrounds his stipulation with talk of *opposed* (*entgegengesetzt*) truth-values. This suggests that we might read “ $\neg\neg p$ ” as “the opposite of the truth-value that p ”, that we treat Frege’s negation sign as a definite description. Wittgenstein criticizes this view in NL: “One of the most natural attempts at solution consists in regarding ‘not- p ’ as ‘the opposite of p ,’ where then ‘opposite’ would be the indefinable relation. But it is easy to see that every attempt to replace the ab-functions [truth-functions] by descriptions must fail” (NL, p. 107[6]).

There are two difficulties with the approach. First, Wittgenstein would apply Russell’s celebrated recipe to analyze this definite description. This method of analysis would introduce new occurrences of the truth-functional connectives,

including negation, into the analysans, producing a vicious regress in the process of analysis. Application of Russell's theory of descriptions presupposes an account of the truth-functional connectives, one that does justice to their iterability. The second difficulty is more elusive. As Wittgenstein notes in the quoted passage, Frege's talk of opposed truth-values suggests that opposition is a primitive relation over objects. However, relations over objects are modeled by sentences that then agree or disagree with the reality they describe. Objects can oppose each other, as when one chair faces another. But no relation of objects can constitute disagreement, contradiction between sentences. One sentence's contradicting another is a matter of their sense; and the sense of sentences is independent of the facts.

For Wittgenstein, it is the senses of “p” and “~p” that are opposed, and senses are not things. Remark 4.2 says, “The sense of a sentence is its agreement and disagreement with possibilities of existence and non-existence of states of affairs,” so that a sentence is the expression of agreement and disagreement with the truth-possibilities of elementary sentences (see 2.201, 4.031, 4.1, and 4.4). In this way, a sentence determines a location, a region, in logical space. Negation reverses the sense of a sentence (5.2341), reverses its agreement and disagreement with the possibilities of existence and nonexistence of states of affairs: “One could say that the denial [*Verneinung*] is related to the logical place that the denied [*verneinte*] sentence determines. . . . The denying sentence determines a logical place with the help of the logical place of the denied sentence, by describing it as lying outside of the latter place” (4.0641, my translation). The negation of a sentence thus opposes the negated sentence in that it agrees with reality just in case the negated sentence disagrees.

I see in Frege a rhetorical slippage relevant to this point. Frege introduces the notion of a thought as that concerning which the question of truth arises. In a pre-1891 manuscript paralleled in later writings, he says:

Before we judge, we frequently raise a question. . . . We grasp the content of a truth, before we recognize that content as true. But we grasp not merely this, but also the opposed content. For with the question we are caught between opposites [*Gegensätze*]. . . . This opposition [*Gegensatz*] or conflict [*Widerstreit*] is to be understood so that we automatically reject one side as false when we recognize the other side as true, and vice versa. The rejection of the one and the recognition of the other are the same.⁴⁹

This talk of opposed contents or thoughts is striking. It is one of the very few places where Frege speaks seriously about a logical relation among thoughts. Here at the starting point for his philosophy, Frege acknowledges the opposition between a sentence and its negation—he acknowledges sense in something like the way Wittgenstein understands this notion. In Wittgenstein's eyes, this sound beginning is distorted by Frege's function-argument conception of the segmentation of language that assimilates the segmentation of sentences to the structure of complex arithmetical terms. This function-argument segmentation leads him to classify sentences and subsentential proper names together, and to apply his distinction between sense and meaning univocally to both. Thus, having invoked at the beginning of *Grundgesetze* §2 his basic notion of a thought, Frege concludes the section by recharacterizing thoughts as the senses of names of truth-values. His talk of opposed truth-values then seeks to preserve the original insight by remov-

ing the opposition between a thought and its negation from the realm of *Sinn* to the realm of *Bedeutung*.

With this maneuver, sense, as Wittgenstein understands it, disappears. Its absence is most salient in Frege's treatment of negation. Frege's understanding of the judgment stroke as expressing the putting forward of the object designated by a name as the True does not restore it. “ $\vdash 2^2 = 5$ ” puts forward the object designated by “ $2^2 = 5$ ” as the True; “ $\vdash \sim 2^2 = 5$ ” puts forward the object designated by a different name to be the True. There is no opposition, no disagreement here. The last sentence of *Grundgesetze* §6 notwithstanding, Frege's explanation of “ \sim ” gives no grounds for reading “ $\sim 2^2 = 5$ ” as “the square of two is not five.” It is Frege's illustrative paraphrase, not his stipulation of meaning, that leads us to understand the sign he has introduced as a sign for negation. Accordingly, in 4.442 Wittgenstein dismisses the judgment stroke as irrelevant to logic: “(Frege's 'judgment-stroke' ' \vdash ' is logically quite meaningless [*bedeutungslos*]: in the works of Frege (and Russell) it simply indicates that these authors hold the sentences marked with this sign to be true. Thus ' \vdash ' is no more a part of a compound sentence [*Satzgefüge*] than is the number of the sentence. A sentence cannot possibly state of itself that it is true.)”⁵⁰

I began my discussion of Wittgenstein with a description of Russell's shift from the metaphysics of propositions to a metaphysics of facts, and of the enormous problems this shift creates for Russell's philosophy of logic. I urged that Wittgenstein hits on his view of sense in thinking about these problems. From the perspective Wittgenstein acquires in 1913, Frege's and Russell's philosophies of logic alike look hopeless.

From Wittgenstein's perspective, the emphasis on relation-relata structure that underlies Russell's metaphysics of propositions is, after a fashion, correct. Russell goes astray when he reads this structure into molecular propositions—when he treats logical constants as properties and relations. This overgeneralization of relation-relata structure forces the disastrous posit of false complexes to serve as the significata of false constituents of logically compound sentences. As Wittgenstein sees matters, Russell's metaphysics of propositions is incoherent on its face. Russell's shift to the metaphysics of facts opens the way to progress by focusing attention on the question Russell always avoids: “How do sentences, even fully analyzed sentences, represent facts?” Roughly put, Wittgenstein answers this question by finding in atomic sentences the relation-relata structure of the reality they represent.

Frege is much more attentive to notation than Russell, much more attentive to what Wittgenstein calls the logico-syntactic employment of signs. Frege's initial truth-functional account of logical connectives makes their iterability intrinsic to them. He consequently observes that the negation sign—occurring as it must in the clauses of true truth-functionally compound sentences—cannot be conceived as an expression of denial parallel to his judgment stroke. For Wittgenstein, this observation is an exemplary logical analysis.⁵¹ It ought to have led Frege to a thorough reevaluation of the role he assigns to the judgment stroke in his *begriffsschrift*. Or so I conjecture Wittgenstein thought.

Frege goes astray in his uncritical appropriation of the paradigm of a mathematical function to understand this iterability. This move leads him to treat sentences as names of special objects, truth-values. Once Frege takes mathematical func-

tions as his paradigm, he thinks through its consequences with impressive rigor and consistency. His philosophy of logic has a sheen of coherence Russell's lacks. In the September 14, 1914, NB entry, Wittgenstein reflects, "It has been what I should like to call my strong scholastic feeling that has occasioned my best discoveries." He must have seen in Frege's philosophy of logic a similar scholastic feeling at work.

Russell then wrongly generalizes the structure of atomic sentences to logically compound sentences. Frege, by dint of his function-argument segmentation, makes the reverse error. In effect, he finds the iterable structure of logically compound sentences in atomic sentences. As a result of these mistakes, Wittgenstein holds that neither thinker is able to do justice to the logical connectedness of sentences. In NL, Wittgenstein comments: "Frege said 'propositions are names'; Russell said 'propositions correspond to complexes.' Both are false; and especially false is the statement 'propositions are names of complexes' (NL, p. 97[3])."

Notes

I came to the ideas in this essay by thinking through Cora Diamond's paper "Inheriting from Frege" and Peter Hylton's paper "Frege and Russell." I was aided by formative conversations with Hylton. I also benefited from Ian Proops's paper "The Early Wittgenstein on Logical Assertion." I am indebted to Cora Diamond, Burton Dreben, Juliet Floyd, and Warren Goldfarb for comments on earlier drafts of the essay. I had the advantage, in writing this essay, of the presentations and discussions at the 1998 University of California at Riverside Philosophy Conference, "Frege, Russell, Wittgenstein: Shaping the Analytic Tradition"; and, in revising it, of responses from an audience at the University of Virginia.

1. Anthony Kenny (1974, pp. 2–4) alleges five misunderstandings of Frege. In my opinion, none of Kenny's allegations stand up to scrutiny.

2. See Goldfarb, 1997, and Goldfarb, this volume.

3. I will say that I do not think Wittgenstein finds in Frege the subtle conception of truth that I find. Nor would I now say that Wittgenstein and Frege share similar underlying views of judgment, contrary to what I asserted in Ricketts, 1985, p. 3. I thus largely accept the criticisms Goldfarb, this volume, makes of me. As Goldfarb suggests, it may still be the case that Wittgenstein, on his own and not in reaction to Frege, develops a view that both converges at points with Frege's and is responsive to tensions that are present in Frege.

4. Here I believe my interpretive approach differs from Anscombe, 1959, chap. 3 and pp. 105–8, and from Proops, 1997. Anscombe and Proops both tend to view Wittgenstein as pointing out deficiencies in Frege's and Russell's view in order to support and motivate his own alternative. In contrast to me, both see the cogency of some of Wittgenstein's points to be vitiated by misunderstandings of Frege.

5. Here and elsewhere, I have altered both the Ogden-Ramsey and the Pears-McGuinness translations by translating *Satz* by "sentence," not "proposition." See also the entry for 25.12.14 in Wittgenstein, 1979. Hereafter cited as NB with citation of date and paragraph of the entry. My understanding of the *Grundgedanke* builds on the interpretation presented in McGuinness, 1974.

6. Unless otherwise noted, quotations from the Pears-McGuinness translation of the *Tractatus* (Wittgenstein, 1961). Here I have altered the translation of the crucial final sentence of 4.431. The German reads: "Wären 'das Wahre' und 'das Falsche' wirklich Gegenstände und die Argumente in $\sim p$, etc., dann wäre nach Freges Bestimmung der Sinn von ' $\sim p$ ' keineswegs bestimmt."

7. Frege, 1879, §5, p. 5. The similarity to the contemporary truth-table explanation of the material conditional is even closer in "Booles rechnende Logik und die Begriffsschrift," in Frege, 1969, pp. 11, 39–40, and Frege, 1979, pp. 11, 35–36.

8. Frege, 1879, §7, p. 10.

9. See Russell, 1903, §§15–16, 37. The relation of material implication designated by " \supset " relates a proposition p to a proposition q iff it is not the case that p is true and q is false.

10. Whitehead and Russell, 1910, *1, p. 93. In *Principles of Mathematics*, Russell defines " $\sim p$ " as " $(\forall q)(p \supset q)$ " (1903, §19, p. 18). On this definition of negation, because it is not evident that there are false propositions, it is not evident that p and $\sim p$ contradict each other, that their conjunction is false.

11. Hereafter cited as NL by page and paragraph from NB, Appendix I.

12. Frege believes that functions must return a value for any object as argument. In *Grundgesetze*, Frege explains that his negation function maps the True to the False, and everything else to the True (1893, §6). *Grundgesetze* (1893, §12) contains a parallel stipulation for the dyadic conditional function. I do not think this nicety plays a role in Wittgenstein's criticism of Frege in 4.431. So, I shall simply ignore arguments for Frege's negation and conditional functions that are not truth-values.

13. Hylton, forthcoming.

14. Russell, 1904, pp. 521–22; 1983, p. 153. See also Russell, 1906, p. 48.

15. See Hylton, 1993; forthcoming; 1990, chap. 6.

16. As Burton Dreben pointed out to me, this is Russell's self-conception. See Russell, 1959, p. 108. For further discussion of Russell's attitudes toward language, see Dreben, 1996.

17. See Russell, 1903, §46.

18. Russell first formulated, but did not endorse, a version of the multiple-relation analysis in 1906. There is no hint of the multiple relation analysis in Russell's 1908 paper "Mathematical Logic as Based on the Theory of Types." The first endorsement of the multiple relation analysis I am aware of comes in Russell, 1910b. The English manuscript of this essay, "The Theory of Logical Types," appears in Russell, 1992, pp. 3–31. The discussion of the multiple-relation theory is in §ii on pages 10–11. This paper is, as far as I know, Russell's first endorsement of the multiple relation analysis. He presents it in more detail in "On the Nature of Truth and Falsehood" (1910a), published in November 1910. The presentation in *Principia Mathematica*, vol. 1, pp. 43–44, is verbatim identical to "The Theory of Logical Types." I conjecture that the multiple relation analysis is something that Russell grafts onto volume 1 of *Principia*.

19. I have argued that Russell's adoption of the metaphysics of facts is bound up with his critique of Bradley's monism. See Ricketts, 2001. Wittgenstein appears never to have been attracted to Russell's metaphysics of propositions.

20. Russell, 1910a, p. 152. cf. Russell, 1904, p. 524.

21. Wittgenstein to Russell, 22.6.12 (Wittgenstein, 1995, p. 15). For a sketch of Frege's and Russell's universalist approach to logic, see Ricketts, 1996b, §1.

22. For further discussion of this point, see Ricketts, 1996b, §2.

23. Russell, 1983, p. 111.

24. Russell in turn identifies the form of an atomic fact with its existential closure, so to speak. Thus, the form of the fact that Desdemona loves Othello is the fact that something bears some relation to something, the fact that $(\exists x,y,\phi)x\phi y$. See Russell, 1983, pp. 113–14.

25. See Russell, 1983, pp. 117–18, 148–49.

26. My interpretation of Wittgenstein's objection makes good sense of Wittgenstein's mention of negation in connection with criticisms of the multiple relation analysis in NL. See NL, p. 94[1] and p. 95[4].

27. Hereafter cited as MN by page and paragraph from NB, Appendix II.

28. See MN, p. 110[6–8] and p. 113[2]. The second conjunct in the analysis is an expression, as 5.542 puts it, of the coordination of facts by means of the coordination

of their objects. See also the entry for 1.11.14 [15]. It is then not itself a description of reality. As thus analyzed, ascriptions of truth prove to be pseudosentences. See NB 6.10.14 [3–4].

29. See Cora Diamond's penetrating discussion of 5.02 (forthcoming, esp. §§5–6). I am very indebted to this paper, particularly in my discussion of the critique of Russell's view of the logical connectives in this section and in my discussion of 4.431 in the next section.

30. See 3.3411. In colloquial language, definite descriptions present themselves as singular terms whose complexity is relevant to the sense of sentences in which they appear. Wittgenstein embraces Russell's theory of descriptions to analyze away appearances of explicitly or implicitly complex names. This link between the theory of descriptions and the conception of sentences as pictures is one of the reasons Wittgenstein acclaims Russell at 4.0031: "It was Russell who performed the service of showing that the apparent logical form of a sentence need not be its real one."

31. For a discussion of the development of Wittgenstein's view of linguistic representation from NL to the *Tractatus*, see Ricketts, 1996b, §3; see also Kremer, 1997.

32. Frege, 1893, §2, p. 6f.

33. See Frege, 1918a, p. 60; and "Aufzeichnungen für Ludwig Darmstaedter" (Frege, 1969, p. 273; Frege, 1979, p. 253). The original pagination of Frege's published papers is marginally indicated in many German and English collections of Frege's papers, including Frege, 1967, 1984.

34. See Frege, 1918a, p. 60.

35. Frege, 1891, p. 22.

36. See the opening sentence of Frege, 1893, §5. Frege says, "By means of such a sentence, it is asserted that this name means the True" (1893, §32). In "Function and Concept" (p. 22), Frege says of " $\vdash 2 + 3 = 5$," "With it a mere name of a truth-value is not written down, as with ' $2 + 3 = 5$.' Rather, thereby it is also said that it is the True."

37. Frege, 1893, §5, p. 9, and §26, p. 44. The result of prefixing " \vdash " to a latin mark of a truth-value—these correspond in Frege's notation to our free variable generalizations—is also a sentence. Proops (1997, pp. 126–27) notes the importance of Frege's terminology here for understanding Wittgenstein's discussions of the judgment stroke.

38. I have modified the Pears-McGuinness translation.

39. Over the next six paragraphs, I substantiate the suggestion of Max Black (1964, p. 227) that Wittgenstein views Frege's judgment stroke as Frege's attempt "to restore to the propositional sign, which he had degraded to a mere designation, its truth-claiming aspect." Proops (1997, p. 133) follows Black here.

40. Following what I believe is Wittgenstein's view of the matter, I shall henceforth refer to this combination simply as the judgment stroke. Here I agree with Proops, 1997, pp. 126–28.

41. Here I disagree with the discussion of 4.063 in Anscombe, 1959, pp. 105–6. Anscombe sees Wittgenstein's criticisms as turning on the idiosyncrasies of Russell's (mis)reading of Frege in *Principles of Mathematics*, Appendix A, §477. Anscombe is correct to observe that Wittgenstein is influenced by this source in his use of the term *Annahme* (assumption) in connection with Frege, but Wittgenstein's criticisms of Frege do not draw on Russell's interpretation. As I explained, Wittgenstein argues that Frege, by his assimilation of sentences to names, must take the judgment stroke to function as a Wittgensteinian predicate.

42. Indeed, for this reason, Wittgenstein takes his schematic analysis of truth to reveal that even ascriptions of truth to individual sentences are pseudosentences. See the references in note 28.

43. The "etc." here refers to the other logical connectives. See NB 3.11.14 [13].

44. Here I summarize a line of thought Frege presents in "Über Sinn und Bedeutung" (1892, pp. 34–35). See also "Einleitung in die Logik" (Frege, 1969, p. 211; Frege, 1979, p. 194); and "Meine grundlegenden logischen Einsichten" (Frege, 1969, pp. 271–72; Frege, 1979, pp. 251–52). I have argued elsewhere that Frege believes that no sentence

can describe the relationship of thoughts to truth-values, and that this relationship cannot be conceived as a relation (*Beziehung*) between two objects. See Ricketts, 1996a, esp. pp. 133–37.

45. Frege, 1892, p. 35. The German reads, “Das Urteilen ist eben etwas ganz Eigenartiges und Unvergleichliches.”

46. Cf. NB 6.11.14 [2].

47. Here I differ with Anscombe's interpretation (1959, p. 107) of 4.431's criticism of Frege. Anscombe thinks that Wittgenstein's criticism is internal to Frege's own distinction between *Sinn* and *Bedeutung*. Frege's introduction of the negation sign is stated in terms of *Bedeutung*, specifying the value the negation function yields for any argument. Frege, 1893, §32, notwithstanding, Anscombe asserts, “But on Frege's own principles you do not specify a sense by specifying a reference.” I see Wittgenstein's objection as drawing on Wittgenstein's conception of sense, not Frege's. Nevertheless, my interpretation of 4.063 and 4.431 fits well with Anscombe's emphasis on the close connection between Wittgenstein's understanding of truth and his view of sentences as pictures in chapters three and four of her book.

48. Diamond (forthcoming) makes this point. In commenting on 5.02, she urges that Frege's *Grundgesetze* introduction of the logical connectives gives logically compound sentences only the logically insignificant structure of indexed names rather than the logically relevant structure of truth-functions of elementary sentences.

49. Frege, 1969, p. 8; Frege, 1979, p. 8. For similar remarks, see Frege, 1969, p. 161; and Frege, 1979, p. 149; Frege 1969, p. 214; Frege, 1979, p. 198; and Frege, 1918b, pp. 152, 154. Wittgenstein would not have been familiar with any of these sources when writing the *Tractatus*.

50. My translation. The source of this remark is NL, p. 103[2]. Wittgenstein seems to think that Russell is backed into the same sense-conferring use of the judgment stroke as Frege. For the textual basis for this, see Whitehead and Russell, 1910, *1, p. 92. See also the earlier presentation of the multiple relation analysis in *Principia*, especially Russell's talk of judging as transforming a sentence from an incomplete into a complete symbol on page 44.

51. I believe that this is why Wittgenstein advised Peter Geach to include “Negation” in the collection of Frege's writings that Geach and Max Black edited. Geach's recollection is contained in the preface to Frege, 1977, p. vii.

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Truth before Tarski

*After Sluga, after Ricketts,
after Geach, after Goldfarb,
Hylton, Floyd, and Van Heijenoort*

Cora Diamond

Those pictures which are neither originals nor copies, which
the Italians call *Pastici* . . .

The Art of Painting (1706)

John McDowell has suggested that “one of the benefits of studying a great philosopher from an alien age is that it can help us to see that we do not have to swim with the currents of our own time” (1998, pp. 37–38). The great figures of early analytic philosophy do not belong to a wholly alien age, but for just that reason it may be hard to see their differences from ourselves. The philosophical interest of the differences between contemporary analytic philosophy and its antecedents, and the significance of making those differences clearer, turned out to be a theme of the conference at Riverside for which this essay was written. One of my aims in it is to help sharpen our sense of those differences.

The method of the essay is cut-and-paste, or pasticcio. I take ideas, insights, and remarks of other philosophers and form a composition from them. The underlying conception of the method is suggested by remarks of Wittgenstein’s from 1933, in which he compared the method of philosophy to that of aesthetics. What aesthetics does is “to draw your attention to a thing,” to “place things side by side”; and this may then, he suggested, lead the hearer to see things differently (quoted in Moore, 1959, p. 315). I begin with Hans Sluga’s description of truth before Tarski and with his argument about the importance of attending to the differences between Frege, Russell, and Wittgenstein on the subject of truth.

I. Truth before Tarski, after Sluga

In a recent essay, Sluga writes of how we treat truth now, after Tarski, after the establishing, partly through Carnap’s influence, of Tarski’s sort of approach within

analytic philosophy (Sluga, 1999). The change from earlier analytic philosophy, Sluga says, brought loss as well as gain in the way we think about truth. We have gained in the “real advance in our formal capacity to deal with . . . truth,” but, on the other hand, “some of Frege’s, Russell’s and Wittgenstein’s deepest intuitions concerning the philosophical problem of truth have slipped from view” (Sluga, 1999, p. 28). He describes the loss in even stronger terms at the end of the paper: our current understanding of Tarski’s success reflects our having lost an understanding of what the problem of truth really is (see Sluga, 1999, p. 40; also Putnam, 1988, 1994).

Sluga also argues that, although we owe much to Jean van Heijenoort for bringing to attention the deep contrast between the Frege-Russell-Wittgenstein understanding of logic and the later post-Tarskian understanding, van Heijenoort does not make clear enough how different from each other the views of Frege, Russell, and Wittgenstein were; and he adds that the disagreement is particularly evident in their views about truth. If we want to see what was lost in our understanding of the problem of truth when philosophers took up the formal semantics style, we need to pay particular attention to the *different* views of the three great pre-Tarskians (Sluga, 1999, pp. 28–30); the van Heijenoort essay is the classic (van Heijenoort, 1967).

Sluga picks out as one of the crucial points on which Frege, Russell, and Wittgenstein disagree the possibility of a theoretical account of truth, and in particular the possibility of a correspondence view of truth. So he notes Frege’s clear rejection of a correspondence theory and of alternative theories as well; he describes Russell’s move from holding that truth is indefinable to holding some form of correspondence theory; and he draws attention to Wittgenstein’s statement in the *Tractatus* (Wittgenstein, 1963) of something that looks very like a correspondence view of truth (Sluga, 1999, pp. 30–33). In his essay in this volume, Sluga tells a part of the story of truth in the alien age of early analytic philosophy, focusing on Frege and developments in his thought. In what follows, I tell another part of the story, with a different focus. Although it is meant to be a story about truth, it will also be a story about themes in Wittgenstein’s response to Russell’s philosophical difficulties. I shall weave these stories together, providing an account of Wittgenstein on truth that leads to the question how far he shares Russell’s commitment to a correspondence conception of truth. In the next three sections I sketch some of the material I am borrowing.

II. Borrowings from Geach, Ricketts, and Goldfarb

I have two borrowings from Peter Geach, the first, discussed in this section, from “Kinds of Statement” (Geach, 1979). He says there that he himself is taking from medieval logicians a use of the word “proposition” to mean “a bit of language in a certain logically recognizable employment” (p. 221). To bring out the difference between a proposition in this sense and a mere string, he notes that the proposition “Socrates was bald” does not occur in “A philosopher whose teacher was Socrates was bald” but does occur in “Socrates, who taught Plato, was bald.” So, although he is not using the word “proposition” to mean something nonlinguistic like a Fregean thought, he equally does not mean by it something that can be iden-

tified simply by our eyes or ears, without our use of logic. (An instructive contrast here is with Quine's approach. Philosophers who are, as he puts it, "commendably diffident" about positing nonlinguistic bearers of truth will take the bearers of truth to be sentences regarded simply as linguistic strings. The nonlinguistic proposition and the linguistic string appear as the two principal alternatives.)¹

In the paper from which I borrow this point, Geach does not say that the use in question of the word "proposition" ("Satz") is also found in Frege and Wittgenstein. In a late essay, Frege makes use of a distinction between what grammar counts as a Satz and what logic counts as a Satz (1984, p. 391); but the distinction can be seen earlier, for example, in his classification of subsentential clauses on logical grounds in "On Sense and Meaning." The classification involves identifying certain bits of language as having the logically recognizable employment to which Geach referred. Thus, "Napoleon, who recognized the danger to his right flank, himself led his guards against the enemy position" is described by Frege as containing two assertible Sätze, each of which has "Napoleon" as subject and both of which are asserted if the whole Satz is uttered assertorically (1984, p. 172).² One of the component Sätze appears within the complex sentence in the form of an adjectival clause, but that does not affect its logical character. The same Satz could occur independently.³ That Wittgenstein uses the word "Satz" in the sense described by Geach comes out, for example, in the *Tractatus* at 3.11–3.12: a Satz is a bit of language, a kind of sign, in a certain kind of use. (I shall not here discuss Russell's complex use of "proposition"; see, however, Russell, 1984, pp. 105–6.)

We should note that there is a parallel between Geach's point and a point made by Thomas Ricketts and also by Warren Goldfarb, that the identification of "parts and features of statements as logically functioning units" of various types is something we do on the basis of our grasp of inferential relations.⁴ Thus we may, for example, pick out parts of sentences as proper names in Frege's sense in virtue of the way Leibniz's Law can be applied to the sentence. There is a contrast, then, with two other views of how proper names in a sentence can be picked out: through prior knowledge of the kind of thing they are taken to mean (call it the "ontological" method), or through recognition that they are described in a dictionary as proper names, or that they occur on some list of the proper names of the language, or have been generated in accordance with some procedure (call these "methods of prior specification"). Neither the ontological method nor the methods of prior specification depend on recognizing the logical kind of use to which the bit of language is being put in the sentence.

I have picked up Ricketts's mention of the role of Leibniz's Law in the recognition that such and such word or words in the sentence have the function in it of proper names. But, as he notes, recognition of proper names (in this sense) is not just a matter of Leibniz's Law, but of "an interlocking series of basic inference patterns" (1986b, p. 86). Through our grasp of how a sentence figures in inferences of these various patterns, we can recognize proper names, predicates, relational terms: bits of language that in the particular sentence have this or that logically recognizable type of use. Ricketts's argument concerns how logic enables us to pick out *nonsentential* parts of sentences; but now suppose that what we are concerned with is *sentences* rather than subsentential parts, and that we use "sentence" (or "proposition" or "Satz" or "statement") to mean a bit of language in the logically recognizable use to which Geach referred. If it is via our grasp of logic

that we are able to identify bits of language as sentences, and also to identify something as the same sentence again, or not the same sentence though similar in appearance, what are the logical characteristics involved in those identifications? And what must logic itself be like, how must we be conceiving *it*, if it is thus involved in the identity of sentences?

That question is central for the *Tractatus* and Wittgenstein's earlier thought. It is closely related not just to Wittgenstein's view of truth (which is my concern here), but also to his idea of logic as looking after itself, and of language as preventing any logical mistakes (Wittgenstein, 1961, pp. 2–4, 11; also Wittgenstein, 1963, 5.47–5.476). I shall not attempt to make plain how these interconnections work, how the totality of truth-functional logic is supposed to be involved in what a proposition is. I shall illustrate by an example the connection between the idea of logic looking after itself and the idea of logic as helping us to tell when we have the same sentence, and shall then turn back to questions about truth.

Consider A. N. Prior's case (in Prior, 1960) of the runabout inference ticket. Prior rejects the idea that we can introduce a logical connective by specifying its role in inferences. He introduces a connective in that way in order to show the logical trouble we get into if we take seriously the view he rejects. But Wittgenstein's conception of logic as looking after itself lets us see why Prior's connective does not lead to logical trouble. Here is Prior's story.⁵ We introduce a new connective "tonk" by specifying that, from any proposition P, you can infer the proposition formed by joining P to any proposition Q by "tonk" (which we refer to as the proposition P-tonk-Q), and that from any proposition P-tonk-Q, you can infer the contained proposition Q. This, Prior thinks, makes trouble for any attempt to show that logical connectives can be introduced by giving their role in inferences, since the inferential role of "tonk," as specified, allows us to infer Q from P, whatever propositions Q and P are. The *Tractatus* provides a basis for the following response to this argument: it is through logic that we can identify any proposition as the same proposition as some proposition uttered or written earlier, and therefore we must look to logic to see whether what appear to be two occurrences of the same proposition P-tonk-Q are occurrences of the same proposition, or whether in some inference we have taken P-tonk-Q first as containing one logical connective, and then as containing some other. Logic will show us that the two rules through which the connective "tonk" was introduced (or supposedly introduced) in fact provide partial specifications of two logical connectives; and that a proposition P-tonk-Q inferred from P is not in general the same proposition as the equiform proposition P-tonk-Q used by itself as a premise from which to infer Q. We can get this out of logic if the logic that we need in order to do the job of identifying propositions is truth-functional logic, for that will impose the conclusion, not that the rules for "tonk" are logically wrong, but that they go part of the way toward introducing two logically distinct connectives. I said that Wittgenstein's view can show that the introduction of "tonk" does not give logical trouble. On the story as I told it we might nevertheless find "tonk" troublesome, because it would be a nuisance to have two logical connectives that were not perceptibly different, just as it can occasionally be a nuisance that the English word "or" sometimes functions as does "aut" and sometimes as does "vel" in Latin. But this is not logical trouble (any more than the Anglo-Saxons were in logical trouble if their word for black sounded like their word for white).⁶

Here I return to the question: What must logic be for it to do what it supposedly does on the *Tractatus* view? I believe that Wittgenstein's putting forward of what looks like a correspondence theory of truth is part of his response to that question, and that, on the *Tractatus* view, what we usually think of as correspondence theories of truth are attempts to treat as a theory what is actually a reflection of certain ways we operate with sentences, dependent on their logical character. Wittgenstein's understanding of these issues involves the distinction that, in the 1913 "Notes on Logic" (1961, pp. 93–106), he expresses as the distinction between the role of names and that of forms. Ricketts has pointed out that in this distinction there lies the germ of the picture theory (1996b). But that is getting ahead of the story. At this point we need some more ingredients, including another borrowing from Geach.

III. More from Geach, and More from Ricketts

In "Truth and God" (1982), Geach expounds arguments against a variety of theories of truth, including the correspondence theory, and against any theory that takes sentences to play a semantic role akin to that of names, either when the sentences are standing alone or when they occur in other sentences. (Here he uses "sentence" or "statement-making sentence" as he had used "proposition" in the essay quoted in section II.) In contrast to any such theories, what we need for a correct logical understanding of truth is, he says, the logic of duality. It would not be exaggerating to say that Geach takes the logic of duality to be the key to an adequate understanding of truth; and in explaining what he means by the logic of duality, he draws on *Tractatus* 4.062 and 4.0621. The line of reasoning to which Geach refers is already present in Wittgenstein's "Notes on Logic" (1961, p. 95). What Geach emphasizes, picking it up from Wittgenstein, is the idea that we could communicate using our sentences so that what Wittgenstein calls their sense is reversed: "p" is used in the new system to say what " $\sim p$ " says in ours, and vice versa. An important feature of such a system is that in it the sign of negation has exactly the same meaning it has for us; that is, it is the sign used to form the contradictory of a sentence.⁷ Geach develops the logic of duality in order to bring out how deeply names differ from sentences. A language dual to ours is one in which each sentence equiform to ours contradicts the corresponding sentence in ours. In such a language, the names have the same meaning they have in ours, while sentences "are essentially dual in significance" (p. 93). I shall discuss that point later. But I want now to note Geach's remarks about his own use of metaphor in explaining the duality of sentence-meaning and the contrast between name and sentence. Geach picks up Wittgenstein's metaphor of a reversible arrow: a sentence and its negation are like arrows pointing in opposite directions. What this metaphor is a metaphor for "cannot be informatively explained"; but "an inchoate understanding of these relations is involved in all informative discourse" (p. 94). He adds that this understanding "can be clarified or sharpened by logical and philosophical training, but there can be no question of analysis or explicit definition" (p. 94).

In order to place what Geach is doing, we should consider his remarks alongside part of Ricketts's discussion of Frege on truth. After arguing that, as Frege

sees it, a grasp of the notion of truth comes out in the capacity to reason, “to recognize one truth on the basis of the recognition of others,” Ricketts says that objective truth, as understood by Frege, cannot be split off from *recognition-of-the-objective-truth-of*. We are engaged in an attempt at such splitting off when we try to make truth the content of a predicate. Ricketts also invites us to interpret Frege on thoughts as an attempt to articulate “aspects of objective truth cognized in acts of judging” (Ricketts 1996a, pp. 136–38). Here Ricketts is taking further a kind of reading of Frege developed in some of his earlier essays, in which he sees Frege as redescribing features of the kind of discourse in which we seek to arrive at truth.⁸ We can read Geach as self-consciously engaging in the same activity, and as implicitly taking Wittgenstein to be engaging in that activity, too. That is, we can see Geach on the logic of duality as following Wittgenstein on the logic of duality, where both of them are doing what Ricketts says Frege is doing: articulating what is involved in informative discourse. And, further, we should see the two articulations—Frege’s, and what we might refer to as the Wittgenstein-Geach articulation—as standing in a complex relation, that of alternative articulations, a relation which is different from that of incompatible theories. (Geach’s articulation is not by any means identical with that of the *Tractatus*, but I am concerned here with shared features of the two articulations, which contrast with that of Frege.) But, if we are to take the articulations to be intellectual alternatives (while at the same time we are not taking them to be incompatible theories), we need to see them as having in common some understanding of the answerability of philosophical discussion of logic to what is present in ordinary informative discourse.

There is a connection here with Sluga’s point that we have lost the pre-Tarski understanding of the problem of truth. To show the connection, I shall reformulate Sluga’s claim. On the pre-Tarski view, work in logic is essential in articulating what is involved in informative discourse, and such articulation is answerable to the inchoate understanding we have of such discourse. My restatement of Sluga continues: later analytic philosophy lost its understanding of the problem of truth in losing that earlier conception of answerability to the inchoate understanding of informative discourse. What Sluga calls the problem of truth is a problem only if we take seriously the inchoate understanding, which includes an idea that he puts this way: “our talking is in some way or other responsible to the world.”⁹ (Here there is also a connection with a very interesting issue raised by G. E. M. Anscombe in her *Introduction to Wittgenstein’s Tractatus*. She leads us to see a question about what justifies the complex use of Ts and Fs in the truth-table. She suggests that the usual explanations of the truth-table in logic books draw on our inchoate understanding of informative discourse in putting Ts and Fs down under the component propositions as well as in the final column, but the explanations then given of how the truth-functional calculus works are detached from that inchoate understanding.¹⁰ I believe that something similar is part of Sluga’s idea of the character of post-Tarskian analytic philosophy: it draws on the inchoate understanding of informative discourse in its use of words like “true”, but it does not see itself as thereby bound by the internal logic of that understanding, and therefore as needing to draw out what that internal logic is.)

IV. Russell's Predicament

In this section I sketch what I call Russell's predicament, borrowing from Peter Hylton and Thomas Ricketts elements of their discussions of the problems Russell has with judgment after he drops his earlier metaphysics of propositions (Hylton, 1990, 1994; Ricketts, 1996b). Ricketts explains how the inadequacies of Russell's account motivate the *Tractatus* conception of representation. I should not want to call into question his argument, but I shall put the story in a somewhat different way. Here, then, is Russell's predicament.

1. Russell recognizes that one can understand a proposition (a sentence capable of expressing a statement) without knowing whether it is true or false (Russell, 1984, p. 109). No explanation of this fact is available to Russell in terms of items like Fregean Sinne (see Hylton, 1994, p. 345).
2. If we can understand propositions without knowing whether they are true or false, understanding them cannot be a matter of our having some relation to an entity that exists only if they are true (Russell, 1984, p. 109).
3. If A loves B (say), then the relation *loves* actually relates A to B, in that order. But if A does not love B, then *loves* is not a relation actually relating A to B in that order. Only in the complex that makes it true that A loves B is it the case that *loves* occurs as a relating relation uniting A and B in that order.
4. It seems, therefore, that the understanding of the proposition "A loves B" cannot invoke an account of its content in which *loves* is a relating relation, actually relating A and B (in that order), for *loves* actually relates them that way only if A does love B.
5. If someone believes the false proposition "A loves B", "*loves*", it seems, must occur in the proposition he believes as a genuine verb, that is, as an expression not just for a relation but for a relation actually relating the two terms. But how can that be possible? And if it is not possible, what account can be given of what he believes?

Russell approached the problem in various ways at different stages of the development of his theory, but arguably none of these approaches is satisfactory. In the 1913 "Notes on Logic," Wittgenstein claims that when we say of someone that he judges that such and such, we must specify what is judged by using a *whole* proposition (1961, p. 96). We cannot give the judgment itself in terms of bits and pieces: in terms of the items judged to be related, and then the relation, and then whatever else, as in Russell's 1913 account. But what is it that leads Russell to the account in terms of bits and pieces? It is that he thinks he cannot have an account of the sort that, in some sense, he really wants, which is that, in what is believed, the relation in question actually relates the terms of the relation. If, in an account of the judgment that A loves B, we were to say that "*loves*" is a genuine verb and that the content of the judgment involves *loves* actually relating A to B, we run into trouble; for *loves* does not join A to B unless the judgment is true. Since the content believed is the same whether the judgment is true or false, the content cannot include the relation actually relating the terms. This problem immensely interested Wittgenstein for years: that it should look to us as if the solution we really want here is one which we cannot have.¹¹ He says repeatedly in the 1930s

that it seems to us that the only thing that can explain what it is that we expect (suppose we expect that Smith will come into the room) is the fulfillment of that expectation, for, in what we expect, *coming into the room* (no substitute for it) actually attaches to *Smith* himself. But we do not have *that* unless the expectation *is* fulfilled.¹²

The solution to Russell's predicament that Wittgenstein provides in the *Tractatus* involves the rejection of an assumption on which it rests, namely, that judging and believing are relations: relations either to a *whole* content, or to the things about which we are believing or judging something. On the relational view, if the whole sentence were irreducibly essential in giving what we judge, there would have to correspond to it something that we are related to by the judging relation. In responding to the problems here, Wittgenstein helps himself to the language of relationship between proposition and reality, but he does so in the course of developing a story in which propositions are not themselves relata (i.e., are not in any effable or ineffable relations). He aims to bring out clearly the logic of having content, but the understanding for which he aims does not in the end rely on the structure of relations to which we appeal (implicitly or explicitly) when we attempt to give accounts of judgment and of understanding a proposition.

We should bring to mind here what happens when we philosophize about the relation between thought and reality, or about truth as correspondence to reality, or about propositions agreeing or disagreeing with reality. In all these cases, we think in terms of some *relation* that might be spelled out. Wittgenstein in 1913 is struggling to get out of this mode of thought, and the idea at which he arrives in 1913, of sentences as *facts*, is a part of his breakthrough, as Ricketts emphasizes. I shall look further at that breakthrough, but shall turn back first, in section V, to Sluga's statement about the problem of truth being centrally concerned with the responsibility of talk to the world. It was that idea about which I said in section III that it was part of the inchoate understanding of informative discourse.

V. The Inchoate Understanding of Truth

We have in the *Tractatus*, in the notion of the proposition as a picture, and of a picture as agreeing or disagreeing with the reality it depicts, an expression of our inchoate understanding of the relation between what we say and the world. But how does the *Tractatus* help us to get beyond that still inchoate understanding? I should suggest that it does so in part by redirecting our attention toward logical features of the use of ordinary sentences, logical patterns of use. We can note how talk about agreement or disagreement is connected with repeated uses of the same sentence, or of sentences standing in some logical relation to each other; and we can consider further what is involved in such patterns. Here are specifications of three groups of patterns of the sort I mean.

Group 1 (cf. Wittgenstein, 1961, p. 94)

p.

So "p" is true and "not-p" is false.

not-p

So "p" is false and "not-p" is true.

Group 2

A believes p.

p.

So A's belief is correct.

A believes p.

not-p.

So A's belief is incorrect.

A believes p.

A's belief is correct.

So p.

Group 3

p, and S says that p.

(“S” is here a stand-in for some way of speaking about the sentence.)

A has R to B, and S says that A has R to B.

A does not stand in R to B, and S says that A has R to B.

In, for example, the first and second cases within the second group, we say what someone believes; we say how things are; and then we infer that the belief is correct, or that it is incorrect. In the cases in this group and in the third group, a sentence appears twice, first as giving a content (what is believed), and then as a statement or part of a statement of what is so. But here we should recall the point from section II: that it is logic that tells us when we genuinely have the same sentence twice. We can thus see a question: If our inchoate understanding of truth and informative discourse involves the possibility of sentence-patterns like those above, what logical features of such patterns of use are reflected in that inchoate understanding? What, that is, is involved, what does logic tell us is involved, in the identity of the sentences that appear twice over in the various cases in these groups?

I am suggesting that we should turn our attention away from Sluga's question, the question what it is for belief or discourse to be responsible to the world, to the question what logic enables us to see in the structure of sentence patterns like those in the various groups above. It is not, after all, mere visual similarities that allow implications and inferences like those in patterns 1 and 2. If such visual similarities as “p” turning up three times in the pairs of sentences in Group 1 signal logical characteristics, what are the logical characteristics on which we implicitly rely in those patterns and the others?

Already in 1913, in the “Notes on Logic,” Wittgenstein gives an answer with several interconnected parts. One point he emphasizes concerns the form of description that should be used to replace what I have as the letter “S,” which in the third pattern stands in for a way of speaking of a sentence. But the kind of replacement we need will be dictated by some further features of our inchoate understanding of informative discourse and the place in it of truth and falsity.

A significant feature of this inchoate understanding is reflected in the patterns in the first group: whatever it is that enables us to determine the truth or falsity of a proposition thereby also enables us to determine the truth or falsity of its negation. The truth or falsity of the two propositions is determined together, but that point needs to be treated as part of what is still inchoate. It is not, for example, at

all clear why we should not say that Frege provides an account in which the truth of one thought determines the falsity of another thought, and in which therefore the truth and falsity of two opposed propositions, Sätze, are determined by one thing, the truth of the first thought.

In the "Notes on Logic," Wittgenstein says that the chief characteristic of his theory is that the proposition "p" has the same meaning as "not-p", the meaning being the single fact that corresponds to both propositions.¹³ The editors of the "Notes," Von Wright and Anscombe, point out that the word "meaning" here should be taken as the English of "Bedeutung" (Wittgenstein, 1961, p. 94). Later on, after he had (late in 1914) given up this way of using "Bedeutung", he expressed the same conception by speaking in terms of the reality that is compared with a proposition; the proposition and its negation are compared with the same reality, but in two opposite ways.¹⁴ The difference between the proposition and its negation lies, then, in how they are compared with the reality, or, in 1913 terms, in how they are compared with the fact which is the Bedeutung of both. I shall put the idea now using the language of propositions as relating to a reality or a fact, a way things are, but that is a mode of speech that I shall drop. Using it now to put this element of the inchoate understanding of truth: the idea is of one and the same fact, or one and the same reality, to which both a proposition and its negation are related. The reality determines the truth or falsity of both propositions. This conception has internal to it that the relation to reality is *reversible*. That is, if we take some reality to determine that a proposition "p" is true and its negation false, we could reverse the relation which each of the pair "p" and " $\sim p$ " has to that reality, and each now will be understood in the opposite way from before. Understood in this way, the truth and falsity of "p" and " $\sim p$ " will be reversed.

I hope to show by the end of this essay that talk of a sentence as having a relation to reality (i.e., talk of the sort in which I have just been engaging) is meant in the *Tractatus* to dissolve from within, as one might say. Here I pick up a phrase of Warren Goldfarb's: he speaks of the "dissolution from inside" of *Tractatus* talk about possibility (Goldfarb 1997, p. 66). And I shall borrow again from Thomas Ricketts, who has spelled out in detail how our understanding shifts as we read and think about the sections in the *Tractatus* (the 2s) in which Wittgenstein appears to be presenting a metaphysics of possibility. We can, Ricketts argues, recognize the conflict between such talk about possibility and the conception of facts in the *Tractatus*. Writing about these passages in the *Tractatus*, he says: "Wittgenstein's rhetoric . . . is carefully calculated both to limn a metaphysical picture and simultaneously to cancel the incompatible implicatures that any presentation of this metaphysics carries with it. . . . When we . . . reflect on Wittgenstein's words, on the view we take these words to convey, we realize that, on their own telling, they do not communicate a view at all. Wittgenstein's words pull themselves apart."¹⁵ The argument I shall construct (about talk of the relation of sentences or propositions to reality) is parallel to Ricketts's argument about the *Tractatus* talk of possibility, and its coming apart.¹⁶ Wittgenstein's view in the *Tractatus* and earlier writings is that, if we are genuinely speaking about propositions, the language we use is language for something that cannot be said to be in a relation to anything. So the *Tractatus* talk about propositions as agreeing or disagreeing with a reality, which is deliberately relational language, is meant to be recognized by us, as we read the book, to be talk that reveals itself to be incoherent. If we take Ricketts to

have shown that the *Tractatus* does not give us a metaphysical but ineffable account of possibilities underlying intelligible discourse, the same sort of reasoning will show that it does not give us a conception of propositions as having an effable or ineffable relation of agreeing or disagreeing with reality.

Earlier in this section I said that our understanding of informative discourse includes the idea that the truth or falsity of a proposition and of its negation are determined together; and I added that a Fregean account of this joint determination was available. But I suggested that we consider Wittgenstein's quite different idea of the joint determination, expressed in the "Notes on Logic" in what he refers to as the chief characteristic of his theory, namely, that "p" and "not-p" have a single Bedeutung, determining the truth or falsity of both propositions, where this determination is reversible. The notion of reversibility of sense is, as Geach noted, metaphorical. It cannot be informatively explained, he said, but he added that our understanding of the relations expressed in the metaphor can be sharpened.¹⁷ With the aim, then, of trying to sharpen that understanding, I turn, in the next three sections, to the logic of duality and to the question why Wittgenstein insists (in the *Tractatus* and the earlier writings) that propositions are facts. How is their being facts connected with their reversibility? That question can be expressed in terms of the sentence patterns in Group 3. If the letter "S" in those patterns stands in for a way of speaking of a proposition, a bit of language in a certain logical employment, then to say that propositions are facts is to say that we shall not be speaking of anything that has the logical use characteristic of propositions unless what replaces "S" is the specification of a fact. Why should that be? We need to see how the logic of a sentence's reversible relation to reality fixes what should replace the letter "S."

VI. Duality

Wittgenstein has a characterization of duality in the "Notes on Logic"; he repeats it in the *Tractatus* in the sections to which Geach refers. Here is how it goes in the latter:¹⁸ Wittgenstein begins by asking whether we could make ourselves understood by means of false propositions instead of true ones, so long as we knew that they were meant to be false. No, he answers, for a proposition is true if we use it to say that things stand in a certain way, and they do; and if by "p" we mean $\sim p$ and things stand as we mean that they do, then, construed in the new way, "p" is true and not false. And he adds that it is important that the signs "p" and " $\sim p$ " can say the same thing, because that shows that nothing in reality corresponds to the sign " \sim ". We can see here how closely related are the following two ideas: the idea in the "Notes on Logic" that he calls the chief characteristic of his theory, namely, that the propositions "p" and "not-p" have the same meaning (Bedeutung), and what he called (in the *Tractatus*) his fundamental idea, namely, that the logical constants, so-called, are not representatives. If they were representatives of something, then "not-p", on the line of reasoning we are considering, could not be related in the truth-or-falsity determining way to the reality to which "p" is also related in that way. Putting the point in the terms used in the "Notes on Logic," "p" and "not-p" would not then have the same meaning, since the meaning of "not-p" would include the item corresponding to "not", while the meaning of "p" would not.

There is, then, according to Wittgenstein, and, following him, Geach, the possibility of communicating with sentences looking and sounding just like the sentences we now use, but having reversed sense. I shall use Geach's word "Unglisch" for the language that stands to English in this relation of reversed sense: an Unglisch sentence is used to say the contradictory opposite of what the equiform English sentence says (Geach, 1982, p. 89). We should note first here that names refer to the same thing in English as in Unglisch (i.e., the dual of a name, the Unglisch of an English name, is the name itself).¹⁹ The Unglisch sentence "Rover is spotted" says that Rover is not spotted; the name "Rover" is dual to itself, as are all names. If we were to follow Frege in treating definite descriptions as proper names, then a definite description in Unglisch would refer to whatever the equiform description refers to in English. So, the Unglisch sentence "The present Queen of England was born on April 21, 1926" would mean that the present Queen of England was not born on April 21, 1926; and that indeed would be taken by Frege to be the contradictory opposite of the corresponding English sentence. In this reading of the Unglisch sentence, the function name "the present Queen of ()", which is part of the Unglisch expression "the present Queen of England", means the same function as English "the present Queen of ()". Such function names are then like proper names in being self-dual. That point is reached by drawing on the inchoate understanding of informative discourse, which includes the idea that, if you say the contradictory of what you have said, you speak of the same things and say the opposite about them (see Geach, 1982, p. 90). The logical point that names are self-dual gives an articulation of that idea, and it applies to Fregean function names as logical elements within Fregean proper names as well as to Fregean proper names.

The details of the relation between English and Unglisch can be worked out in various ways (see Geach, 1982 on some controversial issues, including the treatment of intensional contexts). But the one crucial feature we need to note is how the main verb works in simple predication. The main verb will be dual to its negation. I believe that this point gives us part of the logical characterization for Wittgenstein of a verb. A verb is a verb, logically speaking, only if it is *not* self-dual, and no sentence is a sentence, logically speaking, unless it has a verb that is genuinely a verb in this sense.

There are important connections between the point I have just ascribed to Wittgenstein and the thought of both Russell and Frege. What we have seen is that the articulation of what is involved in informative discourse, of our inchoate understanding of truth and falsity, has important implications for a contrast between names and verbs. A connection between the character of truth and falsity and the nature of verbs was already noted, but in a very unclear way, in Russell's discussion of verbs in *The Principles of Mathematics* (Russell, 1903, §§52–55). Russell ties the problem of what it is for a verb to be genuinely used as a verb to the problems of the unity of the proposition and the nature of assertion, and he then simply says that the points he has been discussing lead to logical problems that would have to be gone into thoroughly in a treatise on logic. One is left with the impression that he is glad that he can at that point leave the problems and get on with other questions. He returns to these issues when he speaks, in the *Theory of Knowledge* manuscript, of the difference between relations genuinely relating and relations that are not relating (see, e.g., 1984, pp. 108–10); the matter of relat-

ing relations had also come up in the earlier discussion of verbs in *The Principles of Mathematics*. I shall come back, in section VII, to Russell's problems, but I shall turn first to a connection with Frege.

I argued that a function-expression within any expression taken to have the logical role of a proper name is self-dual. But a verb used as a verb is never self-dual. That pair of points may be involved in Wittgenstein's extremely puzzling criticism of Frege at 4.063 in the *Tractatus*, and also in the "Notes on Logic" (1961, p. 95). According to Wittgenstein, Frege mistakenly believed that the verb of a proposition is "is true" or "is false," but, as against any such view, what is true must already contain the verb. Frege, though, never said that propositions have as their verb "is true" or "is false," and he would not have accepted the ascription to him of that view. What, then, can explain Wittgenstein's criticism? Anscombe suggests that its source is his reliance on Russell's earlier misreading of Frege in *The Principles of Mathematics* (Anscombe, 1963, p. 106). But, in order to explain how Wittgenstein can have ascribed to Frege the view he does, she needs to add to the misunderstanding that Wittgenstein got from Russell a further misunderstanding introduced by Wittgenstein on his own. Her account of Wittgenstein treats his criticism as based upon misunderstandings piled on misunderstandings. What Anscombe thinks there is to be said for her explanation is that no other account of the puzzling criticism at 4.063 is available. She may have the right explanation, but there is at least one other way to understand 4.063, given its place within Wittgenstein's discussion of the logic of duality, and the connection between the logic of duality and sentences not being names. What follows is an alternative account of the criticism of Frege at 4.063.

On Frege's view of sentences as proper names, any function sign occurring in a sentence taken to be a complex proper name would have to be treated as self-dual. That is, no function sign occurring in such a name is a verb in the logical sense (i.e., in the sense imposed by the logic of duality). On this criticism of Frege, there is no implication at all that Frege would have granted that unasserted sentences (which for him are proper names) have no verbs. Wittgenstein is not suggesting that Frege was explicitly putting forward a view like the *Begriffsschrift* view that there is one predicate of all judgments, namely "is a fact", only with two predicates instead of one; rather, he is taking Frege to be committed willy-nilly to a certain view about verbs through his account of how his symbolism works. If Wittgenstein is reading Frege this way, what Frege 'believes' the verb of the proposition to be is read off what, in his symbolism, has a genuinely reversible use. If, on Frege's account of his symbolism, only asserted propositions are used in such a way that they have what works logically (by the standards of the logic of duality) as a verb, then we find out Frege's 'beliefs' about where there are verbs without asking him, without considering what he would say or did say. On this reading of Wittgenstein, he is not misunderstanding Frege's horizontal stroke, and taking it to be the predicate "is true". He is taking Frege's use of the combination of the assertion sign and the horizontal to be a use that has the logic of reversibility. In Frege's treatment of sentences preceded by the assertion sign, and only there, the horizontal stroke (the first horizontal stroke, if there is more than one) works as a genuine verb. In asserted propositions in which the negation sign follows the assertion sign, there is also a genuine verb. If Frege's symbolism allows for a verb "is one and the same as the True" or something like that, in as-

sserted propositions, it equally allows for an opposite verb “is not the True” (or “is the False”, as Wittgenstein has it) in asserted propositions.

(Wittgenstein is not ascribing to Frege the view, later explicitly criticized by Frege himself, that we need two *acts*, assertion and denial. Wittgenstein’s point is rather that it follows from Frege’s view that no unasserted proposition contains what Wittgenstein would recognize as a genuine verb. But it cannot be the case that a proposition has a verb only when asserted. What impedes Anscombe in her reading of Wittgenstein’s criticism may be a particular element in her understanding of Frege’s symbolism. In her account of his symbolism, she says that the sense and reference of the sign formed by completing the horizontal function with the name of a truth value is the same as the sense and reference of the truth-value name on its own. But that description uses a notion of sense that blurs the difference between what Wittgenstein would call a verb and what he would deny was a verb. On his view, nothing that has a verb has the same sense as something that lacks a genuine verb.)²⁰

I am not sure that the account I have given of the criticism of Frege at 4.063 is correct. Maybe the passage does rest on Wittgenstein’s taking over Russell’s misunderstandings, with some further ones thrown in. But it is possible to generate the 4.063 criticism of Frege if we put together three things: (1) an explanation of what it is to be a genuine verb in terms of the logic of duality, which makes a sharp distinction between names and verbs; (2) Frege’s treatment of unasserted sentences as proper names; and (3) Wittgenstein’s tendency to go in for horrendously compressed exposition, so that his talk of what Frege believes is not concerned with what Frege actually says, or even with what he would say, but rather with what logically can be read out of Frege’s account of his symbolism. There are further questions here, which I cannot get into, whether the criticism, understood as I have suggested, is a sound criticism, and whether there is open to Frege any kind of response that would not be what Geach calls *ad hocus-pocus* (1982, p. 92). Another question concerns the bearing of these issues on Wittgenstein’s remark (1961, p. 96) that assertion is merely psychological, a criticism of what Russell had said in *Principles of Mathematics* (1903, §52). Whatever its obscurities, Wittgenstein’s remark has at least this clear implication: we are not indebted to assertion for a sentence’s having a verb that is genuinely a verb. The verb must be as much a verb in unasserted sentences as in asserted ones, a point which is also implied in Wittgenstein’s claim that in “not-p”, “p” is exactly the same as if it stands alone (Wittgenstein, 1961, p. 97). The latter point, which is, he says, “absolutely fundamental,” immediately follows the remark about assertion’s being merely psychological, in the summary that Wittgenstein dictated of the material in the “Notes on Logic.”²¹

VII. On Reversibility of Direction and Propositions as Facts

One more borrowing from Thomas Ricketts will, I hope, put me in a position to answer several interrelated questions: How is Russell’s predicament resolved? Why can propositions not occur as terms of a relation? Why did Wittgenstein attach such great significance, within the body of ideas he was developing in 1913, to the idea that “p” and “not-p” have the same Bedeutung? Why is the logic of dual-

ity important? Section VII starts from Ricketts's account of Wittgenstein's criticism of Russell on judgment, and leads to the point about "p" and "not-p" having the same Bedeutung. Section VIII moves on to the issue of propositions not being relata. (The two sections share another feature of Ricketts's approach, namely, rejection of the view taken by various commentators that the topics of belief and judgment *become* problems for Wittgenstein only after the basic ideas of the *Tractatus* are in place, and only because they seem not to fit into the truth-functional conception.²² This is not to deny that propositions ascribing beliefs and judgments may appear anomalous from the point of view of the *Tractatus*, but only to insist that the importance of the questions they raise was recognized in Wittgenstein's very early thought about Russell's view of judgment. The need for an adequate account of judgment is a constraint on Wittgenstein's thought from the beginning; from the beginning he rejects any relational account.)

Ricketts (1996b, pp. 64–69) examines the shifts that took place in Russell's theory of judgment in the years from 1910 to 1913. In 1910, Russell moves from his earlier conception of judgment as a relation between a judging subject and a (Russellian) proposition to a conception of judgment as a relation that has as its terms a judging subject and items that form a complex of an appropriate sort if the judgment is true (the multiple-relation theory). The important further shift, from our point of view, is the shift from the 1912 version of the multiple-relation theory to the 1913 version in the *Theory of Knowledge* manuscript (Russell, 1984). Ricketts notes that one main failing of Russell's theory of judgment, in the form it came to have in 1913, was its inability to handle asymmetrical relations, like *precedes*. Russell had no adequate account of the directionality of such relations. Earlier versions of Russell's theory had not given rise to such a difficulty, for he had until 1912 continued to hold the view put forward in *Principles of Mathematics*, that a relation and its converse were two distinct relations. "[It] is characteristic of a relation of two terms", he had said in 1903, "that it proceeds, so to speak, from one to the other" (1903, §94; see also §219). The direction in which the relation goes is part of the relation; hence if a relation is one of the terms united in a judgment, it carries with it its way of ordering its own terms. Ricketts explains how a problem about directionality arises for the theory of judgment when Russell denies, in 1913 (in the chapter on relations in *Theory of Knowledge*), that the ordering of argument places is part of the relation, and comes to hold instead that a relation can be identified with its converse. The chapter was written shortly after a conversation with Wittgenstein in which Wittgenstein is known to have raised objections to Russell's previous theory of judgment; and I find particularly suggestive Ricketts's argument (1996b, p. 69) that that conversation was what led Russell to the new view of relations. Certainly Russell's argument in *Theory of Knowledge* (1984, pp. 85–87) that a relation and its converse are not two relations introduces a very Wittgensteinian-sounding element into Russell's account of relations.

I shall ascribe to Wittgenstein a line of reasoning that I take to embody a powerful insight, but only part of that line of reasoning is explicit in the texts we have. That is, although the texts allow me to speak of Wittgenstein on the true-false reversibility of propositions, there is nothing explicitly connecting such reversibility with the partially parallel case of the reversibility of relational propositions, yet I shall argue that the parallel is significant. So, my argument that there

is a significant parallel rests (so far as it ascribes to Wittgenstein an account of relational propositions) on nothing beyond the suggestiveness of the remarks of Ricketts's which I have just mentioned, and which he may not wish to take as I take them.²³

If I want to say that A loves B, I need to be able to refer to A and to B; I need to use expressions that pick them out as opposed to other things that those expressions might have been used to pick out. If I want to say that A *loves* B, expressing that that relation holds is a very different matter from naming anything, but there is nevertheless also in this case an element of arbitrary correlation, though of a different sort. If I express that A loves B by putting one name to the left and another to the right of "loves", I might in that way have expressed not that the person named first loves the other, but instead that he *envies* the other, or that he *despises* the other, and so on. In fixing how my symbolism works, I need to "extract from" all the relations that I might mean the relation *loves*, "as by a name I extract" the object it means from all other things ("Notes on Logic," Wittgenstein, 1961, p. 99). The insight I am ascribing to Wittgenstein here is that there is a vital contrast between the "extracting" of one thing rather than another for some expressive feature of my symbolism to mean (whether it is the extracting of an object from objects or of a relation from relations) and the determining of the direction of a relation.²⁴ This contrast between the way the directionality of a relation is expressed in language and the way a particular item is "extracted" from others of its kind in settling the meaning of an expression is tied to the possibility of a use of language in which the directionality of two-term relational sentences is systematically reversed, that is, in which "A loves B" would say that B loves A, and so on. On the model of Geach's use of "Unglisch" to mean the dual of English, we can use "Conglisch" to mean the language that differs from ours in that two-term relational sentences in Conglisch express the converse of the relations which the equiform English sentences express. Wittgenstein's approach to these issues then would give us, I am suggesting, this point: that this difference in the meaning of English and Conglisch sentences is not a matter of a difference in what is coordinated with any part or expressive feature of the corresponding sentences. (In the language of the "Notes on Logic," it is not a matter of a difference in what is coordinated with any *component* of the corresponding sentences.)²⁵ On this view, when we say "A loves B," our sentence has a complex kind of expressive capacity involving both arbitrary correlations and directionality. That a two-term relational sentence works as it does in English rather than as the equiform sentence does in Conglisch depends on a rule specifying that how things stand in the sentence (e.g., one name to the left of a relation-word and another to the right) says that the named things stand this way to each other, rather than that they stand to each other conversely in that relation. The difference between the English sentence and the Conglisch one is, then, a difference in how the two equiform sentences are used, not in what is "extracted by," or coordinated with, any part or feature.

I believe that there is a very brilliant move on Wittgenstein's part involving a partial analogy between the directionality of relations like *loves* and the directionality of propositions as such, their true-false reversibility. True-false reversibility is partially analogous to A-loves-B/B-loves-A reversibility. In explicating both types of case, we can make use of the distinction between settling whether this item rather than that is meant (is extracted from a range of possible

items that might be meant), and settling *directionality*. In both cases there is the idea that the components of two propositions might be the same, and what the names and forms are coordinated with might be the same, but the propositions might nevertheless differ in directionality. In both cases there is the idea that directionality is determinable via rules in which propositions are themselves treated as facts, that is, their description already has directionality. Just as the difference between English “A loves B” and the equiform proposition in Conglish, or between English “A loves B” and logical-English “A converse-loves B,” is not in anything correlated with any component of these propositions, so the difference between “p” and “not-p”, two propositions with the same Bedeutung but opposite direction of determination of truth and falsity, is not in there being anything that any component of the two propositions is correlated with. (So “not” is not a component, i.e., its role in sentences is not tied to an item of the appropriate sort being “extracted” from some range of such items, for it to mean. There is not something it stands for.)

I said that difference in directionality is a difference in use. Peter Hylton describes such differences as difference in what we *do* (1997, p. 105); Ricketts says that the differences here are differences in how we compare propositions of the relevant sorts with the facts (1996b, p. 71). What that means can best be seen if we turn to the “Notes on Logic” (1961, pp. 98–99), where Wittgenstein says that what corresponds to symbols of the form “xRy” is primarily pairs of objects. Objects of the x,y pairs stand in various relations to each other, with the relation R holding between some and not others. We can now lay down a rule that fixes the comparison with reality of symbols of the form “xRy”. When the object named by the name standing to the left of “R” names an object that stands in relation R to the object named by the name standing to the right of “R”, this “behavior of the facts” shall count as like in sense (*gleichsinnig*) with the proposition “xRy”; otherwise the facts count as opposite in sense to the proposition. We could equally have had a rule laying it down that the condition for agreement between proposition and facts is that the name standing to the right of “R” names an object that stands in R to the object named by the name to the left of “R”. This would be a difference in how we use symbols of the form “xRy”, but not a matter of having “R” stand first for a relation and then for its converse (see Ricketts, 1996b, pp. 71–73.) Writing not long after the “Notes on Logic,” Wittgenstein says of the relational structure of the proposition (which is what expresses this or that relation between objects named in the proposition) that this relation has a quite different relation to the relation it means from the relation which a name has to the object it means (Wittgenstein, 1961, p. 111). Part of meaning a (nonsymmetrical) relation is meaning it in this or that direction: you cannot set up the meaning-connection without setting up the direction involved in agreement between proposition and facts; and that direction is reversible.²⁶

We can now see the significance of Wittgenstein’s claim that the chief characteristic of his theory is that “p” and “not-p” have the same Bedeutung. Wittgenstein’s account of the sense of asymmetrical relations involves the coordination of the propositional component “x loves y” with the same relation, whether propositions “A loves B,” and so on, are taken as they are in English or with the direction of the relation reversed, as in Conglish. If a partially analogous account of directionality can be given of the true-false directionality of propositions, then this can

be explicated through the idea that the truth or falsity of “p” and of “not-p” is determined by the same fact, but in opposite directions. A directional conception of propositional sense requires that a proposition and its negation should be ‘related’ to the same something-or-other, but in opposite directions.

The expressive capacity of a proposition, then, essentially involves directionality, and that directionality itself belongs to the proposition through a kind of use, a rule-governed kind of comparison with reality, which is reversible. The names of the things we speak about have an utterly different kind of meaning; that sort of meaning has no intrinsic capacity for reversal; it has no directionality. A thing, one might say, has no opposite. Relations, which do have directionality, hold between things; nothing with directionality is a relatum. This takes us to the topic of the next section, propositions not being relata.

VIII. Propositions Are Facts, Not Names: How Is This Connected with Russell’s Predicament? How Is It Connected with Truth?

Recall here the Fregean idea that there are ways of attempting to talk about a concept that are self-defeating, in that we put what we want to be an expression for a concept in the logical place for a proper name, whereupon it is no longer the expression for a concept (Frege 1984, pp. 184–86).²⁷ I want to ascribe to Wittgenstein a corresponding idea, namely, that a proposition can be deprived of its logical character if it is treated as a name. When he says that propositions are facts, not names, this is not an ontological discovery. It means, rather, that attempting to speak of a proposition as one can speak of names, or attempting to treat it in other ways as a name, that is, by putting it into a slot where the subject of a predicate goes, or where the name of one of the terms of a relation goes, will result in its no longer being, logically speaking, a proposition. It will lack directionality.

But how does that point help us to get out of Russell’s predicament? Wittgenstein had said in 1913 that when we say that A judges that, etc., we have to mention a *whole* proposition as what A judges (1961, p. 96). Russell had thought that we could not do so, because there would be no entity connecting in the appropriate way the things judged about, if the judgment were false (see section V above, on Russell’s views). If Wittgenstein holds that we do have to mention a whole proposition as what A judges, does that not imply that A stands in some sort of relation to the content that he judges? How can it not do so? In the *Tractatus*, Wittgenstein says that the form of “A judges that p” is “‘p’ says that p” (1963, 5.542). But how is that not relational? How are any of the other patterns specified in section V not patterns in which propositions occur in slots for relata?

A further question: in my explanation of Wittgenstein’s views, I have been taking seriously his claim that a single reality makes true or false two propositions, “p” and its negation. How is that not a case of two propositions having a relation to the reality in question? It is no answer to that question to say that the relations in question are logical relations or projective relations, and that these are not really *relations*; for as long as we go on using the crutch of such expressions as “logical relations”, we are thinking of them through our idea of relations. Confidence that we can distinguish logical relations from relations is part of what stops

us from throwing away the ladder. When we talk about logical relations, we tell ourselves that we mean a special sort of relation—that-is-not-really-a-relation, a relation not expressible by ordinary propositions.²⁸

Here we should turn back to the ideas of Ricketts and Goldfarb, discussed in section V, about Wittgenstein's talk of possibilities in the *Tractatus*. They argue that it would be inconsistent with the conception of fact with which Wittgenstein is working to treat possibilities as part of the ontological structure of the world. Talk about possibility in the *Tractatus* is meant to dissolve into incoherence; it is not meant to give us a conception of a realm of possibilities on which the sense of our language depends. Interpreted in this way, the *Tractatus* leads us to recognize that, as Goldfarb puts it, “our understanding of possibility . . . arises simply from our understanding of and our operating with the sensical sentences of our language” (1997, p. 66). The Ricketts-Goldfarb reading of the *Tractatus* on possibility shows us, I am suggesting, how to read the *Tractatus* on the relation between thought or talk and reality. Just as following out the implications of reified possibilities lands us, as Ricketts argues, in incoherence, so following out the implications of what Wittgenstein tells us about the relation between our propositions and reality lands us, and is meant to land us, in incoherence. Let me spell out once more the reason. All such attempts, even when we tell ourselves that we mean to be speaking of a ‘relation’ of some special sort, not a genuine expressible relation, nevertheless involve thinking of propositions as items going into a relation as its terms, that is, not as genuinely directional, not as genuinely capable of propositional sense. Such talk can be understood to dissolve when we understand that what we are trying to do is to treat a proposition as nameable, as a logical subject, and when we understand the self-defeatingness of that aim. What is left, when we recognize the self-defeatingness of such talk, is our understanding and operating with the sensuous sentences of our language. That point seems to me what we need to focus on in this somewhat different connection.

Look now at the second group of sentence patterns in section V. These patterns represent one important way in which we operate with sensuous sentences. If we say that A judges that p, and we then say that p, we use the same proposition twice, once to say what A judges, once when we say what is so. How is it still a proposition when it occurs after the words “A judges that”, that is, when it is in the slot where we provide a content? That slot has an important kind of reversibility, which you can see most easily if you consider the verb “to doubt”. To say in English that A doubts p means that A rather thinks that p is not the case; to say in Scots (and some archaic forms of English) that A doubts p means that A rather thinks or fears that p is the case.²⁹ The difference between the two uses of “doubts” is nothing but the difference in the rules for how we may infer: from English “A doubts we shall be late” and “We shall be late,” we can move to “A is wrong”; from Scots “A doubts we shall be late” and “We shall be late,” we can move to “A is right”. This is to say that there is no difference between the English and the Scots sentences except the direction of comparison. Propositions about what someone says or judges or believes (etc.) have a kind of logical reversibility over and above that common to all propositions. That is, we can reverse not just the direction of comparison of the whole proposition with reality (which we can do with any proposition); we can, in the case of “A judges that p”, and so on, also reverse the direction of comparison determining what we say about the correctness of A’s

judgment. The slot into which we put the proposition which A judges, after the words “A judges that” or “A doubts that” (etc.) is not a slot for a relatum but for a proposition, as is shown by the fact that, without there being any alteration in the meaning of any component of the proposition in question (a component here being any feature of the proposition that “extracts” an object from objects or a relation from relations), we have two opposite ways of operating logically with the resulting proposition. That is, from “p” we can infer in two opposite directions to propositions about A, about the correctness or incorrectness of what he is said to hold. The difference between the two members of pairs of opposed propositions about A (that he doubts_{English} that p, and that he doubts_{Scots} that p, or that he judges that p, and that he judges that not p, etc.)³⁰ is the difference in the direction of the inference from “p” to the correctness or incorrectness of the belief or doubt or judgment (or whatever) ascribed to A. (It is a consequence of this view that the apparently relational structure of these propositions is not a propositional component, in the sense of “component” that I have picked up from the “Notes on Logic,” and does not mean a relation.)³¹ If we try to speak philosophically about this, and we say that A’s judgment is correct if it agrees with reality, or if it corresponds with reality, all that this means is that from a proposition saying what is so, we infer that A judges correctly, or that A judges incorrectly. The central thing to note is that agreeing with reality or corresponding with reality is not here any kind of predicate. To speak of a judgment as agreeing with reality is nothing but to allude to the kind of operating with propositions that I have just sketched: namely, that if you put a proposition into a content-slot, and use that same proposition to say what is so or what is not so, you can then infer, depending on the rules governing the judgment-verb, either that the judge judged correctly or that he judged incorrectly, and the rules in question are reversible in the sense we have seen. To reverse them involves a shift in direction of comparison, not in anything being coordinated or correlated with something.³² Earlier I quoted Wittgenstein’s remark that “A believes p”, “A says p”, and so on, are of the form “p’ says that p” (1963, 5.542; see also 1961, p. 118). I think it is possible to spell out what is involved in that claim,³³ but I shall leave that aside and move back to the implications for truth and falsity and for talk of correspondence with reality.

A conception of truth as correspondence with reality runs the risk of being subject to the criticisms of Frege (see Ricketts, 1996a; Sluga, 1999, this volume). Wittgenstein’s early work provides a way of considering the *issue* of correspondence, of what Sluga calls responsibility of talk to the world, that is not, I believe, subject to the Fregean objection or anything like it. Our ideas about correspondence are attempts to articulate features of our use of senseful sentences, essential and characteristic features of how we operate with senseful sentences. This sort of talk about correspondence makes use of the language of *relationship*, of our thought or our talk standing in a *relation* to reality. From such talk we can move in two quite different directions. One direction of development takes that mode of talk seriously and develops theoretical conceptions of what must hold of what we call true for the thing in question to be true. The *other* direction of development is Wittgenstein’s: it returns us to the logic of our ways of operating with senseful sentences, that is, with propositions, and specifically to our ways of speaking about content. Any proposition can be used to say what is so. And can therefore be part of how we go from a statement about what was said or thought or judged

to such statements as that A judged *correctly* or that the proposition “aRb” is true or that it is not. Our talk is responsible to the world in that propositions figure in such patterns of operation in two different kinds of place: giving a content and saying what is so. It belongs to propositions to figure in both places.

The Fregean objection to theories of truth applies to such theories because they say of what is judged that it needs to have some feature in order to be true; hence the truth of the original judgment cannot be established without establishing that the judgment has that feature. But any attempt to reach a judgment that the initial judgment *has* the feature leads to the need for the judgment about the first judgment’s having the feature itself to have the feature. The structure of this argument treats that which is said to be true as logically capable of falling under concepts and standing in relations, and so it is not applicable to Wittgensteinian correspondence, which takes as central what is also central for Frege, that the judgment that p is true adds nothing new to the judgment that p. Putting it another way: to know of A that he has judged correctly, we do not need to know that his judgment stands in some relation. If he has judged that p, what we need to know is simply p.

Frege says of the word “true” that its content is unfolded in the laws of logic, the laws governing recognition-as-true (1979, p. 3; cf. also 1984, p. 352). My account of Wittgenstein suggests that he could also allow us to speak of an unfolding of the meaning of “true.” Everything that can be the content of any description, everything that can be said, can also be something that is the case (see *Tractatus* 6.362). The general form of everything that can be sensibly said is the general form of what goes into content-slots like those in patterns 1, 2, and 3, and is also the general form of what goes into the slot in those patterns where we put what is the case, the facts, how things are. The general form of everything that can be sensibly said gives us the general form of operating with sentences comparing what we say to what is so, and reaching conclusions about what so and so said correctly, about the falsity of such and such proposition, and so on. To know how in general to operate with sensible propositions includes all operating with them in inferences, including those to someone’s having spoken truly, or judged correctly, or expected wrongly, inferences to such and such proposition’s being true, or false, and also inferences the other way, from statements that someone said that p, and that he was correct, to p. In the general form of proposition we have the general form of occurrences of propositions in all the slots where they go with “true”, “correct”, “incorrect”, and so on. So truth is thereby unfolded.³⁴

IX. Four Things in Conclusion

1. I hope to have provided a route into one mode of pre-Tarski thought about truth, a mode of thought that is implicitly highly critical of later developments. In this I take myself to have been going on from Sluga’s discussion (1999, this volume). An issue that should be mentioned here is that of compositionality. Wittgenstein’s story about truth gives a particular shape to the idea that propositions are composed of parts whose meaning we know. The implied notion of compositionality is shaped by the determination of what propositions are, as given by logic (including here the logical features that go with reversibility of sense). Wittgenstein has in the *Tractatus* what one might call a use account of compositionality, tied

to the story about sense, which is itself a use story. And this is importantly different from a post-Tarskian conception of compositionality.³⁵

2. In section VIII I took Frege's metaphor of an unfolding of the meaning of "true" and applied it to the *Tractatus*: we might speak of the meaning of "true" as being unfolded in the general form of proposition. This was meant to suggest some questions: Might we contrast Wittgenstein's earlier and later thought by asking what would be, from the point of view of his later thought, an unfolding of the meaning of "true"? And how, then, would such a conception itself constitute a criticism of the earlier view? Would it have a more adequate way of putting before us what it is for our talk to be responsible to the world? And could it do so without going back on the genuine logical insights of the earlier work?

3. I have not explicitly discussed general questions about the interpretation of the *Tractatus*. I intend this essay to be consistent with what Goldfarb (1997, p. 64) has described as a "resolute" reading of the *Tractatus*, and to bear on the questions he has raised about such a reading, but there are still fundamental and unanswered questions that Goldfarb has raised about Wittgenstein's "contrastive" view of meaning (1997, pp. 68–70). It is perhaps worth pointing out that the argument of my essay implies that the so-called 'picture-theory' of the *Tractatus* is not merely something that fails to fulfill some supposed narrow *Tractatus* criterion of sensefulness. The picture-theory is a story about the relation between propositions and reality; it dissolves from within, since we cannot attempt to grasp what it supposedly conveys without using the relational forms of our language, but what we are being shown is that, used in the way we try to use them, they are empty.

4. I have some comments on something that both Sluga and Ricketts have said. The comments depend on a final borrowing, from Juliet Floyd. Ricketts says that Wittgenstein shares with Russell a commitment to a correspondence conception of truth (1996b, p. 64); and Sluga describes Wittgenstein as arguing for a specific version of the correspondence conception, which suggests that we already have some general idea of what a correspondence conception would be, and that various specific versions would differ in the accounts they gave of the relation of correspondence. Thus Sluga sees Wittgenstein's version as making the Russellian account more precise, in that it replaces Russell's talk of correspondence with an account in terms of a picturing or mapping relation (Sluga, 1999, p. 33; see also Sluga, this volume). My immediate response to Ricketts and Sluga might perhaps be that Russell and Wittgenstein do not share a commitment to anything here. There is no conception of truth, that is, about which we could say Wittgenstein and Russell were both committed to it, truth as correspondence. But Juliet Floyd (1995) has shown us how interested Wittgenstein was, later on, in arguments that lead us to acknowledge something as what we had wanted to say, as what we had in some sense wanted all along, or as what we had really had in mind, even though we had never thought of it that way at all. Borrowing this idea from Floyd, we might put the relation between Russell and Wittgenstein this way: Russell wanted a correspondence theory of truth, and, if Wittgenstein's arguments had accomplished what he hoped they would, Russell might have recognized that accomplishment by saying, "Yes, Wittgenstein, the *Tractatus* gives me what I was really after with my talk of correspondence," although what he would thereby have accepted would certainly not have been the kind of thing he had earlier described as what he wanted, when he had laid down the conditions for an adequate ac-

count of truth (Russell 1912, chap. 12). To say that Russell and Wittgenstein *shared* a conception of truth would be misleading, unless we meant that Russell was, in a sense, intended by Wittgenstein to recognize the ultimately nonrelational story about truth as what he had wanted all along. This comment applies also to Sluga's idea that Wittgenstein made Russell's conception more precise. Russell's was a profoundly relational conception; Wittgenstein's was not. If, in the light of that point, we were to say that Wittgenstein made Russell's view more precise, we should ourselves be acknowledging Wittgenstein's view as offering to anyone in Russell's kind of unclarity something that he could acknowledge as a clarification of what he had wanted all along. If we make use of Floyd's discussion of "what I really wanted to say," and think of Wittgenstein and Russell as sharing a correspondence conception of truth, we are taking Wittgenstein to show what might be accepted as giving us what we wanted with correspondence. It is not given in advance what that might be.

Notes

I am indebted to the questions and comments of participants and audience at the conference "Frege, Russell, Wittgenstein: Shaping the Analytic Tradition," held at the University of California at Riverside in 1998; and at the colloquium "Origins of Analytic Philosophy," part of the 1997–98 Boston Colloquium for Philosophy of Science.

1. Quine, 1970, chap. 1. Quine and Geach reject for different reasons the 1950s Oxford conception of statements (made on this or that occasion by the use of sentences) as bearers of truth.

2. I am not suggesting that "Satz", in "Sense and Meaning," can be translated as "proposition"; in many contexts in that essay it has to be translated as "clause." But, in Black's translation, the use of the word "sentence" for "Satz", as opposed to "clause," often indicates that a Satz in that particular context is something that logic counts as a Satz, i.e., a proposition in Geach's sense. I am indebted to Thomas Ricketts for remarks about Frege's use of "Satz".

3. Frege elsewhere denies that occurrences of equiform signs can be spoken of, except inaccurately, as occurrences of the same sign (1903, §§99–100). But in "Sense and Meaning" his account of the replacing of a component Satz by another Satz with the same truth value, where the replacement involves changing the form of the latter, suggests that in informal exposition he is willing to engage in talk that has both the inaccuracy criticized in *Grundgesetze* (equiform Sätze treated as the same Satz) and the inaccuracy of treating nonequiform Sätze, one of which is a grammatical transformation of the other, as the same Satz. Cf. also Frege 1979, p. 261, on the grammatical transformations induced by German "wenn . . . so . . ."; also Geach 1979, p. 222.

4. See Ricketts, 1986a, 1986b; Goldfarb, 1997, pp. 60–61. My formulation of the point is based on, and in part quoted from, Ricketts, 1986b, pp. 85–86.

5. I retell the story using "proposition" where Prior uses "statement." This would create problems if he were using "statement" as it is used in some roughly contemporaneous writing, e.g., Strawson, 1952. But it can be shown that he is not doing so.

6. I believe that that example comes from Geach. A kind of antipsychologism is in play in the response to Prior which I have constructed, as there is in an argument of Quine's on a related question (Quine, 1970, p. 81). Quine rejects the idea that one might mean conjunction by some symbol, and be using it in accordance with deviant rules, say, the rules for disjunction. That suggestion is empty. The idea that "tonk" produces logical trouble rests on an idea closely related to that criticized by Quine, namely, the

idea that one can make a pair of rules *be* rules for a single logical connective by *meaning* them that way.

7. Max Black (1964) misses that point. Geach discusses and analyzes the confusion of Black's account; see Geach, 1982, p. 89.

8. See especially Ricketts, 1986b.

9. Sluga, 1999, p. 40; see also Sluga, this volume.

10. Anscombe, 1963, chap. 3; see also Diamond, "Inheriting from Frege" (Diamond, forthcoming).

11. In "Does Bismarck Have a Beetle in His Box?" (Diamond, 2000), I discuss another case in which Russell's taking us to want something that we cannot have is of great importance to Wittgenstein. See also Wittgenstein's comments on the idea of logical impediments to what we take ourselves really to want or need (1958, §426).

12. See, e.g., Wittgenstein, 1964, pp. 30–43; 1974, part VII; also the various records of Wittgenstein's lectures during the 1930s.

13. See Wittgenstein, 1961, p. 94. Where the use of quotation marks in published versions of the "Notes on Logic" differs from the use of quotation marks in similar material in the *Tractatus*, I follow the latter.

14. For Wittgenstein's 1914 talk of the reality that is compared with a proposition, see Wittgenstein, 1961, p. 22; cf. also the closely related passage at 1961, p. 37. Some years after her translation of Wittgenstein 1961, Anscombe changed her view about how, in translating Wittgenstein's early writings, one should treat occurrences of "*Wirklichkeit*" preceded by the definite article. I follow her later practice, which is foreshadowed in her version of the notebook entry for 23.12.14 (1961, p. 37), in the translation of "*sie*" as "*this reality*." For Wittgenstein's ways of using "Bedeutung" in connection with propositions in 1914, see Wittgenstein, 1961, pp. 17–24. *Tractatus* 4.126, with its talk of various sorts of symbols the Bedeutungen of which fall under this or that formal concept, continues to allow for the idea of propositions as having a Bedeutung, but this usage no longer plays a role in Wittgenstein's exposition of his ideas about propositional meaning.

15. Ricketts, 1996b, pp. 89–90. Cf. also Goldfarb, 1997, pp. 65–66, arguing that the inconsistency between the *Tractatus* talk of possible states of affairs and the conception of fact in the book is intentional, and connecting Wittgenstein's intention with the point at the end of the *Tractatus* about the "overcoming" of the remarks in the book (1963, 6.54). Cf. also Frege's idea that one can overcome a view by following it through with utter seriousness (Frege, 1903, §137, which I discuss in Diamond, forthcoming).

16. The arguments of Ricketts and Goldfarb do not provide a mere parallel to an argument about truth; for the issue with which they are concerned is inseparable from that of truth. Their treatment of the *Tractatus* discussion of possibility provides a strong objection to some ways of understanding the *Tractatus* on truth. For example, James Levine's account (1996) of the *Tractatus* on truth and sense depends on the ascription to Wittgenstein of a substantial metaphysical conception of possibility, and is thus incompatible with recognition that the *Tractatus* talk of possibility is genuinely undercut by the *Tractatus* itself.

17. I should note that, although I borrow from Geach the metaphor of reversibility which he picks up from Wittgenstein, I do not use the metaphor as he does, and he does not use it as Wittgenstein does. That a proposition and its negation 'point in opposite directions', and that the signs involved could be given the reverse use, can metaphorically apply to various conceptions. Geach's account differs most sharply from Wittgenstein's in not appealing to the notion of facts.

18. See *Tractatus* 4.062; cf. Wittgenstein, 1961, p. 95.

19. See Geach, 1982, p. 90. The logical issues here are complex; Wittgenstein returned to them in the 1930s, when he discussed whether names can be negated. Cf. also G. E. M. Anscombe's discussion of F. P. Ramsey on negating names (Anscombe, 1963, chap. 7).

20. I go into some of the issues here, focusing on Wittgenstein's criticism of Frege at *Tractatus* 5.02, in Diamond, forthcoming. There has recently been published another alternative to Anscombe's account; see Proops, 1997. While Proops's account and mine differ in some respects, they agree in taking a central problem in Frege's view, as Wittgenstein saw it, to be that the view does not allow an unasserted proposition to contain what Wittgenstein would recognize as a genuine verb.

21. See Biggs, 1996 on the history of the "Notes on Logic."

22. There is an early and influential statement of this view in Anscombe, 1963, p. 79. Among other commentators taking this view are Peter Winch (1996, p. 8) and P. M. S. Hacker (1972, p. 37).

23. An element of the parallel which there is not space to discuss can be seen if we note a point emphasized by Anscombe (1963, pp. 53, 59, 108), that Wittgenstein rejects views of logic which make it appear a remarkable logical fact that, e.g., for any proposition there is another which is true when the first is false, and vice versa. (She is developing an idea expressed in the *Tractatus* at 6.111.) Russell's pre-1913 view of relations has exactly such a remarkable logical fact at its core: he is committed to the view that, for any two-term relation, there is another relation that holds, but in the opposite direction, between every pair of individuals that stand in the first relation; see Russell, 1903, §94.

24. Throughout my explanation I follow Wittgenstein's use of "extract from," so that we can speak of a sentence form in use as "extracting" a relation from other relations. This way of speaking is misleading; its grammar inevitably carries the suggestion that relations are things, although very different sorts of things from objects. That suggestion is intended to be canceled; for discussion, see Ricketts, 1996b, pp. 71–72.

25. See Wittgenstein, 1961, p. 106, for the distinction between *constituents* and *components*.

26. In Ricketts's discussion of these issues (1996b, p. 72), he emphasizes the contrast between Wittgenstein's view and the Russellian view of the relation between relational expressions and relations, which makes such expressions analogous to proper names of individuals. It was his view of relational expressions and verbs as standing for entities that had led Russell into difficulties that he could not resolve (1903, §§52–55); and it is that view that Wittgenstein rejects.

27. Cf. also Russell's problem in *Principles of Mathematics* (1903, §52). Russell believes that anything can be a logical subject, but also that the attempt to treat an asserted proposition as a logical subject is self-defeating. The difficulties here are, as he recognizes, intimately connected with the nature of truth and falsity.

28. My remarks in this paragraph are not meant to suggest that talk about projective relations or logical relations cannot be useful, but rather that we may misunderstand its role. Questions parallel to those in this paragraph can be asked about my talk of the coordination of the relational structure of a proposition with a relation. I shall not explicitly consider such questions, the resolution of which would follow the same lines as the resolution of the questions about propositions and their 'relation' to reality.

29. The *O.E.D.* gives as an example of this use Scott's "But I doubt, I doubt, I have been beguiled."

30. On the account I am giving, there is no difference between "A doubt_{Scots} that p" and "A doubts_{English} that not-p".

31. Wittgenstein made similar points in a lecture in December 1934. In sentences like "Smith understands that p" the expression "Smith understands that" does not give us a proposition different from "p" by having a new constituent or a new arrangement of constituents. Like the role of "¬" in "¬p", "Smith understands that" changes the use of the proposition without changing the constituents or their arrangement. This is clearest in Margaret Macdonald's unpublished notes; but see also Ambrose, 1979, pp. 110–13.

32. There is a discussion of these issues in terms of the difference between Dummett's and Prior's types of approach to the duality of propositions in oblique contexts, in Geach's

"Truth and God" (1982, pp. 91–93). My account is a version of Prior's type of approach, but see also Geach on the problems with Prior's own version.

33. An important part of what is involved in the claim that a proposition S saying that someone says or believes that p is of the form "p' says that p" is that the proposition "p" appears in the written-out form of S only within "p' says that p'. Take the case of "It says in Larousse that Wilhelm succeeded Friedrich Wilhelm". We need rules of translation here, treating "Wilhelm" as the translation of "Guillaume", "Friedrich Wilhelm" as the translation of "Frédéric-Guillaume," and connecting those sentence forms in which one name stands to the left and a second to the right of "succeeds" with forms in which one name stands to the left and a second to the right of "succéda à". Those rules, together with "That 'Wilhelm' stands to the left and 'Friedrich Wilhelm' to the right of 'succeeds' says that Wilhelm succeeded Friedrich Wilhelm" enable us to derive a description of an arrangement of "Guillaume" and "Frédéric-Guillaume", the arrangement that we need to find in Larousse if the original claim about what it says in Larousse is true. I am here using (besides *Tractatus* 4.025) *Tractatus* 4.0141: the law of projection that enables us to read a musical score enables us to derive the score from the music. Similarly, then, the law of projection that enables us to read a propositional sign as saying such and such can be used to derive a description of a sign-fact from a statement of what it says. In 4.0141, Wittgenstein considers sequences of projective relations, in which we can read backward or forward from score to sounds to grooves on a record, which correspond to the moves backward or forward from French sign-fact to English sign-fact to what is said or what is true. Thus, also, if we know what the French sign-fact is, we can, using the rules given above, derive a description of what must be the case in the Hohenzollern succession for "What it says in Larousse is correct" to be true. As Ricketts notes in a discussion of *Tractatus* 5.542 (unpublished), the general rules for reading sign-facts, the rules that impose interpretations on them, are not themselves genuine propositions with true-false poles.

34. I cannot here discuss the difference between my account and F. P. Ramsey's reading of Wittgenstein on truth (1965, pp. 274–75). My use of Frege's metaphor of "unfolding" is meant to indicate the kind of difference I should want to bring out.

35. It is also different from Fregean and Russellian conceptions of compositionality. On these issues, see Hylton, 1997, esp. pp. 102–5.

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PART IV

EARLY WITTGENSTEIN

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The *Tractatus* on Inference and Entailment

Ian Proops

The aim of this essay is to investigate Wittgenstein's well-known, yet obscure, objection to the views of Frege and Russell on deductive inference and its justification. The objection, such as it is, is set out at *Tractatus* 5.132 (Wittgenstein, 1989):¹

If p follows from q , I can make an inference from q to p ; deduce p from q .
The method of inference is to be gathered from the two propositions alone.

Only they themselves can justify the inference.

"Laws of inference"² [*Schlussgesetze*], which—as in Frege and Russell—are to justify the inferences, are senseless [*sinnlos*] and would be superfluous. (my translation)

The passage raises two immediate questions. First, what does Wittgenstein mean by "laws of inference"? And, second, why would Frege and Russell have thought of these laws as involved in the justification of inference? I will take these questions one at a time.

I. Laws of Inference

The majority view among scholars who have addressed the first question is that when speaking of "laws of inference" Wittgenstein means to be referring to the inference *rules* of a formal system.³ But whatever the merits of this suggestion, it faces the immediate difficulty that neither Frege nor Russell understood the term "laws of inference" so narrowly as to refer *only* to inference rules. In considering whether the basic laws of logic ought to be called "axioms", Frege says:

Traditionally, what is called an axiom is a thought whose truth is certain without, however, being provable by a chain of logical inferences. The *laws of logic*, too, are of this nature. Some people may nevertheless be inclined to refrain from ascribing the name “axiom” to these *general laws of all inference* [*allgemeinen Gesetze alles Schließens*], but rather wish to reserve it for the basic laws of a more restricted field. (Frege, 1984b, pp. 273–84, my translation and emphasis)

Here, clearly, Frege is treating the expression “general laws of all inference” as a variant upon “laws of logic”.

That Russell took “laws of inference” to encompass more than inference rules is suggested by a passage in “Necessity and Possibility,” a paper he gave to the Oxford Philosophical Society in 1905. He writes: “There are certain general propositions, which we may enumerate as the *laws of deduction*: such are “if not-p is false, then p is true”, “if p implies not-q, then q implies not-p” [and] “if p implies q and q implies r, then p implies r”; in all we need about ten such principles [...]” (Russell, 1994, p. 515, my emphasis).⁴ Because Russell says in a footnote to §13 of the *Principles* that “[he does] not distinguish between inference and deduction,” it seems reasonable to suppose that the expression “laws of *deduction*” figures in this passage merely as a stylistic variant upon “laws of *inference*.” But if so, then Russell, too, would appear to be taking “laws of inference” to include logical laws.

(Notice that in both of these passages when Frege and Russell speak of “the laws of logic” or “the laws of deduction,” they have in mind specifically *primitive* or *basic* logical laws. Since at other times they use these expressions to refer to the primitive laws *and* theorems of their systems, we shall need to remain alert to this ambiguity.)

II. The Justification of Inference

Assuming that Wittgenstein is using “laws of inference” to refer to what Frege and Russell mean by this term, our second question becomes: How exactly, in Wittgenstein’s view, do Russell and Frege see logical laws (and possibly inference rules)⁵ as involved in the justification of inference? And what does Wittgenstein find objectionable in this idea? I shall offer my own answers to these questions shortly, but first I want to examine a rival account that has been offered by Thomas Ricketts in his paper “Frege, the *Tractatus*, and the Logocentric Predicament.”⁶ Ricketts takes Wittgenstein’s criticism to be meant to apply in one way to Russell and in quite another to Frege. Each suggestion has considerable initial appeal, but I will argue that as exegesis neither is ultimately successful.

Against Russell, the criticism is supposed to focus on (what Ricketts alleges to be) Russell’s failure adequately to distinguish inference rules from logical laws.⁷ As Ricketts sees the matter, this confusion is bound up with a mistaken view of the way in which (what we would recognize as) inference rules serve to license the inferences they govern. According to Ricketts, Russell sees inference rules—which for Ricketts’s Russell are not to be distinguished from the laws of logic—as conferring their license only by serving as *premises* in arguments.⁸ Accordingly,

Ricketts presents Wittgenstein as seeking to expose the mistake in such a view by means of a (tacit) appeal to Lewis Carroll's well-known regress argument.⁹

To illustrate: suppose *modus ponens* were taken to function as a premise. Then the argument from “p” and “if p then q,” to the conclusion “q” would actually take the following form:

p
 $p \supset q$
 $(p \cdot (p \supset q)) \supset q$
 Therefore, q

Here *modus ponens* is supposed to have been formulated as a proposition rather than a rule. But in this role it will be powerless to license the inference to the conclusion q, since the argument is now of a new form, one not covered by *modus ponens*. Nor may we remedy the situation by appealing to some new inference rule, because its incorporation as a premise will again alter the overall shape of the argument.

Ricketts concedes that this story cannot be the whole explanation of Wittgenstein's critique of “laws of inference”, since there is no reason to suppose that Frege confused inference rules with axioms. He therefore offers a separate account of how the criticism of 5.132 is supposed to apply to Frege. I will consider this account in a moment, but first I want to raise some doubts about the way the criticism is supposed to apply to Russell.

My main concern is that Ricketts's interpretation portrays Wittgenstein as guilty of an uninterestingly uncharitable reading of Russell. For Russell was in fact perfectly clear about the distinction between inference rules and logical laws. Indeed, as we shall see, he was clear about this distinction precisely because he saw that a failure to respect it would lead directly to Carroll's regress.

In the course of an early proof in *Principia Mathematica*, Russell and Whitehead remark: “The principle of deduction gives the general rule according to which the inference is made, but is not itself a premise in the inference. If we treated it as a premise, we should need either it or some other general rule to enable us to infer the desired conclusion, and thus we should gradually acquire an increasing accumulation of premises without ever being able to make any inference” (Russell and Whitehead, 1990, part 1, p. 106). And in the next paragraph, having mentioned that principles of deduction may also function as premises in an inference—that is, when they no longer play their inference-licensing role—they continue: “This distinction between the two uses of principles of deduction is of some philosophical importance, and in the above proofs we have indicated it by putting the rule of inference in square brackets” (106). So for Russell and Whitehead the distinction between the use of a principle of deduction as a premise and its use as a rule is important enough to be embodied in the very symbolism of *Principia*.¹⁰

Making the same point in the earlier *Principles of Mathematics*, Russell says: “We may observe [...] that in a particular inference, the rule according to which the inference proceeds is not required as a premise” (Russell, 1903, part 1, chap. 3, section 45). And he goes on to argue that if we were to require the rule to function as a premise—if we were to do what is “formally necessary”¹¹—we would be faced with a Carroll-style regress. Russell concludes that the rule according to which

the inference proceeds should be seen as “a respect in which the formalism breaks down,” and he recommends that such rules be accorded the status of informal principles that are employed in making inferences but not recorded as lines in proofs.¹²

Ricketts does not say why he takes Russell to have confused inference rules with logical laws, but I suspect that his reading is based on at least one of the following considerations. First, in section 18 of the *Principles*, Russell applies the term “axiom” to inference rules and logical laws indiscriminately.¹³ This might seem to suggest that he is insensitive to the differences between them. It ought not to mislead us, however, for even while speaking this way, Russell is at pains to emphasize the peculiar status of inference rules among axioms. Thus in section 18, while he lists several axioms in the form of statements containing implicitly quantified variables (for example, (6): “If p implies q and q implies r , then p implies r ”), he formulates the one “axiom” that we would call an inference rule very differently: “(4) A true hypothesis in an implication may be dropped, and the consequent asserted. This is a principle incapable of formal symbolic statement, and illustrating the essential limitations of symbolism.” The rule is recorded as a *permission*, rather than as a statement of fact, and its anomalous status is clearly flagged. There may be room for debate about what it means to say that the principle is “incapable of formal symbolic statement,” but a comparison with *Principles* §45 strongly suggests that it means that such rules cannot be recorded as premises in the deductions in which they are used. It seems likely, therefore, that by calling *modus ponens* an indemonstrable “axiom”, Russell merely wishes to emphasize that it forms part of the fundamental basis of mathematics, not that it is to be construed as functioning as a *premise* in arguments.

What I think is most likely to be driving Ricketts’s interpretation, however, is Russell’s speaking in *Principia* of one and the same “principle of deduction” as having two kinds of *use*: a “use for implication” and a “use for inference” (Russell and Whitehead, 1990, p. 132). But, so long as one is clear—as Russell was—that a principle can be “used for inference” without functioning as a premise in that very inference, this way of conceiving the matter will not lead to a Carroll-style regress.¹⁴

Now, since the relevant passages occur quite early in the *Principles* and in *Principia*, and because we know Wittgenstein to have read both works, it seems plausible that he would have known of Russell’s awareness of the need for a distinction between inference rules and logical laws. Certainly, he *must* have seen the passage quoted from *Principles* section 18, since the criticism of 5.5351, which focuses on the *Principles*’ (sporadic) practice of prefixing “ $p \supset p$ ” as an antecedent to logical laws, relates directly to this section. So it seems doubtful that this is the criticism Wittgenstein has in mind. Of course, it is always possible that Wittgenstein might have simply misread, or skimmed over, the relevant passages—or that, having read them, he should have forgotten about them—but if that were so, his criticism would lose much of its interest. If we want to find an interpretation of 5.132 that presents Wittgenstein’s objections as having real bite against a position that some philosopher actually held, we will need to seek its target elsewhere.

This brings us to Ricketts’s reconstruction of Wittgenstein’s criticism of Frege. The reading begins from the observation that for Frege reasoning from certain

nonlogical propositions to others may need to be¹⁵ mediated by laws of logic, if we are to make fully explicit the grounds on which the conclusion rests. Consider, for example, the following argument:

- (i) All whales are mammals;
- (ii) All mammals are vertebrates;
- Therefore,
- (iii) All whales are vertebrates.¹⁶

According to Ricketts's Frege, (i) and (ii) do not exhaust the grounds for the assertion of (iii). Rather, (iii) rests additionally upon certain laws of logic. So in order to render the grounds of our inference fully explicit, we shall first need to prove the conclusion (iii) by deriving the following logical law from basic laws of logic:

- (iv) $(\forall F)(\forall G)(\forall H)[((\forall x)(Fx \supset Gx) \supset ((\forall x)(Gx \supset Hx) \supset (\forall x)(Fx \supset Hx))]$.

We then proceed to instantiate the second-order variables in (iv), to obtain:

- (v) $(\forall x)(x \text{ is a whale} \supset x \text{ is a mammal}) \supset ((\forall x)(x \text{ is a mammal} \supset x \text{ is a vertebrate}) \supset (\forall x)(x \text{ is a whale} \supset x \text{ is a vertebrate}))$.

The conclusion, (iii), then follows from (i), (ii), and (v) by two applications of *modus ponens*.

Ricketts takes 5.132 to be opposing the idea that logical laws are ever needed to mediate inferences in this way. His view certainly provides a plausible sense in which logical laws might be thought to "justify" inferences; for on this account they do so because they form part of the basis for the assertion of the conclusion. It is far from clear, however, that this could be the notion of "justification" Wittgenstein is focusing on at 5.132.

To see this point it is important to appreciate that on Ricketts's reading the dispute between Wittgenstein and Frege is, at bottom, a disagreement over the true *grounds* for the assertion of the conclusion. For Frege, but not for Wittgenstein, our knowledge of logical laws plays a role in mediating inferences between statements within a particular special science. This means that logical laws will have to be cited in giving a full inventory of the grounds on which a particular inference rests. So if Ricketts is right, we should expect Wittgenstein, in correcting Frege, to deny that laws of logic are among the implicit grounds of the inference, and to insist that the true grounds for the judgment of (iii) are just what they seem to be, namely, (i) and (ii). That is, we should expect him to say that it is the truth of these propositions, and of these alone, that provides the justification for the inference.

What Wittgenstein actually says, however, is that laws of inference are superfluous because the justification is afforded by premise *and conclusion* alone. So whatever *Wittgenstein* means by the "justification of an inference", it must be something in which the *conclusion* itself can figure. But since the conclusion will not figure among the grounds of any inference worth making, the dispute about "justification" cannot, after all, be one about what ought to be included in a full articulation of an inference's grounds. So although Ricketts certainly identifies a sense in which logical laws might be held to be involved in the justification of inferences, it cannot be the sense that is relevant to the interpretation of 5.132.

III. Wittgenstein's True Target

To locate the true target of Wittgenstein's criticisms, I suggest we begin by considering his views on what *does* justify inference, for this will give us a sharper sense of the notion of "justification" that is in question at 5.132. Before we address this issue, however, we should consider Wittgenstein's use of the term "inference." As is widely known, Frege and Russell hold that it is impossible to draw an inference from a false proposition.¹⁷ Wittgenstein disagrees: "One can draw *inferences* from a false proposition" (4.023). He is able to maintain this view because he does not share Frege's conception of inference as the *recognition* of the truth of one proposition on the basis of the recognition of the truth of another. For Wittgenstein, inference is something more akin to the assertion of one proposition's truth on the basis of the *supposition* of the truth of another.

Consequently, for Wittgenstein what justifies my inferring q from p, in *his* sense of "inferring", is the fact that q follows from p. And what *this* consists in is nothing more than the obtaining of an internal relation between the forms of p and q. He says: "If the truth of one proposition follows from the truth of others, this expresses itself in relations in which the forms of these propositions stand to one another, and we do not need to put them in these relations first by connecting them with one another in a proposition; for these relations are internal, and exist as soon as, and by the very fact that, the propositions exist" (5.131). The remark suggests that what Wittgenstein means to be challenging at 5.132 is a view about what it is for two propositions to stand in the relation of logical entailment.¹⁸ Wittgenstein takes logical entailment to consist in an "internal" relation between the "forms" of propositions, while his opponent takes it to consist in something more—something Wittgenstein characterizes as "[the putting of the propositions] in these relations [...] by connecting them with one another in a proposition." This remark is darkly metaphorical, and much of this essay will be taken up with illuminating it. The point of immediate importance, however, is that this view of what is at issue between Wittgenstein and his opponents fits well with the positive suggestion of 5.132, namely, that what justifies a particular deductive inference is *nothing beyond the premise(s) and conclusion themselves*. For since the relation between the forms is an *internal* relation, its holding consists in nothing more than the propositions' *being the very propositions that they are*.¹⁹ (I shall return to this point later.)

But if, as I am suggesting, the issue joined at 5.132 concerns the nature of the relation of logical entailment, then why is it presented as a dispute about what is involved in *justifying* an inference? The answer, I believe, is that the notion of "inference" in play at this point in the text is *Wittgenstein's*. This means that the question what justifies the inference from p to q is equivalent to the question what would justify us in asserting that q on the basis of the supposition that p. And *this* question is answered by citing whatever fact one takes the relation of logical entailment to consist in.

If these thoughts are on the right lines, then Wittgenstein would seem to be criticizing Frege and Russell for having given accounts of logical entailment that somehow involve an essential appeal to logical laws (and possibly to inference rules). But do Frege and Russell offer such accounts? I shall argue that in the case of Russell the answer is a clear yes, and in the case of Frege, a more qualified one.

Russell offers no explicit account of logical entailment in the *Principles*, but when a little later on he is forced to think about how to characterize this relation he develops precisely the kind of account we might have expected, given the conceptual resources available to him in this early work. The account appears in the 1905 paper “Necessity and Possibility,” from which I quoted earlier. In this paper Russell is attempting to put flesh on the bones of the deflationary account of necessity given by G. E. Moore in his 1900 essay “Necessity.”²⁰ Russell explains Moore’s view thus: “According to this theory a proposition is more or less necessary in proportion as there are more or fewer other propositions to which it is logically prior, p is logically prior to q if q implies p but p does not imply q ” (Russell, 1994, p. 513). In other words, a proposition is more necessary in proportion as there are more propositions which imply it but which it does not imply. So, for instance, “Someone is clever” is more necessary than “Russell is clever”, since anything “Someone is clever” implies is already implied by “Russell is clever”, but not conversely.

Russell recognizes that this account will have to employ a notion of implication other than material implication. A false proposition materially implies all propositions, and a true proposition is materially implied by every proposition; so using material implication in the account will dictate that all truths are on one level of necessity, and all falsehoods on another.

Because of this problem Russell is prompted to explore the concept of logical entailment, which he terms “deducibility”—and on one occasion “logical deducibility” (Russell, 1994, p. 513). He views this as an ordinary concept in need of precise definition. He views his task as one of demarcating a class of material conditionals—“implications,” as he calls them—in which the consequent is intuitively felt to be a necessary consequence of the antecedent. The relevant cases are those in which the truth of the material conditional, “ p implies q ”, is known independently of the falsity of p and the truth of q . He says: “In the practice of inference, it is plain that something more than implication must be concerned. The reason that proofs are used at all is that we can sometimes perceive that q follows from p , when we should not otherwise know that q is true; while in other cases; ‘ p implies q ’ is only to be inferred either from the falsehood of p or from the truth of q [...]. What we require is a logical distinction between these two cases” (p. 515). Russell is pointing to the need to distinguish the fact that p materially implies q from the fact that q is logically entailed by or, in his terminology, “deducible from,” p . He defines the latter notion as follows: “We may then say that q is *deducible* from p if it can be shown *by means of the above principles* [i.e., the axioms and inference rules of Russell’s system] that p implies q ” (p. 515). So q is deducible from p just in case “ $p \supset q$ ” is derivable in Russell’s system.

Russell observes that the definition captures the extension of the concept he is seeking to define: “This meaning of *deducible* is purely logical, and covers, I think, exactly the cases in which, in practice, we can deduce a proposition q from a proposition p without assuming either that p is false or that q is true” (p. 515). Finally, having defined the notion of “deducibility,” Russell uses it to explain the intuitive notion of a valid argument. He writes: “It is noteworthy that, in all actual valid deduction, whether or not the material is of a purely logical nature, the relation of premise to conclusion, in virtue of which we make the deduction, is one of those contemplated by the laws of logic²¹ or deducible from them” (p. 517).

Notice that Russell regards the relevant material implication as *licensing* the inference. He calls it the relation “in virtue of which” we make the deduction. As long as the implication is one that holds for reasons other than the falsity of the antecedent or the truth of the consequent, it will license inferences from antecedent to consequent. Such cases are those in which the obtaining of the implication is grounded in the laws of inference. In such cases the implication obtains either because it is “contemplated” by the (primitive) laws of logic (i.e., is an instance of one of the axioms) or because it can be derived from these laws using the inference rules of Russell’s system.²²

Although the 1905 paper containing Russell’s account of deducibility is a relatively obscure source, the view it contains is suggested by passages in Russell’s more public writings.²³ At the beginning of part 1 of *Principia*, in a section entitled “The Theory of Deduction,” Russell characterizes the subject treated in *Principia* as “the theory of how one proposition can be inferred from another” (Russell and Whitehead, 1990, section A, p. 90). He says that “in order that one proposition may be inferred from another it is necessary that the two should have that relation which makes one the consequence of the other.” So, given this characterization of the subject treated in *Principia*, it seems plausible that Russell takes the theorems of his system to report the obtaining of the *logical entailment* relation. In *Principia* Russell misleadingly calls this relation “implication,” but in view of the clarity shown in his 1905 paper about the distinction between implication and deducibility, it seems likely that what he has in mind here is the restriction of the implication relation to those implications that can be shown to hold on the basis of logic alone. Such implications are, after all, just the theorems of the “theory of deduction.” If this is correct, then Russell’s view would seem to be that the relation that justifies inference is just the relation of logically grounded implication. (I shall call an implication “logically grounded” just in case it is derivable as a theorem of Russell’s system.)

These later remarks of Russell suggest that he is no longer thinking of the definition given in the 1905 paper as merely capturing the extension of the logical entailment relation. Rather, he now seems to be regarding the relation of logical entailment as *consisting in* a special kind of implication. In this connection it is worth noting that after 1905, Russell no longer treats the relation that obtains between premise and conclusion in a sound and valid argument as a primitive relation—the so-called *therefore* relation of the *Principles* (Russell, 1903, §§15, 38). I would conjecture that this is because Russell has by this point come to regard the 1905 notion of logically grounded implication as providing for a deeper analysis than the 1903 view. (By 1905 the *therefore* relation need no longer be treated as a primitive, since it turns out to be analyzable in terms of the notions of logically grounded implication and truth.)

Russell says that the premises of the theory of deduction must contain “as many of the properties of implication as are necessary to *legitimate* the ordinary procedure of deduction” (Russell and Whitehead, 1990, section A, p. 90, my emphasis). So, assuming, as I have done, that Russell really means to be speaking here of properties of the logical *entailment* relation, we get a clear statement of the view that I have suggested Wittgenstein means to be opposing. For Russell, the laws of inference (understood as the *primitive* logical laws and inference rules of his system) justify inferences because they logically ground the relation of material im-

plication that obtains between premise and conclusion in a valid argument. Strictly speaking, it is the *fact* that this relation is logically grounded that justifies the inference, but because (primitive) laws of inference provide the grounding in question, they can be said to justify inferences in a derivative sense. By contrast, Wittgenstein will say that the “ordinary procedure of deduction” is justified, not by the relation of implication between premise and conclusion being one that is counted logically grounded by Russell’s “theory of deduction”, but by an internal relation between the forms of the propositions involved.

Because the envisaged grounding is grounding by the laws of Russell’s system, Russell’s account of deducibility turns out to be a system-relative one. What it is for p to entail q is for ‘ $p \supset q$ ’ to be derivable from the axioms and inference rules of *Russell’s system*. I suspect, however, that Wittgenstein would not have regarded this feature of Russell’s view as irredeemably problematic. For he appears to have regarded Russell as having a non-system-relative conception of a primitive proposition of logic (or logically primitive inference rule), namely, as one having the highest degree of psychological self-evidence.²⁴ Reading him in this way, Wittgenstein may have charitably reconstructed Russell’s view so that logically grounded implications were understood to be just those that are derivable from intrinsically *primitive* logical laws and inference rules.

A Fregean view that might be the target of Wittgenstein’s criticism is suggested in a passage from “Foundations of Geometry: Second Series,” where Frege is discussing, not logical entailment exactly, but the related notion of the “dependence” of one thought on a group of others.²⁵ He writes:

Let Ω be a group of true thoughts. Let a thought G follow from one or several of the thoughts of this group by means of a logical inference such that apart from the laws of logic, no proposition not belonging to Ω is used. Let us now form a new group of thoughts by adding the thought G to the group Ω . Call what we have just performed a logical step. Now if through a sequence of such steps, where every step takes the result of the preceding one as its basis, we can reach a group of thoughts that contains the thought A, then we call A dependent upon group Ω . If this is not possible, then we call A independent of Ω . (Frege, 1984c, pp. 423–24)

Frege offers a precise characterization of the dependence of a thought on a group of true thoughts, Ω , in terms of the yet-to-be-clarified notions of “logical inference” and “law of logic.” The notion he defines corresponds closely to the idea of provability in an axiom system with sound inference rules, where the axioms are the thoughts in Ω together with the laws of logic. Frege offers what is in effect a (proto-) “proof-theoretic”²⁶ account of what it is for one *true* thought to “follow from” another—an account, that is, not of the relation of logical entailment, but of the restriction of that relation to the class of true thoughts. As Frege admits, this is only a *sketch*: further work will need to be done to clarify the notions of “logical inference” and “law of logic” (p. 429).

The need for further work could be avoided if we were to *stipulate* certain inferences and laws to be primitively logical, but then we would have defined only a notion of dependence-relative-to-a-stipulation.²⁷ On the other hand, there is reason to believe that Frege might have entertained the hope of working toward a nonrelativized, or “absolute”, conception of *dependence*. For, as Tyler Burge and

Robin Jeshion have recently (independently) argued, there is good reason to think that Frege believed in the existence of laws of logic and rules of inference that were by their very nature *primitive*.²⁸

Briefly, Burge and Jeshion each claims that, quite apart from any psychological notion of “self-evidence”, Frege employed a nonpsychological notion that had to do with a proposition’s place in a hierarchy of truths, structured by objective relations of justification. A truth is “self-evident” in this nonpsychological sense if—in Jeshion’s phrase—it is “self-supporting.” That is to say, if by its very nature it derives its justification from no other truths, but contains its own evidence or support within it. When this is so, the truth in question is in an *absolute* sense “unprovable”, but in virtue of its essentially self-grounding character it may be capable of grounding other truths.²⁹ According to Jeshion, a thought’s being psychologically obvious is good evidence that it is self-supporting, but self-supportingness is not constituted by psychological obviousness. And, most important for our purposes, a truth occupies its position in the objective hierarchy quite independently of our decision to choose it as an axiom.³⁰

If something like this story is correct—and Burge and Jeshion have made it very plausible that it may be,³¹ then Frege would, after all, have had at his disposal a conception of intrinsically primitive logical laws and principles of inference.³² Thus equipped, he could have aspired to frame a notion of the *objective* dependence of one truth upon another. The idea would be to say that a thought q objectively depends on another p, if q can be derived from p together with primitive logical laws, using only primitively logical rules of inference.

It is debatable how optimistic Frege would have been about the prospects of clarifying the notions of a primitively logical law and a primitively logical inference,³³ but for our purposes what matters is that Wittgenstein might well have taken Frege to have been fully committed to such a project. He certainly seems to have anticipated Burge and Jeshion in taking seriously the idea that Frege was committed to a category of *essentially basic* logical laws. For such a view is the target of 6.127, which runs: “All propositions of logic are of equal status: it is not the case that some of them are essentially basic laws [*wesentlich Grundgesetze*] and others derived propositions” (my translation).³⁴ Wittgenstein’s reference to *Grundgesetze* strongly suggests Frege as the target of this remark; so whether or not Wittgenstein was right to think that Frege took the degree of (psychological) self-evidence—that is, obviousness—as the³⁵ criterion of the primitively logical (6.1271),³⁶ he does seem to have ascribed to Frege a conception of the basic laws of logic as having a special status—as *essentially* basic.

The important point for our purposes, then, is that as Wittgenstein viewed him, Frege would have been able to say that a true thought q is dependent (in an absolute sense) on another p just in case a proof of q from p can be constructed, each line of which is a member of Ω' , where Ω' is the closure under the “primitive logical consequence” relation of the union of {p} with the set of the primitive logical laws. Wittgenstein could then have seen such an account as easily amendable to yield an account of entailment: one simply understands the notion of proof in terms of Wittgenstein’s notion of inference instead of Frege’s. (That is to say, one does not require that p be true.) The resulting account would portray the laws of inference (both logical laws and inference rules) as essential to the justification of inferences. For it would present the question whether an inference is justified as

turning upon the possibility of linking together the thoughts expressed by the premise and conclusion by a chain of thoughts constructed in the way described above.

Whether Wittgenstein would have known of these ideas of Frege is, admittedly, unclear. We know that he visited Frege and had philosophical discussions with him in the summer of 1911, toward the end of 1912, and shortly before Christmas 1913; so the issue could have arisen in conversation.³⁷ We also know that “Foundations of Geometry: Second Series” was published in the *Jahresbericht der Deutschen Mathematiker-Vereinigung* in 1906, but it is not one of the works that Wittgenstein discusses, or mentions having read.

On the other hand, we do have reason to think that by the time of writing the *Tractatus* Wittgenstein had read Frege’s *Grundlagen*, and this work contains at least one passage that gestures toward the kind of account we have seen outlined above. In the course of an argument against the adequacy of inductively established arithmetical laws—an argument whose details need not concern us—Frege observes: “Instead of linking our chain of deductions direct to any matter of fact, we can leave the fact on one side, while adopting its content in the form of a condition. By substituting in this way conditions for facts throughout the whole of a train of reasoning, we shall finally reduce it to a form in which a certain result is made dependent on a certain series of conditions” (Frege, 1950, §17).³⁸ So when we have a derivation of $p \supset q$ from basic logical laws, we have a case where the chain of deductions beginning with p and ending with q has been “reduced to the form in which” q is *dependent* on p . But since Frege is here in effect (tacitly) assuming one direction of the deduction theorem: (viz., if $p \vdash q$, then $\vdash p \supset q$), this means that for him q is dependent on p whenever q is derivable from p . Moreover, if we regard Frege here as envisaging the process of “substituting conditions for facts” and so on to be reversible (and so as in effect tacitly committed to both directions of the deduction theorem), we shall have a case of the notion of dependence being cashed in terms of derivability: q is dependent on p iff $\vdash p \supset q$, that is, iff $p \vdash q$. So the kind of “proof-theoretic”³⁹ account of dependence we have examined above is at least hinted at as early as *Grundlagen*—a work that the Wittgenstein of the *Tractatus* almost certainly read.⁴⁰

IV. Wittgenstein’s Criticisms

I find it plausible that Wittgenstein’s criticisms at 5.132 are directed against something like the view of the logical entailment relation suggested by these writings of Frege and Russell. For, as I have already argued, they are directed against a certain view of the relation that must obtain between two propositions, p and q , if we are legitimately to infer q from p in Wittgenstein’s sense of “infer”, and these views of Frege and Russell suggest one account of what this relation might be.

Wittgenstein’s main objections to this view are stated at 5.132. First and most fundamentally, in the hands of Frege and Russell this account of logical entailment rests on a faulty conception of the laws of logic as fact-presenting statements. For Wittgenstein, there are no *laws* of logic, and the so-called logical truths, which Frege and Russell regard as instances of these laws, are for him merely sentences that are devoid of sense because they convey no information. Consequently, far

from expressing truths that lie at the basis of all inference-licensing implications—justifying their status as inference-licensing—the primitive laws of logic are to be viewed as expressing no facts of any kind. Second, even if Russell's conception of the nature of logic were correct, even if the laws of logic were not *sinnlos*, the appeal to logical laws in the explanation of logical entailment would nonetheless be *superfluous*. If one proposition entails another, the entailment holds simply in virtue of the propositions' being the very propositions they are. There is no need to invoke any logical framework, so to speak, holding premise and conclusion in place so that the relation of material implication obtaining between them may qualify *also* as a relation of entailment. Something very close to this thought is evident in the “darkly metaphorical” passage quoted earlier: “If the truth of one proposition follows from the truth of others, this expresses itself in the relations in which the forms of these propositions stand to one another, and we do not need to put them in these relations first by connecting them with one another in a proposition; on the contrary, these relations are internal, and exist as soon as, and by the very fact that, the propositions exist” (5.131, my translation). As I read it, this passage is claiming that in order to appreciate that *q* follows from *p*, there is no need to first show that the proposition “if *p* then *q*” can be known to hold on the basis of the laws of logic alone. This is so because we do not need to think of the validity of the inference as owed to the logical truth of the corresponding conditional (which, on the view Wittgenstein is opposing, amounts to this conditional's being derivable from primitive logical laws). Rather, the validity of the inference is owed merely to an internal relation between the forms of the premise and the conclusion. The holding of this relation is not something that can be put into words. It is something that shows itself forth when we recognize the propositions for what they are: “That the truth of one proposition follows from the truth of other propositions, we perceive from the structure of the propositions” (5.13).

This is *relatively* clear, but at 5.11 Wittgenstein muddies the issue by seeming to express a rival conception of logical entailment. He says: “If the truth-grounds which are common to a number of propositions are all also truth-grounds of some one proposition, we say that the truth of this proposition follows from the truth of those propositions.” This remark needs to be handled with some delicacy, since it seems to invite something analogous to the criticism that John Etchemendy has recently leveled against Tarski's analysis of logical consequence.⁴¹ By focusing on the fact that the truth-grounds of the premises are also truth-grounds of the conclusion, we seem to leave out the fact that the truth of the premises *guarantees* the truth of the conclusion. This idea is at least more nearly conveyed by the thought of 5.132 that relations of entailment just consist in internal relations between the forms of propositions. So one wonders why Wittgenstein should have troubled to formulate this apparently less appealing analysis.

I would conjecture that 5.11 is supposed to function as a guide to the adoption of a perspicuous notation—a notation in which internal relations of form between propositions are transparent. Its antecedent is the kind of thing we find ourselves mouthing in the attempt to give voice to what we discern when we recognize the internal relatedness of the forms of premise and conclusion. The antecedent of 5.11 is the attempt to give propositional expression to something that can only be shown, and is, strictly speaking, nonsense. But like much of the *Tractatus*, it none-

theless has value in getting us to *do* something. It moves us to adopt a notation that makes visually apparent the internal relations between the forms of propositions. In what we might call the “truth-table notation”, for example—a notation where propositions are expressed in the form of truth-tables (cf. 4.442)—we can visually read off the relations of form from the notation itself, for these relations are expressed by facts about the distribution of “Ts” and “Fs” among the rows of the propositional signs.⁴² It is because we can think of the rows of a sign in the truth-table notation as corresponding to the unofficial idea of “truth-possibilities”, and the rows on which a “T” is entered as corresponding to the unofficial idea of “truth-grounds”, that we recognize the truth-table notation as a good notation for making clear the internal relatedness of certain propositions.

By framing propositions in a perspicuous notation such as the truth-table notation, we make more plausible Wittgenstein’s idea that propositions contain *their own* resources for revealing themselves to stand in the inferential relationships in which they stand (cf. 6.1265). If *in practice* we sometimes need to derive a conditional in an axiom system in order to recognize the corresponding inference as valid, that is only because we are creatures with quite limited logical capacities. According to Wittgenstein, an axiom system provides for what he calls proof *in logic*, which is to say, the derivation of tautologies from tautologies (cf. 6.126). Unlike the proof of one senseful proposition on the basis of another, this kind of proof serves as no more than a “mechanical aid to the recognition of tautology in complicated cases” (6.1262, my translation). Most important, an axiom system is not to be viewed as systematizing any supposed body of propositionally expressible “logical knowledge.”

The criticisms stated at 5.132 do not, however, exhaust Wittgenstein’s critique. The remark about all propositions of logic being “of equal status” at 6.127 is a further implicit criticism, since it challenges the conception of intrinsically primitive logical laws upon which a non-system-relative conception of entailment rests.⁴³ And there is a further criticism, again not explicitly formulated, but clearly available from the Tractarian standpoint. This is the worry that if we were to take derivability of the conditional ‘ $p \supset q$ ’ from primitive laws of logic as a reductive analysis of the logical entailment of q by p , we would risk making the facts of logical entailment seem to rest upon facts that are not, intuitively speaking, logical. For, from Wittgenstein’s point of view, what Frege and Russell *call* the laws of logic are not intuitively logical laws at all. They are merely peculiarly compendious generalizations, which speak about what is actually the case, and this gives them the appearance of especially general empirical laws—laws that Wittgenstein takes to describe features of reality that might have been otherwise (6.111).⁴⁴

V. Wittgenstein’s Positive View

When we infer from p to q : “The method of inference is to be gathered from the two propositions alone” (5.132). The “method of inference” is the particular form of argument employed in the inference. It is to be gathered “from the two propositions alone” because it is something that can be conveyed only by being exhibited *in* propositions. The point is a corollary of Wittgenstein’s denial of the expressibility of logical form. He says:

Propositions cannot represent logical form: this mirrors itself in propositions.

That which mirrors itself in language, language cannot represent.

That which expresses *itself* in language, we cannot express by means of language.

Propositions *show* the logical form of reality.

They exhibit it. (4.121, my translation)

These remarks apply to “logical form” in a rather wide sense, but it is clear from the comment on this thought at 4.1211 that Wittgenstein is thinking, among other things, of the inexpressibility of the internal relatedness of the forms of premise and conclusion in a valid argument. He says:

Thus a proposition “*fa*” shows that the object *a* occurs in its sense, two propositions “*fa*” and “*ga*” show that the same object is mentioned in both of them.

If two propositions contradict one another, then their structure shows it; *the same is true if one of them follows from the other*. And so on. (4.1211, my emphasis)

As we have seen, Wittgenstein’s positive view is that the inference-licensing relation between the forms of the propositions is an internal one that shows itself in the structures of the propositions. But what does Wittgenstein mean by an “internal relation”? And which view exactly does he mean to be opposing by insisting on the internality of the relation of logical entailment?

Wittgenstein says that a relation is internal if it is unthinkable that it should fail to relate its actual relata (4.123). If one shade of blue is brighter than another, then it is unthinkable that these very shades should not be so related (4.123). Wittgenstein does not explain what it is for a state of affairs to be “thinkable”, but one supposes he must have something nonpsychological in mind—perhaps a state of affairs is thinkable in Wittgenstein’s sense just in case there is a proposition that expresses its obtaining. I shall pursue this idea in more detail later, but first I want to delve a little into the prehistory of Wittgenstein’s discussion of the notion of “internality.”

In presenting entailment as an internal relation, Wittgenstein is issuing a direct challenge to Russell’s view that *all* relations are external. Russell, in turn, adopted this view as a central plank in his rebellion against the monistic idealism of F. H. Bradley. In his 1899 essay “The Classification of Relations,” Russell says: “Mr. Bradley has argued much and hotly against the view that relations are ever purely ‘external.’ I am not certain whether I understand what he means by this expression, but I think I should be retaining his phraseology if I described my view as the view that *all* relations are external” (Russell, 1990, p. 143).⁴⁵ Russell’s view, notice, is that he does not know what Bradley means, but that he nonetheless regards it as wrong. Notice also the comprehensiveness of Russell’s rejection of Bradley. He does not merely assert that *some* relations are external, which is all he needs to say in order to disagree with Bradley; he goes the whole hog and asserts the externality of *all* relations.⁴⁶

The view that all relations are external looks extremely unpromising, if we understand externality in terms of Wittgenstein’s notion of “thinkability.” For we most naturally view propositions as incapable of standing in relations of logical

entailment different from those in which they actually stand. We reason: if a proposition were to stand in relations of logical entailment different from those in which it actually stands, that could only be in virtue of its having a different logical structure (containing different logical constants);⁴⁷ but if a proposition contained different constituents, it would not be *that* proposition.

But it does not seem likely that Russell is taking himself to be committed to the externality of relations in *Wittgenstein's* sense. His view would seem to be, rather, that all relations are “not-internal”, in whatever sense Bradley attaches to the term “internal.” For Bradley, a relation is “internal”—or, in his terminology, “intrinsical”—when it “effects” or “passes into” or, again, “penetrates the being of” its terms (Bradley, 1908, p. 364). How Russell understands this idea can be gathered from his essay “The Monistic Theory of Truth” of 1906–7, where he offers the following characterization of what it means to say that there are *external* relations: “There are such facts as that one object has a certain relation to another . . . [and such facts] do not imply that the two objects have . . . any *intrinsic* property distinguishing them from two objects which do not have the relation in question” (Russell, 1910, pp. 139–40).⁴⁸ If we take “imply” here to mean something stronger than “materially imply,” we might read Russell as denying something close to what we should today call a supervenience claim. To say that there are external relations is to say that there are relations that do not supervene on the intrinsic properties of their relata. The idea Russell is gesturing at might be expressed, using the apparatus of possible worlds, as follows: “A relation R is external (in the sense of not penetrating the being of its terms) just in case there are possible worlds w and w¹ and terms X₁, Y₁, X₂, Y₂ such that X₁ and Y₁ are in w and X₂ and Y₂ are in w¹, neither the X's nor the Y's differ intrinsically, and the relation relates only one of the pairs of terms.”⁴⁹ One might well have reservations about glossing the remark in this way, for the formulation I have given employs modal notions for which, officially at least, Russell had no time.⁵⁰ But we should bear in mind that, in practice, Russell does not consistently maintain his antimodalist stance. He is prepared to invoke modal notions both in 1902, in speaking of logico-mathematical truths as governing “not only the actual world but every possible world” (Russell, 1981, p. 55.); and also in 1905 in characterizing—though not defining—propositions, as *possible* objects of belief (see Russell's *Collected Papers*, vol. 4, p. 495). It does not seem outlandish, therefore, to suppose that Russell might have been employing modal notions, tacitly and unofficially, in trying to make sense of the idea that an internal relation “penetrates the being of its terms.”

Let us suppose, then, that Russell's talk of externality is to be understood in terms of nonsupervenience, and his talk of internality in terms of supervenience. Is this proposal compatible with his claim that *all* relations are external? It seems not. For consider what it would be to assert the externality of, for example, the logical entailment relation. One would have to claim that there are two worlds w and w¹ and two pairs of propositions <P₁,Q₁> and <P₂,Q₂> such that the first pair is in w and the second in w¹, both the Ps and the Qs agree in all their intrinsic properties, and the members of only one pair stand related by logical entailment. But this is plainly false, for the terms in question, being propositions, are items for which a lack of intrinsic difference amounts to a lack of numerical distinctness. If two propositions share all their intrinsic properties, they must contain exactly the same constituents, and must combine these constituents in precisely

the same way; but then they will just be the *same* proposition.⁵¹ So to suppose that the relation relates one pair of propositions at one world, and fails to relate an intrinsically identical pair at another, is just to suppose it equally possible (hence equally thinkable) that the relation of entailment should either relate or not relate one and the same pair of propositions. But that was the very position we were trying to avoid by pursuing the detour by way of Bradley's conception of an internal relation. (We pursued the detour in the belief that not even Russell would wish to allow that a proposition could be conceived as having a range of different conceivable positions in the inferential network.) So Russell must, in the end, deny that there are any such worlds as the ones we have tried to describe; and he must, accordingly, judge the relation of logical entailment to be internal.

There are two very different kinds of conclusion we might draw from these reflections. On the one hand, we might say that once the thesis that all relations are external has been made clear—by giving a precise sense to the notion of externality—it is found to be false. But, alternatively, we might say that because he must reject our construal of the notion of internality Russell has in fact failed to attach any meaning to the expression “external”, and so has failed to invest the sentence “All relations are external” with sense.

Russell seems to want to occupy an impossible space between these two positions. He is not sure what Bradley means by calling a relation “internal”, but he is confident that Bradley must mean *something*, and certain that, whatever that should turn out to be, the truth is that there are no internal relations *in that very sense*. In effect, Russell keeps his thesis viable by deferring scrutiny of its key notions. It is as though he takes himself to see matters clearly enough to know that Bradley is wrong, but not sufficiently clearly to know *what* he is wrong about.

The *Tractatus* contains a strand of thought that is deeply critical of this philosophical attitude. Where Russell would take himself to have propounded a substantive, defensible thesis, albeit one requiring clarification, Wittgenstein would take him to have failed to attach meaning to certain of his words, and so to have uttered nonsense, albeit nonsense with a strong tendency to pass itself off as sense (cf. 5.4733).⁵²

Given that such a style of criticism is so thoroughly Tractarian in spirit, and so plausibly applicable to the case at hand, one might wonder why Wittgenstein refrains from applying it here. Instead of saying that the question whether all relations are external makes no sense because some terms employed in its formulation lack a meaning, he seems to propose a way of making sense of it. That is to say, he appears to give his own characterization of what it means for a relation to be internal, and to claim that the relation that justifies inference is internal in just this sense. Is he, then, disagreeing with Russell in the same spirit in which Russell disagreed with Bradley? Is he claiming to have discovered the core of what Bradley and Russell must have meant by their talk of “internal relations”, and to have a counterexample to Russell's thesis that all relations are external? One might be forgiven for thinking so, for, having drawn the distinction between external relations, which can be put into words, and internal relations, which show themselves in language, Wittgenstein writes: “Now this takes care of the vexed question ‘whether all relations are internal or external’” (4.1251, my translation).

However, the impression that Wittgenstein regards himself as having “taken care of” the question by *solving* it begins to evaporate as soon as one scrutinizes

the notion of “thinkability” appealed to in Wittgenstein’s exposition of internality. Earlier we made do with the following suggestion: to say that a state of affairs (that p) is thinkable is just to say that there is a proposition which says that p. But this proposal needs refining,⁵³ for we shall have to decide whether, for the purposes of this characterization, tautology and contradiction count as “propositions.” And here we seem to be in a bind. If we say they do, we commit ourselves to the thinkability of contradictions. If we say they do not, we commit ourselves to counting the circumstance that either it is raining or it is not, as *unthinkable*. But our intuitive sense is that this is something we *can* think. After all, it is a *logical consequence* of something we can think, namely, that it is raining.

The best way to respond to this dilemma is to blunt one of its horns. We need to insist that it is not, strictly speaking, *thinkable* that either it is raining or it is not raining. To make this ruling plausible we shall have to deny that the technical notion of “thinkability” in play here has very much to do with the intuitive notion of what we can think. Since tautologies do not express “thoughts” in Wittgenstein’s technical sense of the term—that is, *senseful* propositions (4), what they do express, if anything, is not in this sense thinkable.⁵⁴ Taking such a position would allow us to reformulate the explanation of thinkability by substituting “proposition with sense” for “proposition” in the formulation given above. This would yield the mildly satisfying result that the circumstance that p is *thinkable* just in case there is a *thought* (i.e., a *senseful* proposition) which says that p.

But how are we to understand this quantification over thoughts? On the face of it, we are saying that when the state of affairs that p is thinkable, for some q, q is a proposition, q has sense and q says that p, where “q” is a variable which occupies an argument place accessible to names of propositions.⁵⁵ We can make sense of this statement only if we can make sense of such sentences as “*a* has sense”, where “*a*” is a singular term designating a proposition. But because propositions, for Wittgenstein, are facts (cf. 3.12 and 3.14), and because facts cannot be named (cf. 3.144),⁵⁶ we will not be able to treat these singular terms as Tractarian names. It seems that we will need to treat sentences containing singular terms that purport to designate propositions, as admitting of paraphrase by sentences in which no singular terms purporting to designate propositions occur.⁵⁷ However, Wittgenstein provides no proposal that would enable us to pull off this trick. We cannot employ a combination of the description theory of (apparent) names and the theory of descriptions to eliminate the apparent name for a proposition, since, even if we could find a satisfactory description, the resulting application of the theory of descriptions would reintroduce quantification over propositions. Nor can we apply the *Notebooks*’ analysis of a sentence predicating a property of a complex: namely, $\phi a. \phi b. aRb = \text{Def } \phi[aRb]$ (Wittgenstein, 1979, p. 4; cf. 3.24),⁵⁸ for whatever its prospects more generally, the proposal will obviously fail in connection with the kinds of things one will want to say of propositions. (Consider, for example, the result of substituting ‘has sense’ or ‘is a proposition’ for ‘*q*’ in the above definition.) In the end, then, Wittgenstein’s talk of thinkability and his explanation of ‘internality’ turn out to be in the same boat as Russell’s talk of ‘internality.’ In both cases we have promissory notes, but no satisfactorily completed explications.

Why, then, does Wittgenstein trouble to *half* formulate the notion of an “internal relation”? Why does he act as though there is a well-defined question whether all relations are external, upon which to take sides? In this case I believe there

may be something to the idea of Cora Diamond and Warren Goldfarb⁵⁹ that for certain heuristic purposes Wittgenstein sometimes behaves as though certain notions to which he has (as yet)⁶⁰ attached no meaning make full sense. In the present case, the temporary adoption of this stance at very least has a certain *diagnostic* value: it enables us to detect various gradations of philosophical error, and see *why* someone might have taken claims involving internal relations to make sense.

Consider, for example, a paradigmatic report of the obtaining of a supposedly internal relation: "This shade of blue is darker than that one." I take it that the idea behind Wittgenstein's insistence on the "bipolarity" of the proposition in his pre-*Tractatus* writings⁶¹ is that one who took this sentence to make sense could be brought to realize that it fails to say anything by being invited to think through what would be said by its negation. On realizing that there is nothing they succeed in thinking when they attempt this exercise, they will come to see that the original unnegated sentence fails to exclude any coherent state of affairs, and so, after all, fails to express a thought.

Wittgenstein's diagnosis would be that the sentence says nothing because we have failed to attach any significance to the fact that 'is darker than' stands between names of *shades* of color (cf. 5.473 and 5.4733). He would add that we are inclined not to notice this because we tend to construe the sentence on the model of the fully senseful sentence: "This patch of cloth is darker than that one," a sentence that contains an expression for a proper external relation. (In so doing, we will have missed the subtle "shift" from a genuine external relation to a "formal relation" or "pseudo-relation"; cf. 4.123). We may conclude that Wittgenstein has identified a class of utterances that express no thoughts, even though it is quite natural to take them as expressing thoughts, namely, those utterances to which we are inclined to prefix the phrase "It is unthinkable that it should not be the case that . . .".

The notion of an internal relation, which presupposes the notion of thinkability, may also play a role at an earlier stage in our considerations. By temporarily avail-ing oneself of the notion of an internal relation, one may arrive at a more refined—if still ultimately problematic—view of *which* internal relations there are. One might, for example, be persuaded that the only internal relations are ones that hold or fail to hold of *propositions*. Wittgenstein's remark that the only necessity is logical necessity (6.37; cf. 6.375) might be seen as gesturing toward such a view, for it invites us to construe talk of internal relations between *objects* as the expression of an inchoate insight into the obtaining of internal relations between *propositions*. So, for example, while G. E. Moore says that a complex stands in an internal relation to its constituent part,⁶² Wittgenstein says: "A proposition about a complex stands in internal relation to the proposition about its constituent part" (3.24). (The relation in question might be the one "shown" by the *Notebooks'* analysis mentioned above.)

I take it that this unofficial view of which internal relations there are is supposed to play a heuristic role in guiding the project of analysis. Suppose that we help ourselves temporarily to the notion of an internal relation. If we then acquiesce in the *Tractatus*'s view of which internal relations there are—that is, if we come to believe that whatever we formerly regarded as a conceptual connection (e.g., that between being colored and being extended, or being known and being

true) is, in fact, an internal relation of form (i.e., a logical entailment) holding between *propositions*—we will be led to try to analyze all propositions into a form where these connections become fully apparent, and, for Wittgenstein, this will be a form in which propositions reveal themselves to be truth-functions of elementary propositions.

Thus, by taking a stand on which internal relations there are, we motivate and direct the project of analysis. Once this conception of analysis is accepted and incorporated into our philosophical practice, the transitional theoretical talk of “internal relations” can be jettisoned as part of the ladder we are to throw away (6.54).

Notes

My thanks to: Ed Holland, Ori Simchen, Peter Sullivan, Markus Stepanians, Jon Curtis, Stephen Everson, Steven Gross, Robin Jeshion, Oystein Linnebo, Michael Potter, Peter Railton, and Jason Stanley. I owe a special debt of gratitude to Warren Goldfarb, Richard Heck, Charles Parsons, and Jamie Tappenden, each of whom provided generous comments on numerous drafts. Various incarnations of the paper were given as talks at Harvard University, Cornell University, and the University of Michigan. The discussion on each occasion led to extensive revisions. My thanks to all who participated.

1. I have followed one or the other of the following translations, except where indicated: Ogden (Wittgenstein, 1922); Pears and McGuinness (Wittgenstein, 1963).
2. The German contains quotation marks that are missing from Ogden’s translation.
3. See, e.g., Baker, 1988, p. 130; Kenny, 1973, p. 98; Mounce, 1981, p. 46.
4. A point that we should keep in mind throughout our discussion is that often—though, as we shall see, by no means *always*—Russell uses the word “implies” to express the relation of *material implication*, rather than logical entailment.
5. These passages do not *exclude* inference rules from the class of “laws of inference,” and I know of no other that does.
6. Ricketts, 1985.
7. See Ricketts, 1985, p. 7: “Unlike Russell, Frege is absolutely clear on the difference between logical laws and inference rules, as well as the need for both in his axiomatic formulation of logic.” Ricketts cites no texts to support his implication that Russell was not clear on the difference.
8. “A law of inference, a proposition in its own right, could justify an inference only by serving as a premise from which a conclusion is drawn. How else?” (Ricketts, 1985, p. 12).
9. Carroll, 1895.
10. However, for the sake of convenience the formally correct notation is immediately dispensed with. (Cf. the conventions of abbreviation employed with parentheses.)
11. It is important to note that Russell does not himself regard what is “formally necessary” as necessary. Here he means by “formally necessary” what would *seem* to be necessary from the point of view of an overzealous formalizer—i.e., one who wishes the formalism never to break down.
12. Russell in fact overshoots his target, failing to record substitution even as an inference rule in *Principia*. (For Russell’s later acknowledgment of this oversight, see Russell, 1919, p. 151 n. 1.)
13. Russell, 1903, §18.
14. Often Russell employs as rules principles of deduction that occur elsewhere as premises simply in order to avoid the tedium of providing fully rigorous proofs.

See the “Note on the Proof of *2.15” (*Principia*, p. 102) for an explanation of the abbreviative procedure.

15. I say “*may* need to be,” since, obviously, there will be some inferences, such as an inference by *modus ponens*, that will not require such mediation.

16. For an illuminating discussion of this conception of logic, see Goldfarb, forthcoming. Ricketts gives a closely parallel account in Ricketts, 1996, pp. 59–99.

17. See, e.g., Frege, 1984c, pp. 387, 425; 1984e, p.145; 1984a, p. 47. Faced with the objection that at some level this misrepresents mathematical practice, which includes proof by contradiction, Frege offers an alternative story of what is really going on. The objection is raised: “Surely one can make deductions from certain thoughts purely hypothetically without adjudging the truth of the latter.” Frege replies: “Certainly, purely hypothetically! But then it is not these thoughts that are the premises of such inferences. Rather, the premises are certain hypothetical thoughts that contain the thought in question as antecedents” (Frege, 1984c, p. 425). A fuller explanation along the same lines is contained in Frege, 1984d, pp. 245ff.

Paolo Mancosu has made clear that Frege’s view on this point is not idiosyncratic. Frege appears to be following in the footsteps of Bolzano, who held proof by contradiction to be always eliminable in favor of direct proof from true axioms. See Mancosu, 1996, chap. 4. (I am grateful to Jamie Tappenden for bringing Mancosu’s work to my attention.)

That Russell regarded a genuine inference from a false proposition as an impossibility is evident from his characterization of the “therefore” relation, which is the relation that is supposed to hold between premise and conclusion when we infer the latter from the former. Russell says that the [holding of] the relation implies that hypothesis [i.e., premise] and consequent [i.e., conclusion] are true. See Russell, 1903, §15.

18. By “logical entailment” I mean the converse of the relation of logical consequence, but understood in such a way that propositions as well as sentences can properly be said to stand in this relation. (My aim is to have a neutral term for a relation understood to have in its field items that are the bearers of truth and falsity, whatever they may turn out to be.)

19. This formulation is suggested by some remarks in Wittgenstein’s Cambridge lectures of 1931. In the course of some reflections on logical conceptions from the *Tractatus*, Wittgenstein remarks: “Inference is justified by an internal relation which we see; the only justification of our transition is our looking at the two terms and seeing the internal relation between them.” A moment later he continues: “Propositions do not follow from one another as such; they simply are what they are” (Wittgenstein, 1980, p. 57).

20. Moore, 1900, pp. 289–304.

21. Here, once more, Russell is using the expression “laws of logic” for the *axioms* of his system. If he did not mean *only* the axioms, there would be no need to distinguish those cases in which the relation of premise to conclusion is “contemplated” by the laws of logic from those in which it is deducible from them.

22. Here I am simplifying Russell’s account by treating deducibility from the laws of logic as equivalent to derivability from them. These are not strictly the same notions, but it is easy to show that they are extensionally equivalent.

23. It is not known whether Wittgenstein read Russell’s 1905 paper, but it is quite possible that he should have learned of Russell’s views on deducibility during his numerous conversations with him at Cambridge during the years 1911–13. These conversations were long and involved, and the topic was usually the nature of logic.

24. Perhaps because of his awareness of Russell’s own doubts about self-evidence as a criterion of the logically primitive—doubts that arose after Russell’s discovery that the reduction of arithmetic to logic requires the axiom of reducibility—Wittgenstein is more cagey about directly attributing this view to Russell than to Frege. (The attribution to Frege is made at 6.1271.). *Tractatus* 5.4731, however, suggests that Wittgenstein believed that Russell’s most considered view would have been to maintain a tight

connection between self-evidence and the primitively logical. Wittgenstein writes: “Self-evidence, of which Russell has said so much, can only be discarded in logic by language itself preventing every logical mistake. That logic is a priori consists in the fact that we *cannot* think illogically.” The last sentence strongly suggests that Wittgenstein took himself to be replacing Russell’s psychological criterion of primitive logicality (viz., obviousness) with a nonpsychological notion: unthinkableability of the negation. I will have more to say about the notion of “unthinkability” later.

25. I am grateful to Warren Goldfarb for stressing the importance of this passage in this connection.

26. The description of the account as “proof-theoretic” needs to be heavily qualified. Frege, of course, is speaking of thoughts, not uninterpreted symbolisms; so his account is by no means a contribution to “proof-theory” in the sense familiar to contemporary philosophers—i.e., a theory that deals with syntactic objects known as “proofs.”

27. An illuminating discussion of this point is contained in Ricketts, 1997, pp. 169–88.

28. See Burge, 1998, pp. 305–47; and Jeshion, forthcoming. I am grateful to Jamie Tappenden for bringing Jeshion’s paper to my attention, and to Robin Jeshion for allowing me to discuss a prepublication version.

29. Whether a truth has this feature will depend on whether the truth has the generality required for one to be able to derive other propositions from it. (Instances of the law of identity, for example, will not.)

30. Jeshion’s story is actually a little more complicated than this, for it takes the quality of being self-supporting as a necessary but not sufficient condition for being a basic truth. (Basicness additionally requires that the simple constituents of the truth are themselves “basic” in another sense.) The details, however, are less important for our purposes than the idea that Frege has some notion of an objectively basic truth.

31. It is worth mentioning, in support of Burge and Jeshion, that Frege’s conception of an *objective* structure of justification is not unique among nineteenth-century views on the foundations of mathematics. Paolo Mancosu (1996, pp. 102–3) quotes a passage from Bolzano (1810), which contains a striking anticipation of this idea:

Now the further question arises, what should properly be understood by the *proof* of a truth? One often calls every sequence of judgments and inferences by which the truth of a certain proposition is made generally recognizable and clear, a *proof of the proposition*. In this widest sense, all true propositions, of whatever kind they may be, can be proved. We must therefore take the word in a narrower sense and by the *scientific proof* of a truth we understand the representation of the *objective dependence* of it on *other truths*, i.e. the derivation of it from such truths which are to be considered as the *ground for it*—not fortuitously [*nicht zufälliger Weise*]—but in *themselves and necessarily*, while the truth itself must be considered as their *consequence*. (§12)

32. Burge argues that the nonpsychological notion of self-evidence applies to inference rules as much as to laws of logic (Burge, 1998, p. 322).

33. Ricketts (1997) claims that Frege was reluctant to endorse the strategy he sketches in “Foundations of Geometry: Second Series” for giving independence proofs, and attributes this reluctance to Frege’s awareness that he lacked precise characterizations of the notions of “logical law” and “logical inference.” Since such characterizations could be developed from characterizations of “primitive logical law” and “primively logical inference,” I take it that Ricketts would read Frege as similarly pessimistic about the clarification of these notions.

34. The standard translations by Ogden and by Pears and McGuinness are misleading here. Each suppresses the reference to Fregean-sounding “basic laws.”

35. The German is *als Kriterium*. By omitting the article, Wittgenstein leaves it

unclear whether he takes Frege's view to be that psychological obviousness is *the* criterion (i.e., either a necessary and sufficient condition, or, possibly, a sufficient condition) or merely *a* criterion (i.e., either a necessary condition, or, possibly, a defeasible "way of telling"). The context of the remark suggests that he took Frege to regard obviousness as at the very least a necessary condition of being an essentially basic logical law.

36. At 6.1271 Wittgenstein speaks of the degree of self-evidence as supposed to be the criterion of a "logical proposition", not of a "primitive logical proposition", but it seems likely that he has the latter notion in mind here, for the remark begins with Wittgenstein's claim that Frege was wrong to think the number of *basic* laws nonarbitrary, since one could develop logic from one basic law, namely, the conjunction of Frege's basic laws. Wittgenstein then envisages Frege as challenging the status claimed for this conjunction—i.e., of being a primitive proposition—on the ground that it lacks self-evidence. Moreover, we have seen that Frege and Russell both sometimes speak of the "logical" when they mean the "primitively logical", so it would be no surprise if Wittgenstein also had developed this loose way of speaking.

37. See McGuinness, 1988, chaps. 4–6.

38. I am grateful to Jamie Tappenden for bringing this passage to my attention.

39. For the significance of the scare quotes, see note 26.

40. As evidence that the Wittgenstein of the *Tractatus* had read this work—besides his acknowledgment of the "great works of Frege" in the preface of the *Tractatus*—we may cite *Tractatus* 3.3, which loudly echoes the *Grundlagen*'s famous context principle.

41. Etchemendy, 1990, chap. 6.

42. At 6.122 Wittgenstein emphasizes this visual aspect of inference: "[W]e can get on without logical propositions, since we can recognize in a suitable notation the formal properties of the propositions by mere inspection [*durch das blosse Ansehen*]." And there are many other passages that stress this visual theme (see, e.g., 6.1221).

It seems that Wittgenstein is envisaging an adequate notation to be one in which the necessity of the relation of entailment is expressed by visual means. That one *cannot* draw a quincunx without drawing four triangles is visually apparent in the figure itself. I would conjecture that an adequate notation is one that is supposed in a similar way to render visually apparent the necessity of the relation of logical entailment. (I am indebted to Peter Railton for this point.)

43. The notion of psychological self-evidence might also yield a non-system-relative notion of derivability, but, as we have noted, it too is a notion to which Wittgenstein takes exception (6.1271).

44. To require that the recognition of logical inferential relations should not turn on the discernment of any worldly facts is not to embrace a conception of logic as somehow entirely without existential presuppositions, a free-floating entity having its nature quite independently of whether there is a world. As 5.5521 makes clear, such a conception would render mysterious how such a logic could apply to the world: "[If logic were prior not just to how the world is but to the fact of its very existence] how could we apply logic? We could say: if there were a logic, even if there were no world, how then could there be a logic, since there is a world?"

For Wittgenstein, logic is simply a by-product of language designed to represent the world. If there were no world to be represented (whether truly or falsely), then there would be no logic either, because there would be no language. "The logical propositions describe the scaffolding of the world, or rather they present it. They 'treat' of nothing. They presuppose that names have meaning, and that elementary propositions have sense. And this is their connection with the world" (6.124).

45. Given the date of this remark, it is worth mentioning that the bulk of the *Principles* appeared in the draft of May 1901, and that most of it was written during 1900. See Russell, 1993, "Introduction" and "Chronology."

46. Because Russell may still be found speaking of the problems inherent in “the view that relations are not purely external” as late as 1905 (Russell, *Collected Papers*, vol. 4, p. 502), it is plausible to suppose that he continued to subscribe to the contrary of Bradley’s position throughout the period 1900–1905. Against this, it should be mentioned that Russell does once make a passing reference to internal relations in the *Principles* (§412). However, it is far from clear that “internality” in this context is supposed to amount to Bradley’s notion. Russell may merely mean that a relation is “internal” to a progression in the sense that the progression is ordered by that relation, and so in a sense is built up out of it.

47. I would take the intuitionist interpretation of the logical connectives to amount, in a Russellian setting, to an alternative view of which connectives the proposition contained, not to an alternative view of the proposition’s logical relations.

48. There is a confusion over the title of this paper that seems worth clearing up. The chapter of *Philosophical Essays* from which this remark is drawn is entitled “The Monistic Theory of Truth.” The material of the chapter comprises the first two sections of a three-section paper, which Russell delivered to the Aristotelian Society on December 3, 1906. The paper was originally entitled “On the Nature of Truth.” The editorial note in *Philosophical Essays* (p. 131), however, incorrectly cites it as “The Nature of Truth.” See Russell, 1907.

49. We might be tempted to say: “just in case it is possible that X_1 and Y_1 should stand related by R and X_2 and Y_2 not do so, when neither the X’s nor the Y’s differ intrinsically,” but this formulation counts as internal, relations that we would intuitively consider external, for the definition only considers pairs of terms contained in the same world. However, a relation that related one pair of items at one world, but failed to relate another intrinsically identical pair at another world, is one we should intuitively wish to call external. The point is due to David Lewis. See Lewis, 1986, pp. 14–17.

50. The formulation also presupposes the availability of a workable distinction between intrinsic and relational properties; and that distinction is notoriously hard to formulate.

51. We might argue for the view that there cannot be two distinct propositions with the same intrinsic properties. Suppose there are two such. Then what rendered them distinct would have to be their occupying distinct positions in a network of inferential relations, for there would be nothing beyond their relational properties to which we could appeal to ground their distinctness. But that would not be possible without an intrinsic difference in the propositions. Intrinsically identical propositions must have the same inferential relations to other propositions (though the converse need not hold).

52. The conception of nonsense I am alluding to here is one that has received considerable illumination from the writings of Cora Diamond. See Diamond, 1991a, especially chap. 3.

53. I am grateful to Jamie Tappenden for getting me to rethink this issue.

54. If they express anything, it is only by showing it forth. What they show is “the logic of the world” (6.22).

55. Wittgenstein’s own account differs from this one insofar as it treats existential quantifications as (in effect) analyzable as (possibly infinite) disjunctions. However, I will continue to use this proposal for purposes of illustration, for the problems it raises are also problems for Wittgenstein’s account.

56. Cf. these remarks from the “Notes on Logic”: “Facts cannot be named” (Wittgenstein, 1979, p. 107) and “Frege said ‘propositions are names’; Russell said ‘propositions correspond to complexes.’ Both are false; and especially false is the statement ‘propositions are names of complexes’” (p. 97).

57. It is clear that in the “Notes on Logic” Wittgenstein envisaged paraphrastic analysis as necessary to eliminate what he called, following Russell, “incomplete symbols” (*unvollständige Zeichen*). These include such phrases as “the sense of p ”

and “the meaning of *p*” (Wittgenstein, 1979, p. 102). In the same connection see Wittgenstein’s quotation (of Russell’s quotation) of the original German text of the remark from the “manuscript” section of “Notes on Logic,” and Wittgenstein’s comments upon it, in Wittgenstein’s second letter of November 1913 (Wittgenstein, 1995, p. 50). The first letter of that month makes clear that Wittgenstein envisaged a similar “analysis” of (contexts containing) the word “fact” (p. 48). To cement the connection with paraphrastic analysis, we should note that the phrase “incomplete symbol” is Russell’s. In chapter 3 of the introduction to *Principia*—a chapter entitled “Incomplete Symbols”—Russell and Whitehead characterize the notion thus: “By an ‘incomplete symbol’ we mean a symbol which is not supposed to have any meaning in isolation, but is only defined in certain contexts” (Russell and Whitehead, 1990, p. 67).

58. Note that Wittgenstein gives definitions in the reverse of their now conventional order.

59. See Diamond, 1991a; Goldfarb, 1997.

60. It remains a question for further inquiry whether in adopting this stance Wittgenstein abandons the hope that signs to which he has given no meaning in isolation might later be given a meaning through an unforeseen contextual definition.

61. In the 1913 “Notes on Logic,” Wittgenstein says that every genuine proposition is “bipolar” in the sense of being “essentially true-false” (Wittgenstein, 1979, p. 98). He means that every proposition is, by its very nature, both capable of truth and capable of falsity. He makes clear that bipolarity also involves the idea that to understand a proposition I must know, and have a full conception of, what would have to be the case for it to be true and what would be the case for it to be false (pp. 94–95, 98–99). Although bipolarity is not explicitly mentioned in the *Tractatus*, Wittgenstein is unlikely to have changed his mind about it. He mentions the requirement as late as the *Notebooks* entry for June 2, 1915 (p. 53). If McGuinness is right, this would have been only a matter of months before he set about writing the first seventy sides of the *Prototractatus*, which differs from the *Tractatus* only in minor respects. See McGuinness, 1989, pp. 35–47.

62. Moore held the relation of a (spatially extended) whole to its part—but not of the part to the whole—to be an internal relation. See Moore, 1965, p. 88.

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Number and Ascriptions of Number in Wittgenstein's *Tractatus*

Juliet Floyd

How are we to place the *Tractatus* remarks on arithmetic within the development of early analytic philosophy, in relation to the work of Frege and Russell? Heretofore the understandable focus of most readers has been Wittgenstein's characterization of the nature of logic and of the fundamental logical notions. The logical positivists took Wittgenstein's characterization of logic as "tautologous" or "analytic" to be the crux of his philosophical contribution, the key to the turning point in modern philosophy. Since they also held that Frege and Russell had demonstrated that arithmetic is a branch of logic, they happily applied the terms "tautologous" and/or "analytic" to mathematics as well, ignoring the fact that Wittgenstein never characterized logic and mathematics as a unity.

Indeed, in the *Tractatus* Wittgenstein explicitly rejected the Frege-Russell attempt to prove that such a unified characterization is mandatory, holding that mathematics consists, not of tautologies, but of equations. Wittgenstein questioned the notion that Russell's axioms of Infinity and Reducibility are fundamental principles of logic (5.535, 6.1232, 6.1233),¹ asserted that the Frege-Russell definition of the successor relation "suffers from a vicious circle" (4.1273), and remarked that "the theory of classes is superfluous in mathematics" (6.031). Ramsey felt that Wittgenstein's account of logic should be used to render the logicism of Frege and Russell "free from the serious objections which have caused its rejection by the majority of German authorities," but also held that the *Tractatus* view of mathematics as consisting, not of tautologies, but of equations, "is obviously a ridiculously narrow view of mathematics, and confines it to simple arithmetic" (Ramsey, 1960, pp. 1, 17). Of the many philosophers who took Wittgenstein's terms "tautology" and "analytic" to appropriately characterize the nature of logic—both positivists and antipositivists—nearly all shared Ramsey's dismissive attitude toward Wittgenstein's treatment of arithmetic.²

Thus the problems of analysis that centrally occupied Frege and Russell in their works—the objectivity of arithmetic, the grammatical structure of ascriptions of number, explicit definitions of the natural numbers in logical terms, derivations of fundamental principles of arithmetic from logical axioms—were (until recently)³ treated as peripheral to Wittgenstein's main philosophical concerns in the *Tractatus*.

This essay will attempt to remedy this tendency to pass over the *Tractatus* remarks on arithmetic in silence by exploring their relation to the philosophies of Frege and Russell. I haven't the space to do full interpretive justice to these remarks here. Nor shall I even attempt an exhaustive characterization of Wittgenstein's responses to Frege and Russell's philosophies of arithmetic. Instead I shall argue that one cannot appreciate the full philosophical force of Wittgenstein's treatment of logic without taking his remarks on mathematics into account. The structure of this essay is then as follows. I first argue for the centrality of Wittgenstein's *Tractatus* remarks on mathematics to his early philosophy (section I) and probe the origins and significance of the terminology he uses in the *Tractatus* to characterize mathematics (section II). Next I discuss what I take to be Wittgenstein's recasting of Frege's analogy between the grammatical structures of sentences and the grammatical structures of number words (section III). This sets the stage for an explication of Wittgenstein's un-Fregean, un-Russellian handling of the grammar of ascriptions of number (section IV), and his idiosyncratic treatment of the variable (section V). Finally (in section VI), I characterize Wittgenstein's formal specification of the natural numbers in the *Tractatus* and his conception of the grammar of mathematical sentences.

I. Antilogicism in the *Tractatus*

An obvious question about the *Tractatus* treatment of number words concerns what Wittgenstein has to say about the logic of ascriptions of number, statements such as "there are three men on the street," and inferences such as "Tom has two peanuts and Larry has four peanuts, so together they have six peanuts." An analysis of the deductive structure of inferences involving such statements was arguably one of the principal glories of Frege and Russell. Max Black held that Wittgenstein failed even to address this issue (1964, p. 313). Others disagree, arguing that the *Tractatus*'s treatment of arithmetical equations offers at least the beginnings of an account of the deductive structure of ascriptions of number: equations may be seen to express rules licensing the interchange of certain definite descriptions and numerical terms in nonarithmetical propositions (Hacker, 1986, p. 124; Frascolla, 1994, pp. 20–23). Yet such a response fails to address the question of how Wittgenstein treated the grammar of extramathematical ascriptions of number ("Tom has two peanuts") unless we (wrongly) assume that he took these to be equations as well. Wittgenstein took equations to be neither identities nor genuine propositions. But extramathematical sentences ascribing number would seem to be genuine propositions: that Tom has three peanuts is either true or false. So a failure to account for the grammar of such sentences would appear to be a failure on Wittgenstein's part to account for the nature of the proposition. It should certainly incline us to think that Wittgenstein embraced a kind of formalism about mathematics of just the sort which Frege and Russell so effectively criticized in

their works. Frascolla remarks that “[Wittgenstein]’s brief outline [of mathematics], without further elaboration, is not enough to account for numerical specifications in ordinary language asserting the number of the elements of the extension of a material concept and for the formal relations between these statements” (Frascolla, 1994, p. 23).⁴ If this is correct, however, we are faced with a puzzle. For Wittgenstein had familiarized himself with Frege’s *Grundgesetze*, Russell’s *Principles of Mathematics*, and Whitehead and Russell’s *Principia Mathematica* well before writing the *Tractatus*. He knew at first hand Frege’s and Russell’s emphases on the importance of applications of number and arithmetic. Frege and Russell begin by analyzing statements of cardinal number; they emphasize that only their analyses show how to represent the use of elementary arithmetic in ordinary inferences. Indeed, Frege’s primary argument against formalism is that it cannot account for the logic of such inferences, hence it cannot account for the content or “applicability” of arithmetic, that which differentiates arithmetic from a mere game like chess. Frege wrote in the *Grundgesetze*: “Why can arithmetical equations be applied? Only because they express thoughts. How could we possibly apply an equation which expressed nothing and was nothing more than a group of figures, to be transformed into another group of figures in accordance with certain rules? Now, it is applicability alone which elevates arithmetic from a game to the rank of a science. So applicability necessarily belongs to it” (Frege 1903, p. 291). Wittgenstein explicitly remarks in the *Tractatus* that mathematics consists essentially of equations that, as pseudopropositions (*Scheinsätze*), “express no thoughts” (6.2–6.21). Since it is wholly unlikely that Wittgenstein simply ignored Frege’s and Russell’s treatments of the application of arithmetic—this is too central to their writings—it may seem inevitable that we must conclude, as many readers do, that Wittgenstein was quite self-consciously adopting what may be legitimately described as a version of the philosophical doctrine of formalism.

At stake is not merely the question of the adequacy of the *Tractatus*, or the relative impact on Wittgenstein of Frege and Russell. Even more, there is the question of the sense, if any, in which Wittgenstein may be said to have shared Frege’s and Russell’s aims and ideals, their conception of what philosophical logic can and should set out to accomplish. This question cannot be answered apart from detailed consideration of the extent to which the *Tractatus* takes account of the internal workings of Frege’s and Russell’s philosophies. As Michael Dummett has written,

The arguments Frege uses in favour of his answer to the question “What is a number the number of?” and against answers proposed by others, are arguments for adopting his analysis of ascriptions of number. Since that analysis is both syntactic and semantic in character, they are also suasions in favour of his semantic theory: it is, among other reasons, because that theory is capable of giving a convincing account of ascriptions of number, and rival semantic theories are not, that we now take for granted the correctness of a semantics at least generally along Fregean lines, and do not so much as stop to consider one of those implicitly underlying the views Frege . . . so decisively refutes. (Dummett, 1991, p. 74, last emphasis mine)⁵

We, however, should stop so to consider. As I shall argue, Wittgenstein did treat the logic of ascriptions of number in the *Tractatus*. Yet ironically, his treatment

further served to undercut the philosophical purpose and interpretation that Frege and Russell imposed on the (different) logical systems each of them devised. Thus in questioning the adequacy of their purported analyses of the grammar of number words and of arithmetic, Wittgenstein is questioning their analyses of logic itself. The *Tractatus's* treatment of arithmetic is thus a crucial index and expression of Wittgenstein's philosophical stance toward Frege and Russell, a small part of the book that (like so many of its parts) exemplifies the whole. It is not the mathematics of Frege and Russell *per se* to which Wittgenstein objected: nothing he wrote could ever hope to jettison any genuine result about the mathematical structure of, say, quantification theory. Instead, he took Frege's and Russell's purported accounts of logic and arithmetic to mislead in purporting to provide a logical framework that could be used to resolve the philosophical questions, How does arithmetic apply to the world? What gives our number words definite meaning? and How is it that arithmetic is more than a mere formalistic game with symbols? For Wittgenstein, Frege's way of construing the question, What is a number? makes no sense. It is not that Wittgenstein proposed an alternative foundationalist or reductive solution to the question. He was neither a formalist nor an antiformalist. Rather, he challenged both the Frege-Russell and the formalist's conceptions of how logic might figure in resolving the philosophical debate.

Not surprisingly, the *Tractatus* has usually been read through the lens of the new logic, through the eyes of Frege and/or Russell. We see Wittgenstein making remarks about variables, functions, names, objects, concepts, truth-functions, numbers, and quantifiers, and we suppose that we know whereof he speaks. On the contrary, what the *Tractatus* remarks on number and arithmetic show, I believe, is that we misread the *Tractatus* when we impose either a Fregean or a Russellian conception of such notions on its terminology. If we focus on Wittgenstein's treatment of number in the *Tractatus*,⁶ we can see that he rejects Frege's and Russell's conceptions of what logical analysis can accomplish.

Consider, then, that in the *Tractatus* Wittgenstein never once applies the term "function" to what we would ordinarily call mathematical functions. What could be more striking—or more misleading for his readers? Few are prepared to follow him here. Commentators standardly assume that Wittgenstein's notion of (propositional) function may be explicated by appealing to mathematical examples, just as Frege did in explaining his notion of function (Black, 1964, pp. 129–30; Anscombe, 1971, pp. 102ff.) Wittgenstein himself suggests that his use of "function" is reminiscent both of Frege's and of Russell's uses of the term: he writes in the *Tractatus* that "I conceive the proposition—like Frege and Russell—as a function of the expressions contained in it" (3.318). But this is misleading, and not only because Frege's and Russell's conceptions of functions differ from each other. Mathematical functions are not functions in the sense of the *Tractatus*. By insisting that function and operation never be confused with one another (5.25), and then calling mathematics essentially a calculus of operations (6.233–6.234), Wittgenstein is rejecting, not only Frege's and Russell's conceptions of mathematics, but also their (respective) conceptions of the notions of *function*, *proposition*, and *sentence*.

From a mathematical point of view, one main task of building arithmetic up out of "logic"⁷ was to show how to define arithmetical functions and numbers in terms of purely logical concepts or functions (for Frege), or in terms of propositional functions (for Russell). Frege and Russell took themselves to have clarified

our notion of a mathematical function by analyzing it in purely logical terms. But to so analyze, they generalized the notion of *function* beyond its customary mathematical use, applying it uniformly to the structure of all sentences, in order to call it a logical notion. One point I take to be central to the *Tractatus*: Wittgenstein is resisting this sort of generalization. He is denying that one function-argument scheme is adequate to an analysis of logic and mathematics. In particular, he denies that number words and nonmathematical names (or adjectives) must be given the sort of unified logico-grammatical treatment that Frege and Russell gave them. He denies both that number words are names (are object expressions) and that they are adjectives (are function expressions). He refuses to treat those ascriptions of number which he takes to express genuine propositions as identities. As he sees it, not all complexity in the structure of sentences is function-argument complexity. To assume so is to fall prey to the ghost of what he calls “the old logic,” the fantasy of a generally applicable subject-predicate distinction.⁸

Attention to the use of language in philosophizing is as basic to the *Tractatus* as it is to Wittgenstein’s later thought. Yet it has been held—for example, by G. E. M. Anscombe—that there is nevertheless a distinction between the kind of use on which Wittgenstein focused in the *Tractatus* (Anscombe calls this “logico-syntactic use”) and the kind on which he focused in his later philosophy (Anscombe, 1971, pp. 91–92, labels this “role in life” or “practice”). But I think we do better to say that in the *Tractatus*, just as in his later philosophy, Wittgenstein focuses on use in order to wean us from misconstruing the character and scope of those distinctions we are inclined to enshrine as “logical”—especially the distinctions drawn by Russell and by Frege.⁹

Wittgenstein’s categorial (logical) distinctions thus fail to coincide with those of Frege and Russell, despite the coincidence of their terminology. This is best viewed, I believe, as the manifestation of his desire to expose the apparent clarity of their analyses as illusory, to show that their systems of logic, when treated as “great mirrors” of thought and reality (5.511), distort like the mirrors of a funhouse. It is not that Wittgenstein purports to correct Frege’s and Russell’s logic, as if he sees himself having discerned the true categorial reality lying behind the functioning of language when he draws un-Fregean and un-Russellian categorial distinctions among kinds of expression. There are no arguments in the *Tractatus* as to the *correctness* of such distinctions. Instead, in drawing the distinctions he does, Wittgenstein is attempting to recover ordinary modes of speaking about language—about mathematics in particular—in the face of what he regards as Frege’s and Russell’s missystematization. His denial that numerals are names, that numbers are objects or (second-order) properties, is best read as recasting the whole idea of what the drawing of categorial or logical distinctions can accomplish. He is attempting to shift our attitude toward (pre- and post-Fregean) categorial talk, toward our use of notions such as *name*, *subject*, *predicate*, *object*, *number*, *concept*, *function*, *proposition*, and so on. By fashioning his own categorial distinctions and notational proposals for a *Begriffsschrift*, he does without the analyses of Frege and Russell, and thus shows us how misleading is their categorial talk. He builds his own funhouse of mirrors to show us the way out of theirs. His aim is to unmask metaphysical idolatry of notation.

Wittgenstein is quite clear that his own Tractarian remarks mislead as to their categorial status: though they appear to make sense, to satisfy all ordinary gram-

matical criteria for propositionhood, they fail to find uses as genuine propositions according to the lights of the *Tractatus* itself (6.54). This was the price Wittgenstein paid for attempting to unmask the purported analyses of Frege, Russell, and others as nonsensical rather than false. But Wittgenstein's deepest philosophical insight in the *Tractatus* was to try to get us to see that not every grammatically well-formed propositional sign, not every sentence, finds a use in thinking, and, conversely, that there is thinking by means of sentences that are not propositional signs, that fail to fit purportedly universal logical categories of propositionhood. The best commentators on *Philosophical Investigations* and related works all agree that this is a key insight of the later Wittgenstein. Implicit throughout this essay is the claim that this insight is already present in the *Tractatus*.¹⁰ For already in the *Tractatus* Wittgenstein is questioning the idea that any one logical grammar is adequate to guarantee or represent all forms of sense. Proofs within a formalized, pure logic, just as proofs within elementary arithmetic, are mere mechanical expedients, mere calculations (6.126ff. 6.2, 6.2331ff.). But in the *Tractatus* Wittgenstein is trying to show that there are no such mechanical expedients in philosophy, no general logico-syntactic means for showing that an apparently grammatically well-formed sentence does or does not have (a certain kind of) use. Some thinking cannot—except misleadingly—be captured by a logic that represents all inferences as movements from truth to truth (from proposition to proposition) in accordance with generally applicable logical laws. Some insights (including the insight that a particular grammatically well-formed propositional sign finds no use in thinking) take thinking, but no particular thoughts, to see (cf. Floyd, 1998b).

Thus in the *Tractatus* Wittgenstein is showing us that no fixed logico-grammatical criteria—even those of the *Tractatus* itself—are sufficiently rich to explicate the meaningfulness or meaninglessness of any particular configuration of signs in a generally applicable way. This is how I interpret Wittgenstein's rejection of Frege's and Russell's view that pure logic and pure mathematics consist of propositions in a paradigmatic sense of "proposition" extendable uniformly across all of language. His treatment of basic logical notions takes place in a context intended to complicate our conception of what it is to express or fail to express a proposition or thought, of what it is to think, in ways Frege and Russell could not accept. Indeed for Wittgenstein—unlike for Frege or Russell—a pure logic, a *Begriffsschrift*, is a mere *Schrift*, a mere script of signs used to keep track of wholly formal operations. In application to genuine propositions—by way of genuine concept words and names—it is a mechanical expedient in certain circumstances (when, for example, the generality sign is not present, as Wittgenstein explicitly remarks at 6.1203) for helping us to recognize that certain propositional signs that meet the rules of ordinary grammar nevertheless fail to express propositions or thoughts (6.1263). But pure logic does not in itself limn the underlying structure of all thoughts. Insofar as it is framed and treated as a maximally general science of the laws of truth or thought, as a universally applicable framework within which generally applicable logical distinctions are enshrined and a single interpretation of the signs is fixed, in which *thought as such* is characterized, it is *unsinnig*, nonsensical, its structure misleads. Differently put, Wittgenstein believes that Frege and Russell assimilated the logic of our language much too closely to the (mechanical) workings of a systematic notation such as the decimal system, in which every numeral can be generated by a mechanical operation from its prede-

cessor, and in which all properly spelled numerical signs signify—automatically, as it were—numbers. In so doing, Frege and Russell misrepresented both the nature of language and the nature of arithmetic.

II. The Terminology of “Operations”

Here are some of the terms used by Wittgenstein in the *Tractatus* to characterize mathematics:

- 6.2 Mathematics is a logical method.
The propositions [*Sätze*] of mathematics are equations, and therefore pseudo-propositions [*Scheinsätze*].
- 6.21 Mathematical propositions express no thoughts.
- 6.211 In life it is never a mathematical proposition which we need, but we use mathematical propositions *only* in order to infer from propositions which do not belong to mathematics to others which equally do not belong to mathematics.
- 6.2323 The equation characterizes only the standpoint from which I consider the two expressions, that is to say the standpoint of their equality of meaning [*Bedeutung*].
- 6.233 To the question whether we need intuition for the solution of mathematical problems it must be answered that language itself here supplies the necessary intuition.
- 6.2331 The process of *calculation* brings about just this intuition. Calculation is not an experiment.
- 6.234 Mathematics is a method of logic.
- 6.2341 The essential of mathematical method is working with equations. On this method depends the fact that every proposition of mathematics must be self-evident.
- 6.24 The method by which mathematics arrives at its equations is *the method of substitution*.
For equations express the substitutability of two expressions, and we proceed from a number of equations to new equations, replacing expressions by others in accordance with the equations. (last emphasis mine)
- 6.241 Thus the proof of the proposition $2 \times 2 = 4$ runs:

$$\begin{aligned} (\Omega^v)^{\mu'}x &= \Omega^{vx\mu'}x && \text{Def.} \\ (\Omega^{2x2'})x &= (\Omega^2)^{2'}x = (\Omega^2)^{1+1'}x = \Omega^{2'}\Omega^{2'}x = \Omega^{1+1'}\Omega^{1+1'}x \\ &= (\Omega'\Omega')'(\Omega'\Omega')x = \Omega'\Omega'\Omega'\Omega'x = \Omega^{1+1+1+1'}x = \Omega^4x. \end{aligned}$$

As I explain below (in section V) “ Ω ” is a variable ranging over what Wittgenstein calls “operations”.

The terms Wittgenstein uses to characterize mathematics in the *Tractatus*—as a “calculus”, an art of manipulating “substitutive signs” in “equations”, in abstraction from their “meaning”, by the use of “operations” according to “the method of substitution”—were familiar in the late nineteenth and earlier twentieth centuries, especially in algebra. They appear, for example, in Whitehead’s *Universal Algebra* (1898) and *Introduction to Mathematics* (1911), works it is not unlikely that Wittgenstein saw.¹¹ In these remarks Wittgenstein is rejecting Frege’s and

Russell's stretching of the notion of *function* by exploiting this traditional terminology and adapting it to apply to the new logic.

The term "operation" was primarily used to characterize any algorithmic manipulation of signs in a purely formal manner, in abstraction from any particular interpretation of the symbols. "Operation" thus came to be associated with (what Frege and Russell would later criticize as formalist) conceptions of logical and mathematical symbolism as calculi, as algebras in which logical signs are to be treated as variables, as empty formulas awaiting interpretation and/or reinterpretation. When in his *Universal Algebra* Whitehead defines the term "operation", he applies it, not only to mathematical operations such as addition and multiplication, but also to deductive inference, footnoting Bradley's *Principles of Logic* treatment of inference (Whitehead, 1898, pp. 7–9). Not surprisingly, like the other post-Kantian Idealists, Bradley viewed deductive logical inference as empty, merely formal thought. Chapter 1 of Whitehead's *Universal Algebra*, "On the Nature of a Calculus," begins with words that echo those of the *Tractatus*:

1. SIGNS. Words, spoken or written, and the symbols of Mathematics are alike signs. Signs have been analysed [by Stout and Peirce¹²] into (α) suggestive signs, (β) expressive signs, (γ) substitutive signs.

A suggestive sign is the most rudimentary possible, and need not be dwelt upon here. An obvious example of one is a knot tied in a handkerchief to remind the owner of some duty to be performed.

In the use of expressive signs the attention is not fixed on the sign itself but on what it expresses; that is to say, it is fixed on the meaning conveyed by the sign. Ordinary language consists of groups of expressive signs, its primary object being to draw attention to the meaning of the words employed

A substitutive sign is such that in thought it takes the place of that for which it is substituted. A counter in a game may be such a sign: at the end of the game the counters lost or won may be interpreted in the form of a money, but till then it may be convenient for attention to be concentrated on the counters and not on their signification. *The signs of a Mathematical Calculus are substitutive signs.*

The difference between words and substitutive signs has been stated thus, "a word is an instrument for thinking about the meaning which it expresses; a substitute sign is a means of not thinking about the meaning which it symbolizes [n. Cf. Stout, 'Thought and Language,' Mind, April, 1891]."

2. DEFINITION OF A CALCULUS. In order that reasoning may be conducted by means of substitutive signs, it is necessary that rules be given for the manipulation of the signs. *The rules should be such that the final state of the signs after a series of operations according to rule denotes, when the signs are interpreted in terms of the things for which they are substituted, a proposition true for the things represented by the signs.*

3. EQUIVALENCE. In a calculus of the type here considered propositions take the form of assertions of equivalence. (Whitehead, 1898, pp. 7–8, my emphases)

Whitehead explicitly denies that such assertions of equivalence are mere identities. As for the term “operation”, Whitehead takes it in its traditional sense, linking it to what he explicitly calls “the method of substitution”:

OPERATIONS. Judgments of equivalence can be founded on direct perception, as when it is judged by direct perception that two different pieces of stuff match in colour. But the judgment may be founded on a knowledge of the respective derivations of the things judged to be equivalent from other things respectively either identical or equivalent. *It is this process of derivation which is the special province of a calculus. The derivation of a thing p from things a,b,c, . . . , can also be conceived as an operation on the things a,b,c, . . . , which produces the thing p.*

. . .

Instead of reasoning with respect to the properties of one scheme in order to deduce equivalences, we may substitute the other scheme, or conversely; and then transpose at the end of the argument. This device of reasoning, which is almost universal in mathematics, we will call the method of substitutive schemes, or more briefly, *the method of substitution*. (Whitehead, 1898, pp. 8–9, my emphases).

When in the *Tractatus* Wittgenstein calls all the sentences of logic “tautologies”, he is denying, both that every grammatically well-formed sentence expresses a proposition, and that there are any genuine logical *propositions*: even the Idealists and the early Moore and Russell had denied that tautologies are genuine propositions (see Dreben and Floyd, 1991, p. 27). When he remarks that the essence of mathematics is work with equations according to “the method of substitution” by way of “calculation” (6.23–6.24), and when he calls the same method in logic a “mechanical expedient” (6.1262), he is resuscitating traditional algebraic terminology in order to undercut, not Frege’s and Russell’s mathematical logic *per se*, but their claims for its contentfulness and success in simultaneously analyzing both logic and mathematics. There can be little doubt that he chose his terminology with care. For when in 1923 Ramsey ventured to Lower Austria to discuss the *Tractatus* with Wittgenstein, and they went through the book line by line,¹³ there were just a few marginal remarks Wittgenstein wrote into Ramsey’s copy of the *Tractatus* as suggested revisions, and these surrounded the treatment of number. Wittgenstein suggested adding to the text (at 6.02, see section V below). “The fundamental idea of mathematics is the idea of *calculation* represented here by the notion of *operation*"; “number is the fundamental idea of calculus and must be introduced as such”; and “the beginning of logic presupposes *calculation* and so number.”¹⁴ Something like Poincaré, Wittgenstein is suggesting that in setting out the formal language by means of which logic and arithmetic will be analyzed, Frege, Whitehead, and Russell have already invoked the use of a specifically mathematical procedure, that of operation.¹⁵ Unlike Poincaré, he is denying that this shows that mathematical procedures (e.g., mathematical induction) require special mathematical insight or intuition (cf. 6.233). For on his view, in both mathematics and logic, *calculation* is fundamental.

In both the *Principles of Mathematics* and in *Principia Mathematica* (aided by Whitehead), Russell himself reverts to old-fashioned talk of “operations” (applying it to, e.g., addition and multiplication), but when and only when he

is bracketing his logicistic analysis of number and arithmetic, when and only when he is abstracting away from what is symbolized or denoted, and focusing on the symbols alone.¹⁶ Of course Frege and Russell never apply the notion of *operation* either to the logical connectives or to logical inference. This is the hallmark of their philosophies: logic, on their view, is a universal science, something more than an artful manipulation of uninterpreted signs, something more than a *mere calculus*. In logic, they hold, we express thoughts and propositions, and our words have meaning. Once the logicist definitions of the natural numbers are in place and the basic theorems about the natural numbers are derived from logical laws via these definitions, there is no excuse other than convenience for supposing that we may continue to hold meaning or interpretation in abeyance when we speak of mathematics. The notions of *calculus* and *operation* do not really apply, except misleadingly, either to logic or to mathematics. Since mathematics is a branch of logic, it is part of a universal science, and in it there is genuine truth, genuine content expressed at each step of a deduction. For Frege, the *Begriffsschrift* is not a mere uninterpreted formalism. It is a genuine (formalized) language. Similarly, for Whitehead and Russell (what Wittgenstein calls (e.g., at 3.325)) the *Begriffsschrift* of *Principia Mathematica* is not a mere uninterpreted formalism. These *Begriffsschriften* are taken by their framers to express thoughts and meanings, to involve us in the recognition of truth. Insofar as mathematics enjoys the same status as logic, for Frege and Russell it is no calculus.

It is, however, a remarkable historical irony that in spite—or perhaps even because—of their philosophical commitments, Frege, Russell, and Whitehead provided us with quite formidable tools for forging a mechanistic conception of logic and mathematics. Their formalisms—as opposed to what they conceived their formalisms to express—are a means by which thought may apparently be made irrelevant, not of course to the discovery of formal derivations, but to the formal assessment of a logical structure as indeed a derivation in accordance with the rules of the system. For the remarkable thing about the new logic is that transitions from one step to the next in a formal derivation may be checked and even generated by a machine, without regard to the meanings of the various expressions at work in the proof it may be said to represent. Both Frege and Russell attacked the idea that thinking is calculating. Both attacked the idea that mathematics and logic involve a mechanical manipulation of signs. Yet, arguably, what they *did* was to produce what might be called the first computer program, the first truly mechanical way of handling the patterns of inference characteristic of logic and mathematics. Wittgenstein characterizes mathematics in the ordinary terminology of his day to suggest that Frege and Russell, who aimed to bring arithmetic under the auspices of their logic, give the wherewithal for a philosopher to claim instead that their logic is just another sort of calculus. Wittgenstein was not alone in suggesting a mechanical metaphor for characterizing the logic of *Principia Mathematica*—at about the same time C. I. Lewis (1918, pp. 354ff.), quickly followed by E. L. Post (1921), suggested it too¹⁷—but Wittgenstein made the move with the philosophies of Frege and Russell as self-consciously constructed targets of his criticisms, the only immediate student of both to do so. In the *Tractatus* Wittgenstein revitalizes the standard algebraic talk of “operations”, hurling the old-fashioned language back at Frege and Russell—its greatest critics—while at

the same time transforming it to meet the Frege-Russell demand that the logical form of number words be accounted for.

III. Differing Grammatical Analogies between Arithmetical Terms and Sentences

Wittgenstein writes that “in order to recognize the symbol in the sign we must consider the sign’s significant use [*sinnvollen Gebrauch*]” (3.326)—that is, its use in expressing propositions, true or false. A proposition is itself a “symbol” or “expression” as opposed to a mere sign; it expresses a sense, truly or falsely (3.31). We can separate the sign from the symbol—the propositional sign from the proposition, or the numeral from the number—but neither a particular sign token (e.g., *two, man*), nor that token’s type (e.g., “two”, “man”) can be considered expressions apart from considering their contributions to the articulation of propositions on given occasions. Different (types of) signs may be used to express the same symbol: I might express the same proposition in either English or German. Conversely, the same (type of) signs may be used in different ways, to express different symbols or expressions, as in (the ambiguous) configuration of signs “Green is green” (3.321ff.). Yet a sign is also a symbol or expression. A propositional sign, for example, is just that which is perceptible *in* the proposition, *in* the symbol, that which in use *exhibits* the symbol (3.32). Wittgenstein’s way of speaking could be cleaned up and systematized: propositions might be spoken of as propositional signs appropriately used in thinking, while propositional signs might be spoken of as that which is perceptible *in* such uses. But he prefers to rest with a constant ambiguity in his remarks, inviting, for dialectical purposes, a metaphysical misreading. This is part of his funhouse, part of his subversive treatment of Frege’s and Russell’s terminology for their most basic logical notions. By an “expression” or “symbol” he means, not merely a (part of a) sign, but a sign that aids in expressing a sense. This makes it sound as if symbols, expressions, propositions, and senses are entities shown or reflected in our uses of signs (cf. 2.22). But I believe that Wittgenstein himself draws no hard-and-fast distinction between what is articulated and articulation, between what is exhibited by a sign and the sign doing the exhibiting, between thought and thinking. Symbols and expressions are aspects or patterns of the uses of signs in thinking; propositions, senses, and thoughts are nothing but appropriate *uses* of particular propositional signs, and propositional signs are perceptible facts on which we operate (3.12, 3.14).

The apparent relation of the signs to what they express is “internal” or “formal” or “operational”. For Wittgenstein, thinking goes on, but there are no (imperceptible, abstract) thoughts or propositions that are somehow mirrored or shown in propositional signs. A thought is nothing but an applied (*angewandte*), bethought (*gedachte*) propositional sign (3.5); it expresses itself perceptibly in thinking (3.1). This Frege did not and would not have said (cf. Frege, 1979, p. 206; 1984, pp. 159–60, 354). Wittgenstein thus exploits the ambiguity of the German word *Satz* (sentence/proposition) by inflicting an ambiguity on the words *Ausdruck* (expression) and *Symbol*. Sometimes he speaks of these as linguistic categories, types of sign. Sometimes he speaks of them as what signs in these categories reflect. Inevitably he makes remarks that appear to reify proposi-

tions, symbols, expressions, thoughts, and senses. But this is part of the reason Wittgenstein takes his own remarks to be—if construed as propositional signs—nonsensical (*unsinnig*), potentially misleading. Although one might try to say that thought expresses itself perceptibly in a propositional sign (3.1)—as if there are thoughts or senses that are shown—one must also see that thinking is nothing but a way of using a configuration of sensibly perceptible signs, in the first instance nothing but the appropriate use of a sentence-sense (*Satz-Sinn*) to project a state of affairs (*Sachlage*) by operating with a propositional sign (3.11).

Wittgenstein's refusal to sharply distinguish use from mention, linguistic from extralinguistic, is an index of how deeply he rejects Frege's conception of the kind of articulateness that sentences have. As has often been emphasized, by comparing propositions with pictures, Wittgenstein sharply distinguishes propositions from names, as Frege had not. After 1891 Frege explicitly assimilated the grammatical structure of *all* sentences to that of his conception of arithmetical terms (e.g., “ 2^3 ”, “ $8 + 14^4$ ”).¹⁸ He thereby came to construe all sentences as (functionally complex) names of truth-values, just as he had always construed arithmetical terms as (functionally complex) names of numbers. In this way Frege came to apply to sentences his post-1891 distinction between the *Sinn* and the *Bedeutung* of a name. Frege's analysis of number in his *Grundlagen der Arithmetik* in no way depended upon his lumping sentences and names together in one logico-grammatical category. But the lumping is a natural outgrowth of his original (*Begriffsschrift*) idea that he could use what he took to be the notion of *function* (naturally applicable to the grammatical structure of arithmetical terms) to account for the logical structure of all sentences—including, of course, extramathematical ascriptions of number and sentences of arithmetic—in a uniform way.

In the *Tractatus*, I claim, Wittgenstein does not wholly reject Frege's notion that the grammatical structures of (at least some) sentences are analogous to those of arithmetical terms. He grants Frege's analogy but turns it against Frege, uses it to abandon the assumptions Frege always had made, that numerals and arithmetical terms are names and that arithmetical sentences (arithmetical equations) express (functionally complex) propositions. Wittgenstein's unwillingness to apply a univocal notion of *function*—his sharp distinction between functions and operations—is thus crucial for understanding how he differs from Frege (and from Russell) on the nature of the proposition.¹⁹ For to try to break the hold of their conceptions of logic, he reconceived the relation between the logic of the sentence and the logic of the arithmetical term.

Wittgenstein's anti-Fregean analogy between propositions and numbers has several facets.²⁰ The first concerns notation and the grammar of elementary propositions. The decimal Arabic notation in which arithmetic is ordinarily carried out is, as a symbolism, a *systematic notation*, a system of picturing or representing numbers. In this system, a fixed alphabet is set out, and spelling rules are set down, so that just ten separate numerals may be used to express any number whatsoever. Configurations of these numerals in immediate juxtaposition represent (or “picture”) natural numbers through column positionality—positionality that makes any numeral arithmetically (operationally) articulate, even, for example, the numeral “2”. Such configurations (e.g., “372”) may be calculated with, expanded by means of arithmetical operation signs in a systematic way (“ $300 + 70 + 2$ ”, “ $(3 \times 10^2) + (7 \times 10^1) + (2 \times 10^0)$ ”) in order to show what they express.

Now, in a similar way, in the *Tractatus* Wittgenstein construes that portion of our language used to express propositions as consisting in the first instance of a fixed stock of names, each of which has, in the context of (positioned within the expression of) a genuine proposition, a Bedeutung. The logically simple names of the *Tractatus* are analogous to numerical digits, and their modes of possible combination with one another—the function expressions—like alphabetical spelling rules, like the positionality convention of the decimal notation. Vacuous names and redundant names (differing signs with the same Bedeutung) are idealized away, as actually happens in the decimal notation. Elementary propositional signs are configurations of names that, in immediate juxtaposition, express functionally articulate propositions (4.22), just as configurations of numerals in the decimal notation are arithmetically articulate. Any such elementary proposition may be expanded by means of (truth) operations to *show* what it expresses (“p” by “p v p”, “~pɔp”, and so on), just as any numeral may be expanded by means of arithmetical operations into polynomials of powers of ten. Of course the analogy is not perfect, for the alphabetical rules of the decimal system are fixed once and for all via column positionality, whereas the alphabetical rules of a language capable of being used to express propositions are not so rigidly fixed.

That we are using the decimal system with the numerals “1”, “2”, “3” . . . (and not some other), and *that* the use of this notation is adequate to express every natural number, come out in our uses of the notation, in the ways we operate with the numbers, both within and outside of arithmetic. But we cannot use the decimal system in and of itself to *say* that these things are so. Similarly, in the *Tractatus* Wittgenstein is suggesting that we cannot *say* within our language (of propositions) that our names have the Bedeutungen they do, or that our (grammatically well-formed) sentences are adequate to express all propositions, thoughts, senses; these, too, are matters shown in our uses of sentences, in the ways in which we operate with them systematically. It might seem that Wittgenstein’s distinction between what can be shown and what can be said (4.1212) may be escaped or defeated by bringing in the notion of a metalanguage, or a hierarchy of languages. This was what Russell suggested and Carnap thought.²¹ But, as we shall see, Wittgenstein’s Tractarian notion of operation portrays any such move as already surrendering the philosophical quest for an overarching function-argument scheme in terms of which we may represent the logic of all terms and sentences, both mathematical and nonmathematical.

I have just argued that, like Frege, Wittgenstein draws an analogy between the way in which numerals may be conceived to signify and the way in which elementary propositional signs may be conceived to signify—though I have also insisted that for Wittgenstein, unlike for Frege, the point of the analogy is to bring us to see that neither numerals (qua pictures of numbers) nor elementary propositions (qua pictures of states of affairs) play the same role as digits or names do. A further, deeper aspect of Wittgenstein’s recasting of Frege’s analogy concerns the structure of logically complex, molecular sentences. For Wittgenstein, as for Frege, the grammatical structure of arithmetical terms (e.g., “1+4”, “ 5^2+3^{4+2} ”) is analogous to that of molecular propositional sentences. But for Wittgenstein, unlike for Frege, this is so only insofar as each sort of sign reflects, not functional, but operational complexity—arithmetical operations in the case of arithmetical terms and logical operations in the case of molecular sentences. Wittgenstein constructs

a parallel between the (arithmetical) operation by means of which a number is constructed from another and the (logical) truth-operation by means of which a new proposition is constructed from another.²²

Operation signs—number words and truth-operation signs—figure in the articulation of propositions. But the occurrence of a sign for a particular operation in a sentence is never essential for characterizing the sense, if any, of the sentence (5.25). The capacity of operations to be iterated and combined with one another, their specific mutual interplay, is what any adequate notation for them must capture; arithmetic and truth-operational logic *demand* systematic notations for their articulation. This capacity for iteration—for “self-reference”, if you like—is what sharply distinguishes operations from functions on Wittgenstein’s view. If, for example, we tried to make a particular kind of structural configuration of names express the operation of negation (e.g., writing a propositional sign upside down, or writing it in red), we would not have succeeded in devising a notation capable of expressing every proposition of our language unless we could see how this particular way of expressing negation could be iterated and combined with disjunction, conjunction, and so on. Like operation signs in the language of arithmetic, truth-operation signs form a system, a formally integrated network, as names and (material) function expressions do not. When Wittgenstein remarks that elementary propositions are logically independent of one another, he means to say that names and function expressions do not form such a systematically interconnected network, as a system of operation signs always does.

Wittgenstein attempts to articulate this distinction in several ways. Operations may be iterated to form significant expressions (as in “ p , $\neg p$, $\neg\neg p$ ” (propositional signs) or “ $2, 2 \times 2, 2 \times 2 \times 2$ ” (numerals)), but a name or function expression can never be so iterated (neither “HarryHarry is a man” nor “is a man Harry is a man” is a significant expression). This unbounded capacity for iteration goes hand in hand with the systematic interconnectedness of signs in a notation capable of expressing operations. Neither names nor function expressions can ever cancel out the contributions of other names or function expressions to the articulation of a proposition. But an operation sign can do so. Thus, for example, in the tautology $p \vee \neg p$ the logical operators for negation and disjunction cancel out the expressive contributions (the senses) of p and $\neg p$. It is intrinsic to operations that their effects on the articulation of propositions can be captured by using an alternative configuration of operations belonging to the same system. “ 2 ”, “ $+$ ”, “ $2+4$ ”, “not”, “or”, “all”, and “some” are all operation signs. Wittgenstein remarks that such signs are nothing but punctuation marks, like commas or semicolons or periods (5.4611, 1994, vol. 1, p. 41), because any particular configuration of such signs used to express a proposition can always be systematically eliminated from its expression, systematically replaced with another equivalent configuration, and the same thing still said. In the system of arithmetical operations, a single number can be expressed in an unlimited number of ways, each systematically interconnected with every other (“two” may be expressed by “ $1+1$ ”, by “ $0+2$ ”, “ $3-1$ ”, and so on). This is essential to the number’s numberhood, to its operational character. Every proposition ascribing a particular number may be expressed in an unlimited number of ways (the “three” in “three apples” may be expressed by “ 3 ”, “ $2+1$ ”, “ $21/7$ ”, “ $4-1$ ”, and so on, or even simply shown in the use of three distinct names; see section IV below). Such a system of alternative ways of ex-

pressing ourselves is not, on Wittgenstein's view, characteristic of our uses of names or function expressions.

Differently put, Wittgenstein deems it essential to the expressive power of our language that differing configurations of truth-operational signs can be used to express the very same proposition, and, conversely, that no single configuration of the standard truth-operational signs is ever necessary to the expression of any proposition. It is essential to any proposition p that p may be expressed by " $p \vee p$ ", " $p \vee (q \& \neg q)$ ", " $(q \vee \neg q) \vee p$ ", and so on (5.515). If we focus just on the possibilities of the truth and falsity of each of these propositions based on the possibilities of truth and falsity of their components p and q , each way of expressing p says exactly the same thing, expresses, on Wittgenstein's view, the same sense (*Sinn*). Indeed, each mode of expressing a sense presupposes all its other modes of expression. What Wittgenstein calls a "logical form" is that which is common to all these different ways of expressing the same proposition, that which is essential to the expression of its sense (cf. 5.42, 5.441). This is something we *show* in our use of a system of operations, not something we *say* in a proposition. Logical form in Wittgenstein's sense cannot be expressed through a single configuration of truth-operation signs; it is not a genuine property of sentences to be formalized with reference to any unique (truth-operational or quantificational) structure. Logical form emerges instead through a system of operations used to express propositions. Operation signs are thus construed by Wittgenstein—as they were by the algebraists of his day—as variables of a certain special sort (5.24ff.).

IV. Propositional Ascriptions of Number in the *Tractatus*

In Wittgenstein's denial that numbers are objects and that arithmetical terms are names, there is something reminiscent of Russell, who also resisted both Frege's assimilation of sentences to names and Frege's construal of arithmetical terms as names. Neither Frege's *Grundgesetze* nor Russell and Whitehead's *Principia Mathematica* postulates any primitive arithmetical terms; that is the point of the program of reducing arithmetic to logic. But unlike Frege, Russell construed arithmetical terms as definite descriptions, hence, as expressions contextually eliminable, via the theory of descriptions, in favor of certain canonically related, functionally complex proposition expressions. For Russell, after 1905, the logistic reduction consists in explicitly defining (nondenoting except in context) "descriptive" terms for mathematical entities and functions in terms of (genuinely denoting) expressions reflecting propositional functions.²³ As he writes in the *Principia*:

The functions hitherto considered, with the exception of a few particular functions . . . have been propositional, i.e., have had propositions for their values. But the ordinary functions of mathematics, such as x^2 , $\sin x$, $\log x$, are not propositional. Functions of this kind always mean "the term having such and such a relation to x ." For this reason they may be called *descriptive* functions, because they *describe* a certain term by means of its relation to their argument. Thus " $\sin \pi/2$ " describes the number 1; yet propositions in which $\sin \pi/2$ occurs are not the same as they would be if 1 were substituted for $\sin \pi/2$. This appears e.g. from the proposition " $\sin \pi/2 = 1$," which conveys valuable information,

whereas “1=1” is trivial. Descriptive functions, like descriptions in general, have no meaning by themselves, but only as constituents of propositions. (*Principia Mathematica*, *30)

Russell thus takes his theory of descriptions to obviate the need for an account of the informativeness of denoting phrases in terms of what they denote. It thereby obviates the need for anything like Frege's (post-1891) attempt to account for the informativeness of nontrivial identity statements (e.g., “ $2^2 = 4$ ”) through a distinction between the *Sinn* and the *Bedeutung* of names and definite descriptions. It allows Russell to introduce new (arithmetical) terms into his system without extralogical ontological commitment, for example, without independent commitment to the existence of classes.

Like Russell, Wittgenstein rejects Frege's (post-1891) distinction between *Sinn* and *Bedeutung*, but not on the basis of the theory of descriptions (6.232ff.). Wittgenstein does allude to the theory of descriptions in the *Tractatus* (3.24), but he never proposes applying it as it stands. Russell's analysis relies on using identity, which Wittgenstein will not countenance. As we shall see, Wittgenstein does fashion an alternative treatment of descriptions for what he would consider to be genuine propositions. But unlike Russell, Wittgenstein never suggests applying this treatment to arithmetical expressions. For he rejects, both Russell's treatment of arithmetical terms as functionally complex expressions replaceable by propositional signs, and Russell's treatment of arithmetical sentences as propositional signs. Wittgenstein does not take there to be any one logico-syntactic category of “description” in the way Russell does. In *Principia Mathematica* a key constraint on the logistic reduction is the ability of the axioms to support derivations of such (typically ambiguous) propositions as “There is a unique square of three.” For Wittgenstein, “there is a unique square of three” is, like “there is only one number one,” nonsensical (*unsinnig*) if construed as an ascription of number like “there is only one man in the room” (4.1272). So construed, such expressions vacillate or “dither”, to use a phrase of Cora Diamond's, between treating an expression as a variable and treating it as a constant (Diamond, 1997, pp. 78ff.) This is simultaneously to assimilate numerals both to function expressions and to names. Frege and Russell both tried to use phrases like “the square of three” and “there is only one number one” to articulate constraints on what would count as a proper analysis of arithmetic. Wittgenstein takes the fact that Frege and Russell's logic make these look like genuine (constituents of) propositions, like any (logical part of a) proper ascription of number, to be a mark against their analyses.

As is evident from ordinary first-order quantification theory, number words are not needed to articulate propositions ascribing particular finite natural numbers—if, that is, one is prepared to use identity. For “there are at exactly three women” write “ $(\exists x,y,z)(Wx \ \& \ Wy \ \& \ Wz \ \& \ x \neq y \ \& \ y \neq z \ \& \ x \neq z) \ \& \ \sim(\exists x,y,z,w)(Wx \ \& \ Wy \ \& \ Wz \ \& \ Ww \ \& \ x \neq y \ \& \ x \neq z \ \& \ x \neq w \ \& \ y \neq z \ \& \ y \neq w \ \& \ z \neq w)$ ”; for “b follows a by two in the R-series” write “ $(\exists x,y,z)(xRy \ \& \ yRz \ \& \ x \neq y \ \& \ y \neq z \ \& \ x \neq z \ \& \ x=a \ \& \ z=b)$ ”. It was in order to be able to speak more generally about (ascriptions of) *any*, *some*, or *all* finite number(s) n that Frege's and Russell's (second-order) analyses were proposed. Their analyses lean essentially on the use of identity. For Frege analyzed ascriptions of cardinal number (“the number of Fs is n ”) in terms of equinumerosity (“there are just as many Fs as Gs”), which he in turn analyzed by means of the notion of

a *one-to-one correlation*, hence, in terms of identity. Furthermore, he took number words to be substantivals: ascriptions of cardinal number, he argued, should be construed as identities (e.g., “there are two cups on the table” as “the number belonging to the concept *cup on the table* = 2”). The Frege-Russell definitions of the individual natural numbers also rely on identity (Frege defines “1” as “the extension of the concept *is a concept gleichzahlig to the number zero*”; Russell defines it as “the class of all 1-membered classes”).

In 1923 Wittgenstein wrote the marginal remark “identity” in Ramsey’s copy of the *Tractatus* beside 6.031 (“the theory of classes is altogether superfluous in mathematics”). Identity is indeed key to his anti-Fregean, anti-Russellian treatment of ascriptions of number in the *Tractatus*.²⁴

- 5.53 Identity of the object I express by identity of the sign and not by means of a sign of identity. Difference of the objects by difference of the signs [. . .].
- 5.5303 Roughly speaking: to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing [. . .].
- 5.531 I write therefore not “ $f(a,b).a=b$ ”, but “ $f(a,a)$ ” (or “ $f(b,b)$ ”). And not “ $f(a,b) \cdot \neg a=b$ ”, but “ $f(a,b)$ ”.
- 5.532 And analogously: not “ $(\exists x,y) \cdot f(x,y).x=y$ ”, but “ $(\exists x).f(x,x)$ ”; and not “ $(\exists x,y).f(x,y).\neg x=y$ ”, but “ $(\exists x,y) \cdot f(x,y)$ ”.
(Therefore instead of Russell’s “ $(\exists x,y).f(x,y)$ ”: “ $(\exists x,y) \cdot f(x,y) \cdot v \cdot (\exists x) \cdot f(x,x)$ ”).
- 5.5321 Instead of “ $(x).fx \supset x=a$ ” we therefore write e.g.
“ $(\exists x) \cdot fx \cdot \supset fa: \neg(\exists x,y) \cdot fx \cdot fy$ ”.
And the proposition “only one x satisfies $f()$ ” reads:
“ $(\exists x).fx:\neg(\exists x,y).fx.fy$ ”.
- 5.533 The identity sign is therefore not an essential constituent of logical notation [*Begriffsschrift*].
- 5.534 And we see that apparent propositions [*Scheinsätze*] like: “ $a=a$ ”, “ $a=b \cdot b=c \supset a=c$ ”, “ $(x).x=x$ ”, “ $(\exists x) \cdot x=a$ ”, etc. cannot be written in a correct logical notation [*einer richtigen Begriffsschrift*] at all.

Here Wittgenstein appears to be proposing certain analyses—laying down conditions on what is to count as a “correct logical notation”—using Russell’s notation. But his remarks undercut the possibility of his formulating a formal system in either Russell’s or Frege’s sense. For Wittgenstein proposes to formalize the notions of *identity* and *numerical individuation* by means of an anti-Russellian, anti-Fregean reading of the variable. Wittgenstein is not defining these notions—as one might define a name or function expression—but treats them as operations by absorbing them into the interpretation of the notation’s form.²⁵

For consider Wittgenstein’s proposal for writing in logical notation an ascription of number such as “there is exactly one F ”: $(\exists x) \cdot Fx \cdot \& \cdot \neg(\exists x,y) \cdot Fx \cdot Fy$. This does not succeed in saying what Wittgenstein takes it to say unless we interpret it so that the range of “ y ” is restricted to names that differ in Bedeutungen from whatever names might be taken to instantiate the second existentially quantified “ x ”. The ranges of significance of these two variables are “exclusive”, to use Hintikka’s phrase (Hintikka, 1956). Now Hintikka distinguished two sorts of

ways in which we might interpret Wittgenstein's "exclusive" construals of the variable: the "weakly exclusive" reading would apply only to variables within the scope of a (sequence of) quantifier(s), whereas the "strongly exclusive" reading would demand that every distinct bound variable throughout the whole sentence have a range restricted by all the previously occurring quantifiers. The latter reading might take various forms (Hintikka, 1956, p. 230 n. 11). For example, if the same variable letter is bound by different quantifiers in different parts of one sentence—as "x" is in the example above, and also in 5.5321—one might in effect read it as a different letter in its two differing bound occurrences, and hence, as being instantiable only by names with differing Bedeutungen. Alternatively, one might require that the same letter, even if bound in different occurrences by distinct quantifiers, be instantiated by names with the same Bedeutung throughout the sentence as a whole. (On the weakly exclusive and the first of the strongly exclusive readings, the restrictions on instantiation of distinct bound variables begin over again from scratch after the sign for conjunction in the above formula; on the second of the two strongly exclusive readings, these restrictions would apply throughout the whole sentence, and the range of the second bound occurrence of "x" would be restricted to the particular instance picked out for the first occurrence: the existential quantifier would function like a descriptive phrase, or constant.) On all these readings, bound variables occurring within sequences of more than one quantifier (whether in a subformula or in a sentence) are interpreted as having ranges of significance that are restricted as we move from left to right through the sentence. In the second half of 5.5321, the variable "y" expresses "any y except the previously chosen value for x," however we interpret the phrase "x".

Thus we should ask ourselves how Wittgenstein would interpret a sentence such as " $(\exists x)Fx \ \& \ (\neg\exists y)Fy$ " (equivalently, " $(\exists x)Fx \ \& \ (y)\neg Fy$ "). This is a contradiction, both in standard logic and on the weakly exclusive interpretation of the variable, but is not a contradiction according to either one of the strongly exclusive readings of the variable. For on the latter reading, " $(\exists y)Fy$ " (or " $(y)\neg Fy$ ") is taken to mean "every y but the previously chosen x." Now Hintikka has argued that Wittgenstein should be read as having advocated a weakly expressible interpretation of the variable in the *Tractatus* (Hintikka, 1956, p. 230). But it seems to me that Wittgenstein's remarks in the *Tractatus* are insufficiently precise to decide this interpretive question. This in itself, if true, is significant; it tells us something important about Wittgenstein's philosophical aims. For it indicates that on Wittgenstein's way of construing the variable—however we might try to make it precise—the application of logic to sentences that express genuine propositions takes priority over the formal systematization of pure logic (i.e., logic which consists of sentences that do not express propositions, but instead contradictions and tautologies). To take just one example: Wittgenstein sharply distinguishes between the role of the *Satzvariable* "x loves y" and that of the *Satzvariable* "x loves x". "Harry loves Harry" expresses a proposition falling within the range of the latter, but not the former. Although "x loves Harry", "Harry loves x", "y loves Harry", and "Harry loves y" are functional expressions that contribute to articulating the proposition that Harry loves Harry, "x loves y" does not. Thanks to the restrictions on instantiation Wittgenstein imposes on the variable, on his view Harry's loving Harry can have nothing to do, logically speaking, with the proposition expressed by " $(\exists x)(\exists y)(x \text{ loves } y)$ ", as it does for both Frege and for Russell. For Wittgenstein takes the latter to say that someone

loves someone *else*, someone different from him- or herself. For Wittgenstein, “ $(x)(y)(fxy \& fyx) \supset (x) fxx$ ” does not express a truth of logic, as it does in the logic of Frege and Russell. Instead, it expresses a proposition. If true, it tells us something informative about the relation *f*.

Thus Wittgenstein's remarks drastically limit the usual ways in which quantification theory is presented.²⁶ Indeed, this is evidence that a smooth-running system of pure logic holds no interest for Wittgenstein. He wishes to explore patterns in the ways in which differing kinds of sentence contribute to our expressive capacities, but he has no interest in presenting a systematic way of deriving patterns of quantificational structure from other patterns of quantificational structure.

5.5321 (quoted above) only suggests a notational technique for representing ascriptions of finite cardinal number without identity in those cases in which a (genuine) function expression is involved. It thereby provides an alternative to Russell's treatment of definite descriptions, but only in these cases (for “the so-and-so” write “there is exactly one so-and-so”; cf. White, 1979, pp. 164ff.). The notational proposal presupposes, however, that cardinal individuation is given immediately with the forms of elementary propositions. For what Wittgenstein has done is to build cardinality and ordinality directly into their forms of representation. That is, he takes numerical individuation as a primitive notion. Nothing could be further from Frege and Russell. It requires Wittgenstein to reject the Frege-Russell claim that number words are to be construed as constants or substantivals, either as names or as function expressions.

It should come as no surprise that Wittgenstein's restrictions on (what we would call) instantiations of quantifiers reflect aspects of the representational form of elementary propositions. This is one hallmark of his treatment of logic, that all propositions may be conceived to result from successive application of his operator *N* to the elementary propositions (6; see section V). Since number words figure in genuine propositions—in ascriptions of number—their expressive power must also so result. But how is this possible? Answer: If “*a*” and “*b*” are names with distinct Bedeutungen, then the number two is part of the form of representation (*Form der Darstellung*) of any proposition in which they figure, and two is itself represented in the form of the proposition. Wittgenstein conceives of numerals and number words as abbreviations of such depicting features of the symbolism, as forms. Such depicting features do not play the grammatical role that names and functional expressions do, but are variables, operation signs, and thus in a certain sense indefinable (Wittgenstein, 1973, p. 224).

Take, for example, the elementary proposition that *aRb*. The propositional sign “*aRb*” may be used to *say* that *a* bears *R* to *b*. The distinctness of these names in and of itself *shows* (though it does not *say* that there is) the possibility of there being at least two different *R*-relata (Wittgenstein 1997, p. 126; 1994, vol. 1, p. 7). The elementary proposition *aRb* also *shows* the possibility of there being at least *one R relatum* of (the object) *a* distinct from *a*, the possibility of *b*'s being the relatum under *R* of at least *one* distinct object, and the possibility of there being at least *two* distinct objects whose names are “*a*” and “*b*”. These “possibilities” are not ineffable, nonfactual possibilities. They are instead reflections of the use of the propositional sign to say what it says, true or false (that *aRb*).²⁷ Such possibilities are *shown* by *aRb*'s being in the range of the *Satzvariable* “*xRy*”, the variable Wittgenstein's notational proposal would use to articulate the ascription

of number $(\exists x,y)xRy$, which *says* that there are at least *two* distinct R-relata.²⁸ For “ xRy ” does not range over any proposition of the form “ xRx ” according to Wittgenstein’s restricted use of the variable.

That number is built as a primitive into the “form” of the elementary propositions may be seen in a different way by considering Wittgenstein’s differences with Frege and Russell over the nature of identity. At *Principia* *13, “ $x = y$ ” is defined by what Russell takes as a version of Leibniz’s law of the identity of indiscernibles: “ $(\phi)(\phi x \equiv \phi y)$ ”, where “ ϕ ” is understood to range over all predicative functions. To this Wittgenstein objects that “Russell’s definition of ‘=’ won’t do; because according to it one cannot say that two objects have all their properties in common. (Even if this proposition is never true, it is nevertheless *significant*.)” (5.5302). Suppose that we take a language with the names “a” and “b” and the function expression “ $f(x)$ ”, and suppose, too, that the *Bedeutungen* of “a” and “b” are distinct. In this language there would be two elementary propositions, $f(a)$ and $f(b)$. Each would say something about a, and the same thing about b, that is, something about *two* fs. It would make sense to say in this language that two different fs had all their properties in common. But Russell’s definition of identity systematically rules out any such language: he claims that from the supposition that a and b co-satisfy the propositional function $f(x)$, we can *deduce* that $a = b$. As Russell would later write in objecting to Ramsey’s use of the *Tractatus* treatment of identity, Wittgenstein takes numerical diversity to be a “primitive idea” (Russell, 1996, p. 108). For Wittgenstein, Leibniz’s law expresses a proposition: if it happens to be true, it can only be contingently so, and its making sense at all depends on a range of elementary propositions already in use—a range that presupposes numerical diversity and unity from the outset.

Wittgenstein’s suggested interpretation of the Russellian notation is adequate for expressing all those propositions that ascribe a finite cardinal number via a genuine material function, so long as sufficiently many names with distinct *Bedeutungen* are available in the language. If there were, for example, only two names in the language with distinct *Bedeutungen*, then the language would still contain the grammatical forms of the number words by way of the arithmetical operations that Wittgenstein distills from general form of operation (see section VI), and we would still have arithmetical terms and all the sentences of arithmetic. But we would have no means of using arithmetical terms to construct propositions ascribing numbers greater than two via material functions.²⁹ Furthermore, Wittgenstein’s proposal does not suffice to set out in advance a way of saying that there is *some* number of ϕ ’s for a (genuine, material) function expression ϕ , much less that there are n objects (without functional qualification). For Wittgenstein, to try to say such things would be to try to construe number words as names or function expressions. He prefers to rest content with a specification of the numbers that allows us to write down a *Satzvariable* for the formal series of propositions “there is one ϕ , there are two ϕ ’s, there are three ϕ ’s, and so on,” up to the point where names with different *Bedeutungen* are exhausted. This is not an explicit definition of the kind Frege and Russell demanded. But, as we shall see, Wittgenstein did not think his definition worse off than theirs on that score.

This will perhaps become clearer if we focus on the basic logical notion of the *variable*, a notion essential to both Frege’s and Russell’s articulation of their respective notions of *function*. We have seen that Wittgenstein insists that any ele-

ment of a systematic notation—any operation sign—is expressed by a variable, and in so insisting he follows at least verbally the manner of speaking he inherits from the nineteenth-century algebraists. Conversely, he treats variables as themselves operation signs. Thus he extends the algebraical notion of operation back across all the most basic notions of Frege's and Russell's (respective) philosophies of logic. Since it is by means of variables that Frege, Russell, and Wittgenstein articulate categorial (logical) distinctions among kinds of signs (e.g., operation signs versus function expressions versus names), we should expect that fundamental differences among these philosophers about the aims, scope, and character of logical distinctions would be reflected in differing conceptions of the variable. That this is so is the argument of the following section.

V. Variables as *Satzvariablen*: Operations and the Limits of Expression

For Wittgenstein—unlike for Frege, unlike for contemporary logicians—variables are *not* letters of the alphabet. They are what he calls *Satzvariablen*.

- 3.311 An expression presupposes the forms of all propositions in which it can occur. It is the common characteristic mark of a class of propositions.
- 3.312 It is therefore presented by the general form of the propositions which it characterizes.
And in this form the expression is *constant* and everything else *variable*.
- 3.313 An expression is thus presented by a variable whose values are the propositions which contain the expression.
Every variable can be conceived as a *Satzvariable*.
(Including the variable mane [*der variable Name*]). (my emphasis)

“The variable name”—for example, “x”—appears in, for example, “x is a man.” One might suppose that the variable name “x” marks out a form, namely, the class of all names, those elements from which all propositions are composed (3.202). But on its own, “x” does not mark out or range over anything. On its own it is the sign (not a symbol) for “the pseudo-concept *object*” (see 4.1272)—which is to say that in a concept-script, it has no significance on its own. Differently put, “x” makes no contribution (essential or otherwise) to the articulation of any proposition, but only serves to help mark out a class of propositions when it is written beside a function expression (or variable function expression) in the context of a *Satzvariable*. Then it aids in marking out a range (or category) of expressions—that is, *names*. To Frege it makes sense to ask what sort of entity a variable is: it is a letter of the alphabet used to indicate argument places. To Wittgenstein it makes no sense to pose or to answer this question.

The *Tractatus* remarks about *Satzvariablen* are much more closely linked to Russell's talk about the variable than to Frege's, and specifically to what Russell calls “propositional variables” and “propositional functions” in *Principia Mathematica*. An expression like “x is a man” is, according to Whitehead and Russell, “such that it becomes a proposition when x is given any fixed determined mean-

ing" (1910, p. 14), and the range or collection of values of a propositional variable consists "of all the propositions (true or false) which can be obtained by giving every possible determination to x " (1910, p. 15). For Russell, as for Wittgenstein, an expression like " x is a man" is not on its own either a propositional sign or a sign for a propositional function, and each bound or "apparent" variable occurring within the scope of a quantifier has a determinate but restricted "field" or range.

Russell allowed each propositional variable to be used to refer "ambiguously" to "an arbitrary member of the class of propositions it demarcates" (*Principia*, vol. 1, pp. 14–15). Thus he took as a primitive logical idea the notion of asserting a linguistic form with a "real" variable in it, calling this "ambiguous assertion" but insisting that it "cannot be defined in terms of the assertion of propositions" (*Principia*, vol. 1, p. 17). Russell held that the notion of ambiguous assertion is required in order to analyze mathematics, and specifically (1) reasoning by means of representative particulars (e.g., reasoning in geometry about a particular, arbitrarily chosen triangle in the course of proving a general theorem about all triangles of this kind); (2) reasoning by means of equations; and (3) expressions of general truths about numbers (1910, p. 18). The claims are tied, as we shall see, to Russell's difficulties with the theory of types and worries about vicious circles.³⁰

As early as 1913, in the "Notes on Logic," Wittgenstein had written that "there are no propositions containing real variables," for

those symbols which are called propositions in which "variables occur" are in reality not propositions at all, but only schemes of propositions, which only become propositions when we replace the variables by constants. There is no proposition which is expressed by " $x = x$ ", for " x " has no signification; but there is a proposition " $(x).x = x$ " and propositions such as "Socrates = Socrates" etc. (Wittgenstein, 1979, p. 98)

In books on logic, no variables ought to occur, but only the general propositions which justify the use of variables. (p. 100)³¹

At this stage of his thinking, Wittgenstein is anticipating the *Tractatus* conception of Satzvariablen. He dismisses Russell's notion that formulas with real variables must sometimes be asserted (as if they expressed genuine propositions) in logical deductions (cf. Hylton, 1997, p. 98 n.). A Tractarian Satzvariable simply goes proxy for (*vertritt*) the collection of propositions it determines; it cannot be used to name a class ("the theory of classes is superfluous"), nor can it be used to reach through to the Bedeutungen of any of the particular expressions over which it ranges (cf. 3.317). We operate *with* it systematically.

Wittgenstein preserves his fundamental distinctions between names, function expressions, and operation signs in the notation he proposes for Satzvariablen. If ' O ' stands for any operation sign and " $O' x$ " stands for "the result of applying operation O to x ," then the variable for the operation takes on a special form:

5.2522 The general term of the formal series $a, O'a, O'O'a, \dots$ I write thus: "[$a, x, O'x$]." This expression in brackets is a variable. The first term of the expression in brackets is the beginning of the formal series, the second the form of an arbitrary term x of the series, and the third the form of that term of the series which immediately follows x .

5.2523 The concept of the successive application of an operation is equivalent to the concept "and so on".

The "and so on", essential to the formal potential of any operation sign, is expressed by Wittgenstein's square bracketed Satzvariablen. These variables are proxies for collections of propositions ordered internally by the repeated application of an operation to its own result ad infinitum, beginning from a particular basis. They capture the systematic quality of the notation for any operation.

Wittgenstein specifies three sorts of ways in which a Satzvariable may go proxy for its values:

5.501 An expression in brackets [*Klammerausdruck*] whose terms are propositions I indicate—if the order of the terms in the bracket is indifferent—by a sign of the form " $\bar{\xi}$ ". " $\bar{\xi}$ " is a variable whose values are the terms of the expression in brackets, and the line over the variable indicates that it stands for all its values in the bracket.

(Thus if ξ has the 3 values P, Q, R, then $\bar{\xi} = (P, Q, R)$.)

The values of the variables are determined.

The determination is the description of the propositions which the variable stands for [*vertritt*].

How the description of the terms of the bracket expression takes place is unessential.

We may distinguish 3 kinds of description:

1. Direct enumeration. In this case we can simply set out its constant values instead of the variable.
2. Giving a function f_x , whose values for all values of x are the propositions to be described.
3. Giving a formal law, according to which those propositions are constructed. In this case the terms of the bracket expression are all the terms of a formal series.

"How the description of the terms of the bracket expression takes place is unessential": this means that Satzvariablen of the first and second kind may be used within square bracketed Satzvariablen. In this way square bracketed Satzvariablen may be used to collect together more than one formal series (or: formal series with multiple series of complexity). For example, if we let $\xi = (p, q)$, for some definite collection of propositions p and q, then " ξ " is a *Satzvariable* of the first kind (an enumeration). And " $[\xi, x, \neg x]$ "—or alternatively, " $\neg(\xi)$ "—collects together two formal series, namely, " $p, \neg p, \neg\neg p, \dots$ " and " $q, \neg q, \neg\neg q, \dots$ ". As for generality, Wittgenstein's generalized Sheffer stroke of joint denial—operator N —may be used to jointly deny all values of a Satzvariable by use of the bar notation, so that (to stick to our example) " $[\bar{\xi}, \bar{x}, N\bar{x}]$ " ranges over " $N(p, q), N(N(p, q)), N(N(N(p, q))) \dots$ ", or equivalently " $\neg p \& \neg q, \neg(\neg p \& \neg q), \neg\neg(\neg p \& \neg q) \dots$ ". If we let our basis be the Satzvariable f_x , then the Satzvariable " $[f_x, \bar{\xi}, N\bar{\xi}]$ "—alternatively " $N(\bar{f}_x)$ "—ranges over the propositions in the formal series " $N(\bar{f}_x), N(N\bar{f}_x), N(N(N\bar{f}_x)) \dots$ ", or equivalently " $\neg(\exists x)(f(x)), \neg(\neg(\exists x)(f(x))), \neg(\neg(\neg(\exists x)(f(x)))) \dots$ ".

This process of generalizing the square bracket notation by means of Satzvariablen within Satzvariablen culminates in what Wittgenstein calls "the general propositional form." Let 'p' stand for any elementary proposition. Then

6 The general form of truth-function is: $[\bar{p}, \xi, N(\xi)]$.
 This is the general form of proposition.

6.001 This says nothing else than that every proposition is the result of successive applications of the operation $N'(\xi)$ to the elementary propositions.

In the appendix I give several examples of how (what we would call) first-order quantificational schemata may be formally constructed through a finite number of applications of operator N to a basis (collection) of elementary propositions.³²

Wittgenstein had earlier remarked that all propositions are truth-*functions* of elementary propositions (5). Yet he holds that the truth-functions are not really functions, are not “material” functions (5.44). As the *Tractatus* unfolds, he shifts his way of speaking, trading away the phrase “truth function” in favor of “truth operation”. This divides Frege’s and Russell’s notions of function up into kinds, into material or genuine functions and merely formal operations, restricting the application of the notion of function. All propositions are on this view results of truth-*operations* on elementary propositions (5.3), the elementary propositions being limiting cases of empty operations on themselves. On Wittgenstein’s view, the generality of the general form of proposition is not quantificational, but operational. No *proposition* talks about the general form of proposition; the square bracketed Satzvariable is not a propositional sign. Its values are determined through (operations on) elementary propositions. Indeed, none of the three kinds of Satzvariablen Wittgenstein mentions (in 5.501) can be used to express a proposition, except in the limiting case of a lone propositional sign: (1) to enumerate propositions is to list, but to list is not to say that anything is the case, it is not to articulate a genuine proposition, true or false; (2) an expression such as “ x is a man” cannot be used to assert a proposition, true or false; (3) no square bracketed expression such as “ p , $O'p$ and so on” can be used to assert a proposition, true or false.

Neither Frege nor Russell took the notion of an operation as a basic notion. This is no accident, for the Satzvariablen that express them thwart the general applicability of a function-argument scheme. The “and so on” was the very notion Frege and Russell wished to eliminate from the foundations of logic (and mathematics) by means of their ancestral construction. Wittgenstein is insisting against them that ellipsis is essential to our uses of the signs of logic and mathematics, and therefore that operations—not only truth-operations, but also arithmetical operations—figure essentially in the power of our language to articulate propositions (cf. Wittgenstein, 1979, pp. 89–90).

This has special force against the presentation of the theory of types in *Principia Mathematica*, which depends essentially on the use of such an iterable mode of expression. The so-called logical constants—used, for example, in expressing what Russell conceives of as general logical laws—“must,” Russell explains, either be confined “to disjunctions and negations of elementary propositions, or we must regard them as really each multiple, so that in regard to each type of propositions we shall need a new primitive idea of negation and a new primitive idea of disjunction” (*Principia*, vol. 1, p. 128). Russell claims that “by merely repeating the process” he sets forth, “propositions of any order can be reached” (*Principia*,

vol. 1, p. 128), but this requires the use of systematic ambiguity, for logical laws such as *modus ponens* must be assumed afresh in Russell's shift from elementary propositions to quantified propositions. In addition, every enunciation of a logical law requires ambiguity of type in its variables. (Wittgenstein explicitly objects to this at 6.123.) Furthermore, like Frege, Russell and Whitehead have no general treatment of relations of arbitrary n-adicity: the *Principia* proceeds without the Wiener-Kuratowski reduction of order to class, so that their theorems must be proven afresh for unary, dyadic, and triadic relations, *and so on* (cf. Wittgenstein 1994, vol. 1, pp. 27–28).³³ Furthermore, the definition of the natural numbers within type theory stratifies the universe, so that new numbers must be defined afresh at each step in the progression of the hierarchy of types. That there are a particular number of first-level individuals cannot, as is explicitly said, "be proved logically; . . . it is only ascertainable by a census, not by logic" (1912, vol. 2, p. x). Even after such a census, however, there is no way to assert the cardinal number of things in the universe. Nor can one *say* whether or not there is a unique number of classes at each particular type (p. xiii). At best what Russell and Whitehead can write down are "symbolically identical primitive propositions" (p. ix); at best they can indicate mere constancy of symbolic form, form that may be *shown* or *seen* but not said. Thus:

From symbolic analogy we can "see" that the process can be repeated indefinitely. This possibility rests upon two things:

- (1) A fresh interpretation of our constants—v ~, !, (x),. ($\exists x$).—at each fresh stage.
- (2) A fresh assumption, symbolically unchanged, of the primitive propositions which we found sufficient at an earlier stage—the possibility of avoiding symbolic change being due to the fresh interpretation of our constants. primitive propositions.

. . . if, at any stage, we wish to deal with a class defined by a function of the 30,000th type, we shall have to repeat our arguments and assumptions 30,000 times. But there is still no necessity to speak of the hierarchy as a whole, or to suppose that statements can be made about "all types".

. . . we "see" that whatever *can be proved* for lower types, whether functional or extensional, can also be proved for higher types [. .]. Hence we assume that it is unnecessary to know the types of our variables, though they must always be confined within some one definite type.

. . . when we have proved a proposition for the lowest significant type, we "see" that it holds in any other assigned significant type. Hence every proposition which is proved without the mention of any type is to be regarded as proved for the lowest significant type, and extended by analogy to any other significant type.

By exactly similar considerations we "see" that a proposition which can be proved for some type other than the lowest significant type must hold for any type in the direct descent from this

To "assert a symbolic form" is to assert each of the propositional functions arising for the set of possible typical determinations which are somewhere enumerated. We have in fact enumerated a very limited number of types starting from that of individuals, and we "see" that this process can be indefinitely continued by analogy. The form is always

asserted so far as the enumeration has arrived; and this is sufficient for all purposes, since it is essentially impossible to use a type which has not been arrived at by successive enumeration from the lower types. (pp. ix-xii, my emphases (but not scare quotes) on "see")

This is the *Principia's* way of drawing the distinction between showing and saying. In order to present arithmetic without "having to repeat our arguments and assumptions 30,000 times," it fashions a way of speaking about symbolic form and symbolic analogy apart from particular interpretations or "meanings". This involves the authors, as they fully realize, in what can only be seen, and not couched or asserted in propositions. Whitehead classifies "formal numbers" as those that are "constant" (those numbers σ for which there is a symbol α such that whenever α is determined as to type, σ is identical with $Nc'\alpha$, the cardinal number of all objects at α), and those which are "functional", or functionally complex.³⁴ Distinctions among formal numbers depend, he says, "on the symbolism and not on the entity denoted, and in considering them it is symbolic analogy and not denotation which is to be taken into account" (p. xiv). So he adopts the algebraic terminology of "operations" and "symbolic forms": "Addition, multiplication, exponentiation, and subtraction will be called the arithmetical operations; and in $\mu +_c v$, $\mu \times_c v$, μ^v , $\mu -_c v$, (and (will each be said to be subjected to these respective operations" (p. xv). In the context of the theory of types, ordinary rules of substitution in equalities must be restricted. Whitehead distinguishes between kinds of occurrences of formal numbers in symbolic forms: some are argumental, some are equational, and some are logical. For example, in $*100.511. \vdash: \exists! Nc'\beta \supset sm''Nc'\beta = Nc'\beta$, the formal numbers are $Nc'\beta$, and $sm''Nc'\beta$. The first occurrence of " $Nc'\beta$ " is logical, the second argumental, and the third equational, and the only occurrence of " $sm''Nc'\beta$ " is equational (cf. 1912, vol. 2, p. xix). Whitehead defines an "arithmetical equation" as an "equation between purely arithmetical formal numbers whose actual types are both determined adequately" (1912, vol. 2, pp. xxiv–xxv). This allows us to "pass with practical immediateness" from a typically ambiguous equation to the substitution of one symbolic form (one formal number) for another. Whitehead dubs this the "Principle of Arithmetical Substitution" (p. xxivff.). In this way he allows that "all discrimination of the types of indefinite inductive numbers may be dropped; and the types are entirely indefinite and irrelevant" (p. xxxi).

Whitehead has wiped out the expressive difficulties facing the theory of types by reverting to his older algebraical language of "operations", "symbolic form", and "substitution". Naturally Whitehead and Russell take this as a mere convenience, rather than a conceptual analysis of the nature of mathematics and logic. But when Wittgenstein takes the notion of *operation* to be basic in the *Tractatus*, he is construing their appeal to what we can *see* if we proceed to go on "in the same way"—that is, what they *do*, as opposed to what they (try to) say they do—as fundamental to logic and mathematics:

5.252 Only [by means of an operation] is the progress from term to term in a formal series possible (from type to type in the hierarchy of Russell and Whitehead). (Russell and Whitehead have not admitted the possibility of this progress but have made use of it all the same.)

VI. Wittgenstein's Formal Specification of the Natural Numbers, Mathematical Sentences, and Ascriptions of Number in Mathematics

Space permits only a very brief characterization of Wittgenstein's treatment of mathematical sentences, including ascriptions of number within mathematics.

Wittgenstein's treatment of arithmetical terms turns Whitehead's appeal to "symbolic analogy" and "showing" on its head by taking the notions of a *formally iterable operation* and of a *rule of substitution* as primitive notions. The difference between Wittgenstein's use of the notions of *operation* and *symbolic rule* and the ordinary algebraist's uses of these terms is that Wittgenstein ties his notions directly to his characterization of the general form of proposition, and hence, to the forms of the elementary propositions. For Wittgenstein constructs a Satzvariable for his notion of *operation* on the basis of his Satzvariable for the general form of proposition:

6.002 If we are given the general form of the way in which a proposition [*Satz*] is constructed, then thereby we are also given the general form of the way in which by an operation out of one proposition another can be created.

6.01 The general form of the operation $\Omega'(\eta)$ is therefore:

$$[\xi, N(\xi)]' (\bar{\eta}) (= [\bar{\eta}, \xi, N(\xi)]).$$

This is the most general form of transition from one proposition to another.

According to Wittgenstein, every operation gives us a way of making systematic ("formal") transitions from one proposition to another. In "[$\bar{\eta}, \xi, N(\xi)$]", " $\bar{\eta}$ " ranges over all possible bases for operations, " ξ " over all propositions (i.e., all possible results of steps in the development of a formal series), and " $N(\xi)$ " over all propositions (i.e., all possible results of applying operator N). Wittgenstein does *not* mean that operator N is the only operation, but is instead stipulating that nothing he counts as an operation has a basis or a result that cannot be used—whether directly or indirectly—to make a systematic sort of formal transition from one proposition to another.

The *Tractatus* characterizes the natural numbers by extracting from Wittgenstein's notation for the general form of operation a notation to express the notion of a result appearing at a particular stage (after a particular *number* of steps) in the development of a formal series.

6.02 And thus we come to numbers: I define

$$x = \Omega^0 x \text{ Def. and}$$

$$\Omega' \Omega^v x = \Omega^{v+1} x \text{ Def.}$$

According, then, to these symbolic rules we write the series

$x, \Omega' x, \Omega' \Omega' x, \Omega' \Omega' \Omega' x \dots$ as:

$$\Omega^0 x, \Omega^{0+1} x, \Omega^{0+1+1} x, \Omega^{0+1+1+1} x \dots$$

Therefore I write in place of "[$x, \xi, \Omega' \xi$]"

$$["\Omega^0 x, \Omega^v x, \Omega^{v+1} x]".$$

And I define:

$0+1 = 1$ Def.
 $0+1+1 = 2$ Def.
 $0+1+1+1=3$ Def.
and so on.

6.021 A number is the exponent of an operation.

“ Ω ” is a variable ranging over operations, “ v ” is shorthand for finite sequences of this variable. Thus “ $\Omega'\Omega''$ ” stands in for two steps in the development of any formal series, “ $\Omega'\Omega'\Omega'''$ ” for three such steps, and so on. “ x ” ranges over all possible bases of formal series, “ Ω^0x ” over results of first steps in the development of any formal series, “ $\Omega^{0+1}x$ ” over the results of second such steps, and so on.

As Frascolla (1994) sets out in admirable detail, Wittgenstein construes the natural numbers as abbreviations of sequences of variables, variables that range over all operations. Natural numbers are thus “exponents” or “pictures” (not “indices”)³⁵ common to the development of any formal series, to the iteration of any operation, though of course numerals and arithmetical terms “picture” numbers in a different way than propositions picture states of affairs.³⁶

Numerals are operation signs. For Wittgenstein’s specification of the numbers uses the *and so on*, and is thus itself equivalent to a square bracketed *Satzvariable*. It would not pass muster with Frege as a “definition”; it is not explicit. But it *shows* that the series of natural numbers ($0, 1, 1 + 1, 1 + 1 + 1, \text{ and so on}$) models or pictures (shares the form of) the iterability of any operation, and therefore that each natural number models or pictures (shares the form of) the generation of a formal series up to a certain point. It *shows* that any notation for the system of numbers must be a systematic notation. Differently put, there could be no notation for the natural numbers that construed them as proper names or function expressions, any more than there could be a notation that construed the truth-operations as proper names or function expressions (cf. Wittgenstein, 1973, p. 226). “[$0, \xi, \xi+1]$ ”, a variable ranging over exponents of operations, *shows* the general form of natural number (6.03). Like the general form of proposition, it is a (very general) *Satzvariable* that is not a propositional sign. Just as no proposition can be framed using the general notion of proposition, no proposition can be framed using the general notion of natural number.

We have already seen that Wittgenstein takes equations to be essential to mathematics. Equations do not in his view express propositions (about, say, all numbers); they are operationally, but not functionally, complex. They set forth rules for the substitution of one numerical (or operational) sign for another, either in other mathematical equations or in genuine propositions (extramathematical ascriptions of number). Mathematical equations contain no signs used in the way function expressions and names are used; they contain only (abbreviations of) variables and signs for equality. “ $3 + 1 = 4$ ” might be shown (or operated on) in any one of an unlimited number of ways (e.g., “ $3 = 4 - 1$ ”, “ $3 + 1 - 4 = 0$ ”, “ $3 + 1 + 1 = 4$ ” and so on). Each result of operating on the equation yields a different aspect of the equation, a different representation or standpoint from which to consider the operation signs figuring in it (6.2323; cf. 2.173, 1994, vol. 2, p. 56).

Ascriptions of number within mathematics (e.g., “there is only one solution to $x + 1 = 1$ ”) take a special grammatical form: they are not propositions, true or false,

but rather grammatical rules of variables:³⁷ they show us ways we may interchange numerical operations in genuine propositions without affecting the sense expressed. Arithmetical “proofs” of equalities between particular number words are calculations with operation signs by means of the method of substitution (6.241, quoted above).

In the *Tractatus* Wittgenstein never excludes the possibility that his remarks might be used to develop a notation that would interweave signs for quantifiers and truth-operations with arithmetical operation signs.³⁸ But not until he returned to England in 1929 did Wittgenstein begin seriously to investigate the question of whether and how such interweaving might be accomplished in an unmisleading way, without glossing over his sharp distinction between function expressions and operation signs. It was only then that he began to try to develop his view of mathematics beyond elementary arithmetic, to the point where he considered quantification over numbers. Even then, however, he still treated the roles of quantifiers and truth-operations in equational contexts as utterly different from their role in nonmathematical propositions, and he never did accept Frege’s and Russell’s uses of the ancestral construction to account for the logic of ascriptions of number, not because he was a finitist or a constructivist, but because he rejected their conceptions of what the basic logical notions are.

I am not claiming that Wittgenstein’s treatment of number owes nothing to Frege and Russell. Like the logicians, he takes cardinal number words to reflect an aspect of the logical form of propositions; he construes our method of representing the numbers in language as part and parcel of our use of sentences to express propositions.³⁹ Like Frege and Russell, he attends carefully to the grammar of sentences involving number words—including extramathematical ascriptions of number—and eschews the quest for a philosophical account of number that depends upon any sort of psychological or transcendental account of the mind, including mental processes of abstraction, or synthetic a priori intuition. His notational proposal for representing the grammar of number words might even be said to give expression to Frege’s idea that we may “characterize as a concept that which has number” (Frege, 1984, p. 114): according to the *Tractatus*, it is part of what it is to be a genuine concept word (a genuine function expression) to be configurable in propositional signs that are used to ascribe particular finite numbers and in elementary propositional signs, in which concept words aid in the representation of numbers (as forms). Furthermore, his handling of the grammar of ascriptions of number might be formulated in a Fregean way, as Wittgenstein himself wrote in 1929: in a sense he construes an ascription of number as an assertion about a concept (Wittgenstein, 1994, vol. 1, p. 8). Finally, Wittgenstein himself remarks in the *Tractatus* that “mathematics is a method of logic” (6.2, 6.234, quoted above).

For all these reasons, it may still seem appropriate, as Frascolla has maintained, to call Wittgenstein a “logician” about mathematics (Frascolla, 1994, pp. 25–26, 37–39). But I have argued that Wittgenstein’s differences with Frege and Russell on the respective natures (the respective grammars) of logic and mathematics evince a conception of the fundamental notions of logic that is fundamentally at odds with theirs. It is quite misleading to label him a “logician”, as the tradition has so often done.

VI. Conclusion

In closing, I would like to make a few remarks about why I believe it is important for readers of the *Tractatus* to be willing to scrutinize the inner details of the text with the kind of circumspect attention I have paid to them in this essay.

The *Tractatus* counts as one of the most influential works in all of twentieth-century philosophy, certainly within anything one would be willing to count as the “analytic” tradition. We therefore cannot understand recent philosophical history unless we come to see precisely what Wittgenstein’s interpreters did (and do) with his writings: how they read him selectively and partially, according to their philosophical needs and demands, and how he in turn so read his own philosophical predecessors, especially Frege and Russell. If we look at the history of interpretations of the *Tractatus*, it may be said (speaking very generally) that since at least Elizabeth Anscombe’s *Introduction to Wittgenstein’s Tractatus* (1959), there has been a growing consensus that the early Wittgenstein was not a logical positivist, however influential the *Tractatus* was on that philosophical movement. (This trend has paralleled the wider philosophical culture’s increased distancing of itself from positivism.) Readers interested in situating the *Tractatus* within the history of twentieth-century philosophy are far more likely nowadays to emphasize Wittgenstein’s philosophical debts to Hertz, to Russell, and especially to Frege than they are to emphasize what he had to say about analyticity, verificationism, or phenomenalism. (This trend echoes the fact that Frege is far more carefully and enthusiastically read today than he was before the mid-1940s.) Meanwhile, the resurgence of interest in ontology within the analytic tradition since the 1960s seems to have tempted at least some readers to interpret the *Tractatus* as a metaphysical, rather than an antimetaphysical, work: as defending, for example, either a form of modal realism, or a cognitive metaphysics of thought, or a critical realism of the Kantian variety (albeit one according to which the transcendental standpoint is not expressible in factual language, but can only be shown). There is some justification within Wittgenstein’s text for each of these interpretations, and each has certain merits. Most recently, however, a number of American scholars of early analytic philosophy—most notably Cora Diamond, Burton Dreben, and Warren Goldfarb—have reacted against such metaphysical ways of reading the *Tractatus* and have attempted to defend a more sophisticated antimetaphysical reading of the book, albeit one that still distances Wittgenstein’s philosophy from positivism in key respects.⁴⁰ Their readings of the *Tractatus* have sparked heated yet philosophically fruitful debate,⁴¹ precisely by stimulating readers to reexamine the (frequently highly ambiguous) text of the *Tractatus* itself, exploring the philosophical and interpretative possibilities latent within it.

In this essay, focusing on what may seem rather technical or peripheral parts of Wittgenstein’s early philosophy, I have defended a version of this antimetaphysical yet antipositivist reading of the *Tractatus*. I make much of the fact that Wittgenstein accepted neither the Frege-Russell analysis of the grammar of number words, nor their reduction of logic to mathematics, nor even their account of the application of logic within mathematics. This fact separates Wittgenstein’s philosophy sharply from the positivists as well. But my primary aim has been to emphasize Wittgenstein’s differences with Frege and Russell. This is because most

recent antimetaphysical interpreters of the *Tractatus* have, by contrast, tended to stress affinities between Frege and the early Wittgenstein, partly in order to reassess received readings of Frege (see Floyd, 1998a).

Here Cora Diamond's reading of the early Wittgenstein—to which I am greatly indebted—stands out (Diamond, 1991a, 1991b, 1997). Diamond argues that the early Wittgenstein may be seen to have inherited a distinction between showing and saying from Frege, and then used it (like Frege, on Diamond's reading of him) to resist the attractions of both realist and idealist theories of the logical structure of language and/or world. Diamond's interpretation of the *Tractatus* places heavy weight on what she has called its “framing” remarks: the preface and closing lines of the book, where Wittgenstein suggests that the reader should take the remarks of the *Tractatus* to be purely nonsensical. On her “therapeutic” reading, the *Tractatus* is pure nonsense, but at the same time it is a work of great imaginative and philosophical force, designed to depict the attractions of various sorts of realism and idealism, and simultaneously to show us that every such effort to erect a metaphysical theory of logical structure steps beyond the bounds of sense. She argues (rightly, I think) that the early Wittgenstein—like the later Wittgenstein—aimed to help his readers overcome the felt need to propound theories of necessity, meaning, and ontology. His Tractarian strategy, she holds, is to indulge such felt needs imaginatively by constructing an arrangement of remarks (e.g., “objects form the substance of the world”; 2.021) in such a way as to display their nonsensical character—a character that, Diamond insists, is not nonsense-with-a-certain-kind-of-ineffable-or-poetical-significance, but rather nonsense pure and simple, on a par with gibberish (e.g., “Socrates is frabble”). Just here, she claims, Frege provided Wittgenstein with a model. For Frege took sentential forms resembling those of the *Tractatus* to play a key role in his philosophy. He called them “elucidations”, conceiving of them as constructions that are strictly speaking nonsensical, but that nevertheless serve as hints, aids to help a reader catch on to the use of genuine language—that is, to his concept-script. According to Diamond, it is by means of such elucidatory forms as “There are functions” or “No concept is an object”—neither of which can be expressed in Frege's notation—that Frege tries to inculcate our understanding of that language. Yet it is on her view only by working within genuine, meaningful language—for example, the language of Frege's concept-script—that a reader can come to appreciate the status of Frege's primitive, undefinable logical distinctions. Frege's appreciation of the nonsensical character of his elucidations is thus, for Diamond, part and parcel of his appreciation of the nonsensical character of certain forms of realism. And this, she believes, Wittgenstein took from Frege. As she writes,

Philosophical uses of ‘object’ *dither* between wanting the word to have the kind of logical significance it has functioning as a variable and wanting the word to mean some property, to be a genuine property-word. The idea of a notation in which, instead of the word ‘object,’ we always did just have a variable is then supposed by Wittgenstein to be possibly helpful: we should come to see that the notation enables us to say everything we wanted to say and so we may be cured of the *irresolute dithering* use of ‘object.’ The notation itself then helps achieve the therapeutic goals Wittgenstein describes at *Tractatus* 6.53. The ordinary language use of ‘object’ as a variable goes over into that notation in one

way; and the use of ‘object’ as a genuine property word goes over differently. The translation into the logical notation thus reveals that the use of ‘object,’ as a word logically parallel to ‘potato,’ involves no logical error but does involve using a word which, in that use, has been assigned no meaning. The importance of translation into logical notation is thus that the notation doesn’t, as ordinary language does, make it easy to conceal one’s dithering from oneself. (Diamond 1997, p. 79)

On Diamond’s reading, Wittgenstein’s diagnosis of the nonsense that arises in philosophizing is that it comes from unclarity about the “logical significance” of the terms that we use, an unclarity abetted by ordinary language and (possibly) dispelled by the use of a perspicuous logical notation. Diamond insists that her account of the Wittgenstein-Frege notion of the “logical significance” of an expression is not a semantical or meaning-theoretic one. Instead, it is to be made out wholly in terms of the inferential relations that a judgment voiced using that expression bear to other judgments (1997, pp. 75–76). For example, from “There’s a potato in the window” may be seen to follow “There’s an object in the window,” but in two quite differing ways, ways that would be differently expressed in Fregean notation. Philosophical confusion arises, on the view Diamond ascribes to Wittgenstein and Frege, from the fact that one and the same term of ordinary language (“object”) may be used in expressions of judgments conforming to entirely distinct patterns of inferential relations. And herein lies the therapeutic usefulness of the “notation”, the concept script that perspicuously marks—or attempts to perspicuously mark—all (genuine) inferential distinctions syntactically. As she writes,

If a language is capable of expressing thoughts at all, it must (here Frege and Wittgenstein agree) have as the logical component of its grammar what every language has. The point about *the* logical grammar applies not only to any natural language but also to a concept-script designed with the intention that the logical characteristics of the thoughts expressed in its sentences should be shown clearly in the perceptible structure of those sentences. A concept-script is unsatisfactory when it treats in the same way what is logically different (what *the* logical grammar treats as different) or treats in different ways what is logically similar. (Diamond 1997, pp. 126–27; emphases in orginal)

This conception of a *Begriffsschrift* involves the idea that there is a single inferential order of judgment and thought that it is the aim of a perspicuous logical notation to capture or make explicit. This suggests that Diamond takes Tractarian therapy designed to dispel the power of metaphysics to involve—at least in part—the devising of a *Begriffsschrift*. But not just any old *Begriffsschrift*. A proper *Begriffsschrift* reflects *the* logical order of thought insofar as it prevents us from dithering in the relevant respect when we express thinking in language. A *Begriffsschrift* that failed to reflect this order would be one that encouraged us to “treat in the same way what is logically different,” or “in different ways what is logically similar.”

Yet, as I have argued in this essay, by examining the details of what Wittgenstein actually *did* with the *Begriffsschriften* of Frege and Russell in the *Tractatus*, we can see that he is rejecting this ideal of clarity of expression. According to this ideal—vividly set out by Diamond—we imagine ourselves to be depicting *the*

inferential order among thoughts (or sentences of our language) when we work with a logical notation. But on my reading, one aim of the *Tractatus* is to depict such notions as “*the inferential order*”, “*the logical grammar of language*”, and “*the logical form of a proposition*” as chimeras. In this sense the Frege (Russell) ideal stands as a primary philosophical target of the *Tractatus*, and not just an ideal Wittgenstein inherited from them. For Frege and Russell write as if, at least ideally, there is a single context of expression within which we may discern the structure of thought, a systematically presented *Begriffsschrift* within which we can use logical notation to make perspicuous *the logical order*. In contrast, I have emphasized Wittgenstein’s insistence in the *Tractatus* that no single imposition of a logico-syntactic order on what we say is or can be the final word, the final way of expressing or depicting a thought. On the *Tractatus* view (as I interpret it) there is thinking, but thinking without thoughts, thinking without an inferential order. For Wittgenstein—even in the *Tractatus*—however useful the formalized languages of Frege and Russell may be for warding off certain grammatical and metaphysical confusions, these languages must simultaneously be seen as sources of new forms of philosophical illusion—indeed, the deepest kind of illusion of all, the illusion of having found ultimate clarity. Wittgenstein is certainly indebted to Frege’s and Russell’s work; he himself writes that we need a good logico-syntactic notation in order to avoid certain philosophical confusions (3.325). But I do not think he shared either Frege’s or Russell’s (or for that matter the logical positivists’) conception(s) of what an ideal or formalized language could do for us in philosophy. Unlike these philosophers, he does not think any notation can depict *the grammar of language*, or make clear *the limits of sense, the logical order*.

This, it seems to me, is the best answer that can be given to those critics of Diamond (and other antimetaphysical readers of the *Tractatus*) who find incoherent her insistence that the remarks of the *Tractatus* are no more and no less nonsensical than any other gibberish. It is a good question how Wittgenstein’s *Tractatus* could have been so philosophically influential and insightful if it consists of nothing but gibberish or ironically intended remarks.⁴² The answer, it seems to me, is that Wittgenstein’s use of the term “nonsense” (*unsinnig*)—in the *Tractatus* as in his later philosophy—has no freestanding use, but is instead a kind of dialectical punctuation mark, used in context to stop the reader from imposing a single order of grammar (or thought) upon a particular philosophically minded choice of words. On my view, Wittgenstein’s deepest philosophical insight—even in the *Tractatus*—was not one that concerns the notion of nonsense as a generic grammatical category, or a term of philosophical criticism with a systematic use. Rather, Wittgenstein’s insight was that there is no general category of nonsense to made out by the philosopher, no way for philosophy to achieve a perspective from which to systematically chart the bounds of sense.⁴³ We can appreciate this about the *Tractatus* only by penetrating the so-called framing remarks, and exploring the details of Wittgenstein’s own philosophical remarks. Only in this way may we see that there is no clear distinction to be drawn between the frame of the *Tractatus* and what is inside the frame.

I have argued in this essay that one way to see that Wittgenstein had no ineffable theory of logic, thought, or mathematics is to see that he had no theory—

no notation for—the effability of logic, thought, or mathematics. I am well aware that to emphasize, as I have, the distance separating the philosophy of the *Tractatus* from the philosophies of Frege and Russell is to portray the tradition of early analytic philosophy as a conversation in which parties disagree with one another on fundamentals, rather than finding themselves bound together by a common conception of method (e.g., the use of a system of modern mathematical logic to depict the logico-syntactic grammar of language) or doctrine (e.g., that the structure of thought can, and can only, be gleaned from an analysis of the structure of language) (cf. Dummett, 1993). I am holding that there is a point at which Wittgenstein's criticisms of Frege and Russell are external criticisms, criticisms that evince a philosophical perspective and spirit radically different from theirs—even if at particular points in the *Tractatus* (as Diamond for one has shown) we may also take him to have been working through insights he gleaned from both Frege and Russell, and pointing toward internal tensions in their philosophies that he hoped to resolve (or, better, dissolve). It is for this reason that I have insisted on grappling with the *Tractatus*'s treatment of number in such detail in this essay. Until we see what Wittgenstein asked us to do with the *Begriffsschriften* of Frege and Russell, we cannot see what his aims really were in the *Tractatus*.

Appendix

Wittgenstein holds that a proposition may be conceived as the result of a finite number of successive applications of operator *N* to elementary propositions (*Tractatus* 5, 6). It has been alleged that this is false, since pure quantification theory does not reduce to purely truth-functional logic, but I do not think this reduction is at issue in the *Tractatus*, and therefore I do not take this objection to refute his claim (see note 32). In response, I give below an indication of how it is that Wittgenstein understands his claim, using the notation he proposes in the *Tractatus* to express quantificationally complex propositions as results of the application of operator *N* to elementary propositions. I have departed from the *Tractatus* notation in adding indices to the use of operator *N* whenever it is used to express generality; in such cases, operator *N* is applied to a (potentially infinite) number of elementary propositions all at once, by means of a *Satzvariable*; and the index in my variant of Wittgenstein's notation indicates the *Satzvariable* to which this particular use of operator *N* is tied. Some such system of indexing—some such notational differentiation among uses of operator *N* to express generality—is needed in order to unambiguously express multiple generality, for we need to be able to notationally tie each such use of operator *N* to a unique *Satzvariable*. Any such system of indexing goes beyond what Wittgenstein actually proposes in the *Tractatus*, but not in a way that undercuts his insistence that quantification is an operation on elementary propositions. The fact that Wittgenstein never bothered to propose any such indexing system indicates, it seems to me, just how uninterested he was in developing a formalized quantificational language of the Frege-Russell sort.

In the tables below, rows represent steps in the finite sequence of applications of operator N to elementary propositions. To express generality, operator N applies, not to a (single) set of elementary propositions, but directly to the elementary propositions themselves. I have rendered such applications as two-part steps (e.g., 3a., 3b., in the table below), because I conceive of these as really one application of operator N .

To Express One Universal Quantifier with Operator N :
e.g., "Everything is an apple"

English	Russellian Notation	Wittgenstein's Operator N Notation	Comments
1. a is an apple	Aa	Aa	Elementary proposition, hence no variables and we assume "A", "a" are all simple names
2. a is not an apple	$\neg Aa$	$N(Aa)$	Quantifier-free proposition
3a. x is not an apple \Downarrow	$\neg Ax$ \Downarrow	$N(Ax)$ \Downarrow	<i>Satzvariable</i> from 2, $x \rightarrow a$ \Downarrow
3b. It is not the case that there is a non-apple.	$\neg(\exists x).\neg Ax$	$N_x(N(\neg Ax))$	Quantified proposition
4. Everything is an apple.	$(\forall x).Ax$	$N_x(N(\neg Ax))$	Russellian abbreviation of 3b; quantified proposition

To Express Multiple Dependent Quantifiers with Operator N : e.g., "Someone fathers everyone except himself"

English	Russellian Notation	Wittgenstein's Operator N Notation	Comments
1. Adam fathers Cain	aFc	aFc	Elementary proposition, hence no variables and we assume "a", "c", "F" are all simple names
2. Adam is not the father of Cain	\neg aFc	$N(aFc)$	Quantifier-free proposition
3a. Adam is not the father of y	\neg aFy	$N(aFy)$	<i>Satzvariable</i> , 2, $y \rightarrow c$
	↓	↓	↓
3b. It's not the case that there is someone whom Adam does not father	$\neg(\exists y).\neg(aFy)$	$N_y(\overline{N(aFy)})$	Quantified proposition
4. Adam is the father of all	$(\forall y).aFy$	$N_y(\overline{N(aFy)})$	Russellian abbreviation of 3b Quantified proposition
5a. x is the father of all but himself	$(\forall y):x \neq y. \supset .xFy$	$N_y(\overline{N(xFy)})$	<i>Satzvariable</i> , 4, $x \rightarrow a$
	↓	↓	↓
5b. It is not the case that someone fathers everyone but himself	$\neg(\exists x)(\forall y):x \neq y. \supset .xFy$	$N_x(\overline{N_y(\overline{N(xFy)})})$	Quantified proposition
6. Someone fathers everyone except himself	$(\exists x)(\forall y):x \neq y. \supset .xFy$	$N(N_x(N_y(\overline{N(xFy)})))$	Quantified proposition, negation of 5b.

The General Term for the Successor Series for an Arbitrary Relation R Using Operator N :

English	Russellian Notation	Wittgenstein's Operator N Notation	Comments
1. b immediately succeeds a in the R-series	aRb	aRb	Elementary proposition
2. b succeeds a by one in the R-series	$(\exists x):aRx.xRb$	$N(N_x(\overline{N(\overline{N(aRx)}, N(xRb))}))$	Quantified prop.
2.1	aRc,cRb	aRc,cRb	Elementary props.
2.2	$\neg aRc, \neg cRb$	$N(aRc), N(cRb)$	Quantifier-free props
2.3	aRc.cRb	$N(N(aRc), N(cRb))$	Quantifier-free prop.
2.4a	aRx.xRb	$N(N(aRx), N(xRb))$	<i>Satzvar</i> , 2.3, $x \rightarrow c$
\Downarrow	\Downarrow	\Downarrow	\Downarrow
2.4b	$\neg(\exists x):aRx.xRb$	$N_x(\overline{N(\overline{N(aRx)}, N(xRb))})$	Quantified prop.
2.5 = 2	$(\exists x):aRx.xRb$	$N(N_x(\overline{N(\overline{N(aRx)}, N(xRb))}))$	Quantified prop., negation of 2.4b
3. b succeeds a by two in the R-series	$\exists x \exists y: x \neq y. aRx.xRy.yRb$	$N(N_x(\overline{N(N_y(\overline{N(\overline{N(aRx)}, N(xRy)}, N(yRb))))}))$	Quantified prop.
3.1	aRc,cRd,dRb	aRc,cRd,dRb	Elementary props.
3.2	$\neg aRc, \neg cRd, \neg dRb$	$N(aRc), N(cRd), N(dRb)$	Quantifier-free props
3.3	aRc.cRd.dRb	$N(N(aRc), N(cRd), N(dRb))$	Quantifier-free prop
3.4a	aRc.cRy.yRb	$N(N(aRc), N(cRy), N(yRb))$	<i>Satzvar</i> , 3.3, $y \rightarrow d$
\Downarrow	\Downarrow	\Downarrow	\Downarrow
3.4b	$\neg(\exists y):aRc.cRy.yRb$	$N_y(\overline{N(\overline{N(aRc)}, N(cRy)}, N(yRb)))$	Quantified prop.
3.5	$(\exists y):aRc.cRy.yRb$	$N(N_y(\overline{N(\overline{N(aRc)}, N(cRy)}, N(yRb))))$	Quantified prop.
3.6a	$(\exists y):aRx.xRy.yRb$	$N(N_y(\overline{N(\overline{N(aRx)}, N(xRy)}, N(yRb))))$	<i>Satzvar</i> , 3.5, $x \rightarrow c$
\Downarrow	\Downarrow	\Downarrow	\Downarrow
3.6b	$\neg(\exists x)(\exists y):x \neq y. aRx.xRy.yRb$	$N_x(\overline{N(N_y(\overline{N(\overline{N(aRx)}, N(xRy)}, N(yRb))))}))$	Quantified prop., negation of 3.6b
3.7 = 3.	$(\exists x)(\exists y):x \neq y. aRx.xRy.yRb$	$N(N_x(\overline{N(N_y(\overline{N(\overline{N(aRx)}, N(xRy)}, N(yRb))))}))$	

And so on . . .

Formal Series for Ascriptions of Number: e.g., "There are (at least) n apples"

English	Russellian Notation	Wittgenstein's Operator <i>N</i> Notation	Comments
1. There are no apples	$\neg(\exists z).Az$	$N_z(Az)$	Quantified prop.
1.1	Aa	Aa	Elementary prop.
1.2a	Az	Az	Satzvar., 1.1, $z \rightarrow a$
\Downarrow	\Downarrow	\Downarrow	\Downarrow
1.2b = 1	$\neg(\exists z).Az$	$N_z(\overline{Az})$	Quantified prop.
2. There is 1 apple	$(\exists z).Az$	$N(N_z(\overline{Az}))$	Quantified prop.
2.1	Aa	Aa	Elementary prop.
2.2a	Az	Az	Satzvar., 2.1, $z \rightarrow a$
\Downarrow	\Downarrow	\Downarrow	\Downarrow
2.2b	$\neg(\exists z).Az$	$N_z(Az)$	Quantified prop.
2.3 = 2.	$(\exists z).Az$	$N(N_z(Az))$	Quantified prop.
3. There are 2 apples	$(\exists y)(\exists z):y \neq z.Ay.Az$	$N(N_y(N(N_z(\overline{N(N(Ay), N(Az)))))))$	Quantified prop.
3.1	Aa, Ab	Aa, Ab	Elementary props.
3.2	$\neg Aa, \neg Ab$	$N(Aa), N(Ab)$	Quantifier-free props
3.3	Aa. Ab. a \neq b	$N(N(Aa), N(Ab))$	Quantifier-free prop
3.4a	Aa. Az. a \neq b	$N(N(Aa), N(Az))$	Satzvar., 3.3, $z \rightarrow b$
\Downarrow	\Downarrow	\Downarrow	\Downarrow
3.4b	$\neg(\exists z):Aa.Az.a \neq z$	$N_z(\overline{N(N(Aa), N(Az))})$	Quantified prop.
3.5	$(\exists z):Aa.Az.a \neq z$	$N(\overline{N_z(N(N(Aa), N(Az))}))$	Quantified prop.
3.6	$(\exists z):Ay.Az.y \neq z$	$N(N_z(\overline{N(N(Ay), N(Az))}))$	Satzvar., 3.6, $y \rightarrow a$
3.7a	\Downarrow	\Downarrow	\Downarrow
3.7b	$\neg(\exists y)(\exists z):Ay.Az.y \neq z$	$N_y(\overline{N(N_z((N(N(Ay), N(Az)))))))$	Quantified prop.
3.8=3	$(\exists y)(\exists z):Ay.Az.y \neq z$	$N(N_y(\overline{N(N_z((N(N(Ay), N(Az))))))))$	Quantified prop.
4. There are 3 apples	$(\exists x)(\exists y)(\exists z)(Ax.Ay.Az.x \neq y.x \neq z.y \neq z$	$N(N_x((N(N_y((N(N_z((Ax.Ay.Az))))))))^*$	Quantified prop.

And so on . . .

* For reasons of space I here abbreviate the conjunction of Ax, Ay, Az with the Russellian notation: Ax. Ay. Az.

Notes

Earlier versions of this essay were presented at the May 1996 Brazilian Logic Association meeting in Salvador, Bahia, where I profited greatly from discussions with Professors Itala D'Ottaviano, Pasquale Frascola, Mathieu Marion, Carlo Penco, Richard Valée, and Michael Wrigley; at the philosophy departments at the University of Stockholm and the University of Oslo, where I received equally helpful criticism from Professors Peter Martin Löf, Peter Pagin, Dag Prawitz, Göran Sundholm, and Dagfinn Føllesdal; and in Burton Dreben's fall 1996 seminar at Boston University. Thanks are also due to Rosalind Carey, Arthur Collins, Alex George, Warren Goldfarb, Wilbur Hart, Victor Krebs, Montgomery Link, Andrew Lugg, Matthew Ostrow, Rohit Parikh, and especially Thomas Ricketts and Sanford Shieh for much good discussion of the *Tractatus* and comments on earlier drafts which led to substantial improvement of the essay. The American Council of Learned Societies and the Dibner Institute for the History of Science and Technology at the Massachusetts Institute of Technology provided me with generous financial and intellectual support during the writing of the essay. My debt to Burton Dreben is greatest, for he aided in all these ways and more.

1. Numbered references are to Wittgenstein, 1922.
2. An interesting exception was Waismann, 1951, 1982. On Wittgenstein and Waismann, see Shanker, 1987. For a discussion of the history and reception of Wittgenstein's term "tautology", see Dreben and Floyd, 1991.
3. Important exceptions here are Wrigley 1986, 1987; Frascola 1994; and Marion 1998.
4. Bell and Demopoulos (1996) also take Wittgenstein to have failed to account for the application of arithmetic, finding this failure "ironic" in light of Frege's work.
5. Dummett is writing here about Frege, 1884, which we cannot be sure Wittgenstein had read before World War I. But what Dummett says applies equally to Frege's treatment of ascriptions of cardinal number in the *Grundgesetze*, with which Wittgenstein was acquainted before writing the *Tractatus*.
6. A proper treatment of the *Tractatus* treatment of number words would explore the extent to which Wittgenstein altered, and the extent to which he retained, his early philosophy of arithmetic in the period 1922–34. Such a treatment lies beyond the scope of this essay (though see Gerrard 1991). Many interesting remarks are, however, to be found in Wittgenstein's manuscripts from 1929–34 (Wittgenstein, 1994) that shed much light, I believe, on the *Tractatus* itself. Below I shall occasionally quote from these manuscripts (and the related Wittgenstein, 1979) in order to flesh out my reading, fully realizing that these texts must be used with great care, as they may reflect ideas that are not fully Tractarian in character.
7. I am not going to enter here into the vexed question whether second-order logic is or is not "logic", is or is not "set theory in sheep's clothing," to use Quine's phrase.
8. "The old logic" thus applies even to Frege and to Russell, despite the fact that each of them criticized traditional logic (and metaphysics) for naively assuming the general applicability of a subject-predicate distinction. See 6.125 (Wittgenstein, 1979), p. 109, and Wittgenstein, 1976, V §§13, 48.
9. As Cora Diamond writes, "while there are some *Tractatus* remarks which are apparently intended to give some general guidance [about logical analysis], the bulk does . . . take the guidance necessary for logical analysis to be largely negative. Clarity about logic (of the sort the *Tractatus* aims at) is meant to head us off wrong paths in analysis" (Diamond, 1997, p. 77).
10. This is the heart of what I propose as a development of the insight, most vividly pressed in Diamond, 1991a, 1991b, that there is no ineffable (nonfactual, nonliteral) *content* that is shown in the *Tractatus* (cf. Floyd, 1998b). Both Hacker (2000) and David Pears have questioned the arguments—textual and philosophical—that Diamond and others have offered for such readings, but neither has addressed the question of how Wittgenstein's treatment of mathematics bears on the issue.
11. Burton Dreben suggested to me that Whitehead, 1898, might have been what Wittgenstein was alluding to in his entry of June 22, 1915, in Wittgenstein, 1979, p. 70 (see especially Whitehead, 1898, pp. 9–10). Whitehead, 1911, also contains some remarks about mathematics that may be profitably compared with the *Tractatus*. In what follows I quote from these books in order to show how Wittgenstein is reinterpreting their terminology. Whether or not it was these particular works that stimulated Wittgenstein, I shall take them to be sufficiently representative to illustrate the force of his remarks.
12. Whitehead has a footnote here: "Cf. Stout, 'Thought and Language,' *Mind*, April 1891, repeated in the same author's *Analytic Psychology*, (1896), ch. x. §1: cf. also a more obscure analysis to the same effect by C. S. Peirce, *Proceedings of the American Academy of Arts and Sciences*, 1867, vol. vii, p. 294."
13. See Monk, 1990, pp. 216–17.
14. Ramsey had written his review of the *Tractatus* (see Ramsey, 1960) before meeting with Wittgenstein in Austria. A photograph of the annotated page from Ramsey's copy of the *Tractatus* is reproduced in Nedo and Ranchetti, 1983, pp. 192–93, and the provenance of the annotations discussed in Lewy, 1967, where it is noted that Witt-

genstein seems to have questioned the translation of *Anschauung* as "intuition" at 6.233–6.2331.

15. So also say Bouveresse (1988) and Marion (1998), though they take this to indicate that Wittgenstein is a constructivist or even a finitist, both labels I would resist pinning on the *Tractatus*. Goldfarb (1988) argues that Poincaré's objection has no force against the positions of either Frege or Russell, as it imports a psychologistic conception of reasoning into the argument. That would be one reason for hesitating before we ascribe Poincaré's objection to Wittgenstein.

16. Russell, 1938, pp. 117, 377, the latter reference containing an explicit reference to Whitehead, 1898; Whitehead and Russell, 1910, 1912, vol. 1, pp. 297, 345, vol. 2, p. xv. Cf. section IV below. Russell attributes the remarks in vol. 2 of *Principia* to Whitehead (see Russell (1948)).

17. See the discussion of Post and Lewis in Dreben and van Heijenoort, 1986, pp. 44–48.

18. As opposed to simple numerals (e.g., "2") or arithmetical sentences (e.g., "2 + 2 = 4").

19. Hylton (1997) sheds much helpful light on Wittgenstein's distinction between functions and operations in connection with the truth-operations. I do think, however, that Hylton overspeaks in claiming that the truth-operations are the primary examples of operations on which we should focus in order to gain an understanding of how Wittgenstein's views compare to those of Frege and Russell (Hylton, 1997 p. 92 n. 3).

20. While the explanation of this analogy is mine, and is somewhat conjectural about the *Tractatus*, the analogy itself is made explicit in Wittgenstein, 1973, pp. 225–26. I am arguing that the analogy is implicit already in the *Tractatus*.

21. See Carnap, 1937, pp. 53, 101, 282–84. For a different reading of Wittgenstein's show/say distinction in connection with metalanguages (and Gödel's arithmetization of syntax, which Carnap took to have refuted that distinction), see Floyd, 1995, 2000.

22. At Wittgenstein, 1973, pp. 218–19, the parallel is drawn explicitly.

23. For a discussion, see Hylton, 1994.

24. White (1979) and Fogelin (1987, 1983) deem Wittgenstein's treatment adequate to express ascriptions of finite cardinal number, though they do not explore the deductive power of Wittgenstein's notational proposal (discussed below). Marion (1998) and Sullivan (1994, 1995) scrutinize Wittgenstein's (post-Tractarian) exchanges with Ramsey about identity.

25. It might seem as if Wittgenstein is adopting a view of identity such as Frege had embraced in his 1879 *Begriffsschrift* (§8). There Frege held that identity "necessarily produces a bifurcation in the Bedeutung of all signs," for in identity contexts names "suddenly display their own selves"—i.e., suddenly name themselves. This apparent confusion of use and mention was remedied, in Frege's mind, by his later distinction between the Sinn and the Bedeutung of a name. On Frege's (vacillating) attitude toward the project of logically defining identity, see Dummett, 1991, pp. 141ff.; and Floyd 1998a. I am reading Wittgenstein as altogether denying that names flank mathematical uses of the equality sign. Wittgenstein is explicit in the *Tractatus* that use of the identity sign between names is a mere matter of convenience in genuine propositions, but cannot assert anything about the Bedeutung of distinct signs (4.241ff.). Unlike Frege, Wittgenstein sharply distinguishes between signs that represent forms and signs that are names or function expressions.

26. Were someone to actually try to construct a deductive system of logic on the basis of Wittgenstein's notational proposals—which he never tried to do—it might be that only a decidable fragment of first-order logic would result, since on at least one reading, no co-instantiation of distinct variables is allowed. Such an applied "logic" really would be tautologous. Yet even in this case, the question of a *general* decision procedure for logic as a whole could not be formulated in Wittgenstein's terms, for his notational remarks about identity at 5.53ff. apply only to genuine propositions, and not the sentences of pure logic and mathematics. Dreben and Floyd (1991) argue

(against, e.g., Fogelin, 1987) that even at the time of writing the *Tractatus* Wittgenstein did not hinge his characterization of logic as “tautologous” on the existence of a general decision procedure for “all of logic” as such.

27. For a perspicuous criticism of readings of the *Tractatus* that reify possibilities, see Goldfarb, 1997, esp. pp. 65–66.

28. The falsity of an ascription of number lies in there being no way to instantiate the Satzvariablen at work in the propositional sign expressing the ascription and get a truth. If we wrongly suppose that two names, say, “Morning Star” and “Evening Star”, have distinct Bedeutungen, then we shall be wrong in our counting of astronomical bodies and wrong in other propositions as well. But for Wittgenstein this would be as much a matter of our being mistaken about the significance of names and Satzvariablen—i.e., being mistaken about which symbols these signs may be used to express—as it would be an error about heavenly bodies. That there are two distinct astronomical bodies called “the Morning Star” and “the Evening Star” is not necessarily false: in a language in which these names did not share a Bedeutung, it would make sense (and be true) to say that there are two stars that appear in such and such a way in the sky, the morning star and the evening star, while it would be nonsense to try to say that these two stars are one (5.5303). But in a language such as ours, these names share the same Bedeutung, and then according to Wittgenstein’s notational proposal the Satzvariable “ x is a star appearing in such and such a way in the sky and y is a star appearing in such and such a way in the sky” does not range over “the Morning Star is a star appearing in such and such a way in the sky and the Evening Star is a star appearing in such and such a way in the sky.” Every proposition over which this Satzvariable ranges is false.

29. As Anscombe, 1971, pp. 147–49, reports, Ramsey objected to Wittgenstein’s interpretation of the variable, pointing out that apparent ascriptions of number such as $(\exists x,y)(\phi x \& \phi y) \& \sim(\exists x,y,z)(\phi x \& \phi y \& \phi z)$ would turn out to be problematic if the number of objects in the universe were exceeded by the number of distinct variables in the language. To Wittgenstein the above quantificational form would be a structure with symbolic redundancy, in the case of a language in which there were only two distinct names, for there would be no way to find distinct instances of the variables x,y,z in the second half of the conjunction. Ramsey suggested that in this case the statement would imply a contradiction at the point at which two distinct variables were instantiated with the same name. But in the *Tractatus* we are dealing with a picture of language in which we may not so instantiate, because all redundancy in names has been eliminated, and the values a variable may take on are determined by the range of available distinct names. In the kind of case Ramsey imagines, what appears to be a grammatically well-formed sentence, what appears to meet all logical criteria for counting as a propositional sign, is not a propositional sign after all. (Fogelin, 1987, pp. 70–71, also holds that the extra variable [“ z ” in the above example] would be, because useless, meaningless.) I should add that Wittgenstein does not rule out the possibility of an infinite number of names in the language (5.535), but it is difficult to see how he would set out such a stock of names without a general formal rule. Any such rule would of course treat names as part of a system, hence, as operation signs, undermining the *Tractatus* distinction between operation signs and names and his claim for the logical independence of elementary propositions. This is what appears to happen in Wittgenstein, 1929.

30. A proper treatment of Russell’s use of systematic ambiguity to present the theory of types lies outside the scope of this essay. But see Hylton, 1990, for an excellent treatment.

31. The example of “ $x = x$ ” is treated in Whitehead and Russell, 1910, 1912; see vol. 1, pp. 18, 39, vol. 2, pp. vii–viii.

32. I agree with Geach, (1983), Soames (1983) and Sundholm (1992) that Fogelin (1983, 1987) errs in holding that Wittgenstein failed to specify the totality of propositions by means of a finite number of applications of operator N to the elementary propositions. The key to Wittgenstein’s construction is to allow arbitrary collections of ele-

mentary propositions to begin formal series generated by means of operator N, and then to index applications of N by way of a Satzvariable. My proposed construction is closest to the one proposed by Sundholm, which I take to be closest to Wittgenstein's intent; for unlike Sundholm, Geach and Soames build reference to classes into their respective constructions. Bell and Demopoulos (1996) go so far as to credit Wittgenstein with having invented the algebraic notion of a free generator of a logical calculus. Fogelin (1987, p. 82) argues, against all such interpretations, that Wittgenstein failed to specify any such construction on the ground that Wittgenstein is "plainly [...] committed to a decision procedure for the propositions of logic." Dreben and Floyd (1991) argue that Wittgenstein is not so committed.

33. A treatment of relations of arbitrary n-adicity could be obtained for the *Principia* by adding different primitives to the object language, as in Quine, 1932 (cf. Dreben, 1990, for a discussion). Russell's and Whitehead's appeal to "symbolic analogy" in connection with the theory of types is, by contrast, intrinsic to its presentation and could not be eliminated by building stronger primitive notions into the object system.

34. This distinction between "constant" and "functional" formal numbers is not exclusive, for " $1 + \epsilon 2$ ", the cardinal sum of 1 and 2, is both constant and functional (Whitehead and Russell, 1910 and 1912, vol. 2, p. xiv).

35. Like operation expressions, indices are contrasted by Wittgenstein with arguments, with names and function expressions (5.02). But unlike operation expressions, indices are not formally interconnected with other indices in a systematic network; they do not, for example, generate formal series, and do not have to belong to a systematic notation. In calling the numbers "exponents of operations" rather than "indices", Wittgenstein takes number to reflect the process of iterating or applying an operation; he is not thinking of the uses of number words as tags, as in codes, passwords, or bank account numbers.

36. Wittgenstein never explicitly says in the *Tractatus* that numerals and arithmetical terms are used as "pictures". His aim in this work is to stress grammatical differences, to show that our language involves more than one sort of sentence, and he uses the notion of *picture* in part to distinguish genuine propositions—which he explicitly construes as pictures (*Bilder*)—from the sentences (*Sätze*) of pure mathematics, which, to repeat, he views as merely apparent propositions (*Scheinsätze*), not as propositional signs. In the late 1920s, however, he explicitly remarks that a natural number is a form of representing (*eine Art der Darstellung*), and in contexts which I believe are essentially Tractarian in character. For numbers as modes of representing, see Wittgenstein, 1994, vol. 1, p. 7. On construing natural numbers as pictures, see Wittgenstein, 1973, pp. 220–26, esp. p. 223. An analogy between mathematical proofs and pictures remains a governing motif of Wittgenstein's later philosophy, and it, too, I suggest, has its roots in the *Tractatus* treatment of arithmetic. Cf. Frascolla, 1994, pp. 29, 33, 49; Marion, forthcoming; and Floyd, 2000.

37. Wittgenstein 1994–, vol. 4, p. 239: "x² = 1 has two roots versus on the table are 2 apples. The former is a grammatical rule of the variable. . . . Can I determine a variable by saying that its values should be all objects which satisfy a certain function? Not if I don't know this some other way—if I don't, the grammar of the variable is simply not determined (expressed)."

38. A full answer to Frege and Russell would require this, not least because of the need to account for (apparent) inferences like "four apples and three pears make seven fruits."

39. In Wittgenstein, 1973, p. 225, Wittgenstein is reported to have said that "*the method of representing numbers is the method of picturing*" (*die Methode der Darstellung der Zahlen ist die Methode der Abbildung*).

40. See, e.g., Diamond, 1991a, 1991b, 1997; Dreben and Floyd, 1991; Goldfarb, unpublished [1979], 1997.

41. Hofman (1998) gives an engaging account of a recent meeting of the Boston Colloquium for Philosophy of Science attended by Diamond, Dreben, Floyd, Goldfarb, Hacker, Hintikka, Hylton, Pears, and Ricketts at which the *Tractatus* was heatedly

discussed. McGinn (1999) gives an overview of the contrast between what she calls “therapeutic” and “metaphysical” readings of the *Tractatus*, defending an intermediate position between the two.

42. See, e.g., McGinn, 1999; Hacker, 2000.

43. For further discussion of this point, see Floyd, 1998b; Ostrow, 1999.

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Wittgenstein and the Liberating Word

Matthew B. Ostrow

I

A. Introduction

Wittgenstein, in conversation with Moritz Schlick, once characterized his fundamental goal in philosophy as follows: “Everything we do consists in trying to find the liberating word [*erlösende Wort!*]” (VC 77).¹ This remark was made in early 1930, nine years after the publication of the *Tractatus*: with its implication that the proper aim of philosophy is to free us from the hold of philosophy itself, it certainly would not seem out of place in the context of Wittgenstein’s later, explicitly “therapeutic”, thought (cf., e.g., PI 133). But, I claim, this statement could, with equal legitimacy, serve as a description of his early work as well. Indeed, the underlying suggestion here that a Wittgensteinian liberation might wait upon *the word*—that his object is always to find a *single* formulation of the problems of philosophy—makes the remark perhaps even more aptly applied to the *Tractatus*. After all, the idea that there is a kind of essential confusion to the pursuit of metaphysics is, as we shall see, absolutely central to this work. Far from viewing the young Wittgenstein as concerned to offer a new theory of logic, to continue—albeit in revised fashion—the philosophical legacies of Frege and Russell, I shall begin with the idea that he views all such endeavors as the ensnarement of thought. The *Tractatus*, I then claim, is nothing but an attempt at setting down in definitive fashion the way of release.

Still, it is not at once apparent how such “liberation” is to be conceived. For one might well grant that the *Tractatus* involves an attack on the core of metaphysics; such a concern is, after all, already evident in the preface, where Wittgenstein portrays his book as intending to show that “the problems of philosophy” rest on “the

misunderstanding of the logic of our language" (TLP, p. 27).² Nonetheless, the basic form of that attack, if not its details, could be construed in quite traditional terms. That is, it would be entirely possible to imagine as a kind of liberation the *refuting* of the misunderstandings and errors of the past. On this reading, then, Wittgenstein proclaims various philosophical positions to be "nonsense" (see, e.g., TLP 4.003, 5.5351, 6.51) in the sense that so many philosophers in the Western tradition have dismissed their predecessors' claims—namely, as being patently false or absurd. Now such a view has in fact been implicit in much of the literature on the *Tractatus*, beginning with Ramsey's review³ and the responses of the Vienna positivists, and continuing with the work of more contemporary commentators like Black⁴, Stenius⁵, Hintikka,⁶ and Pears.⁷ Characteristic of this approach—which would include a quite diverse set of interpretations—is the insistence on treating the Wittgensteinian attack as if it presented, in opposition to the tradition, a series of straightforward philosophical accounts: accounts of the proposition (the "picture theory"), the tautologous nature of logical truth, the ineffability of logical form, and so forth. These accounts are then criticized or modified by commentators in accordance with the demands that are presumably to be satisfied by a well-constructed philosophical theory.

One rather large obstacle to this approach to the *Tractatus* is represented by remark 6.54. Here Wittgenstein famously declares: "My propositions are elucidatory in this way: he who understands me finally recognizes them as nonsensical [*unsinnig*], when he has climbed out through them, on them, over them." It would appear difficult to treat the author of such a remark as someone who has intended to present a straightforward theory, a series of claims to be evaluated in terms of their truth-value.

What, then, could it mean for Wittgenstein to view his enterprise as a search for the liberating word? This essay is an attempt to address this question, to understand the ultimate purpose of the *Tractatus* in light of its startling declaration of its own nonsensicality. In the first half of the essay I review some of the responses the *Tractatus*'s penultimate remark has occasioned, in particular the influential work of Cora Diamond. I try to show how reflection upon the sorts of difficulties faced by these interpretations points us toward the adoption of what I term a "dialectical" approach to the text. In the second half I bring out what such an approach entails in a particular case through a close examination of the initial part of the *Tractatus*'s so-called picture theory. I attempt to make clear how the central distinction between showing and saying introduced at this point is intimately connected with a dialectical stance and hence with the notion of nonsense itself—that it is only when we see the interrelation of the central notions of nonsense, showing, and dialectic that we begin to understand what a Wittgensteinian liberation comes to in the end.

B. Responses to Remark 6.54

While the *Tractatus*'s attempt to dismiss its own propositions as nonsensical has by no means gone unnoticed in the literature, the responses to this move have been quite multifarious. Perhaps most commonly the tendency among commentators has been, even while noting 6.54, to ignore its consequences with respect to our understanding of the seemingly substantive details of the text. For such

readers, this remark is regarded as striking, but not as a central feature to be accommodated within a satisfactory interpretation.⁸ A second type of response involves an explicit attempt at softening the impact of the text's harsh self-assessment. One notable example of the latter strategy is Carnap's interpretation of the statements of the *Tractatus* as purely linguistic proposals.⁹ For Carnap, while philosophical propositions of the sort espoused by Wittgenstein (and the Vienna positivists) make no claims about the world and thus are not true or false, they are not like many traditional metaphysical assertions in being entirely nonsensical. Instead, legitimate philosophy is to be understood as consisting of elucidations, purely formal assertions that serve to clarify the logical syntax of the language of science. In this sense, they can be seen as having the empty character that Wittgenstein ascribes to the tautological propositions of logic.¹⁰

A second example of an attempt at moderating the *Tractatus*'s view of its own utterances is found in commentators like Anscombe,¹¹ Geach,¹² Hintikka,¹³ and Hacker.¹⁴ The strategy they employ is motivated by remarks such as TLP 5.62 ("In fact, what solipsism *means* is quite correct, only it cannot be said, but shows itself"); 4.115 ("[Philosophy] will mean the unspeakable by clearly displaying the speakable"); and 4.1211 ("Thus a proposition 'fa' shows that in its sense the object *a* occurs, two propositions 'fa' and 'ga' that they are both about the same object"). In these remarks and elsewhere, Wittgenstein seems to suggest that, while the attempt to state what is properly to be *shown* results in what he calls "nonsense", something intelligible is nonetheless thereby expressed. We are then led to suppose that Wittgenstein's propositions—if not the propositions of all metaphysics—are nonsense only in a special sense. To be sure, so such commentators continue, they are not strictly utterable, according to the standards of sensibility established by the *Tractatus*. Still, they somehow manage to convey to us important philosophical truths: at the end of the book we "know" that, in reality, the world is composed of facts, not things, that a common logical form binds together language and the world, that value lies outside of the world, and so on. Except, of course, we cannot actually *say* these things, but must only *think* them, silently to ourselves; or perhaps we may repeat them—grudgingly—to another, but always with the acknowledgment that in so doing we have transgressed the strictly *proper* bounds of sense.

More recently, Cora Diamond, beginning with her important paper "Throwing Away the Ladder," has presented a central challenge to this reading—and, indirectly, to the related interpretation offered by Carnap.¹⁵ Diamond, in effect, attributes to Wittgenstein the position of Ramsey in his oft-quoted *criticism* of the *Tractatus*'s notion of showing: "But what we can't say, we can't say and we can't whistle it either."¹⁶ That is, refusing to countenance the possibility of any sort of meaningful gesture toward the ineffable, she bites the bullet on Wittgenstein's behalf and proclaims that, as far as the *Tractatus* is concerned, its own statements really are nonsense, plain and simple. There is no Tractarian counterpart to the Kantian *Ding an sich*, no deep features of reality that are somehow made manifest in Wittgenstein's utterances. Instead, we must take Wittgenstein at his word at 6.54 and realize that, in the end, all the pronouncements of his text are just so much gibberish.

Now I have a good deal of sympathy for, and have been much influenced by, Diamond's approach, and the elaboration of it provided by James Conant. None-

theless, I think one must take care to be as clear as possible about what this position really comes to, as it can easily serve to mislead. In order to bring into sharper focus the question of liberation with which we began, I want then to consider in some detail the Conant/Diamond interpretation.

C. The Diamond Reading

To begin with, Diamond suggests that Wittgenstein's conception of nonsense and his concomitant show/say distinction have their roots in Frege's so-called concept "horse" problem. Frege, in the article "Concept and Object", dismisses as incoherent Benno Kerry's contention that there can be concepts—like the concept "horse"—that can also function as objects. For Frege, the concept/object distinction is mutually exclusive: a concept by its very nature is predicative or, as he also puts it, "unsaturated"; conversely, the object, as a logical subject, serves necessarily to fill the gap left by the concept. In Kerry's example, then—"the concept 'horse' is a concept easily attained"—the first three words do not designate a Fregean concept, but, as is indicated by the appearance there of the definite article, a Fregean object. The peculiarity of having to maintain that the concept "horse" is not a concept is dismissed by Frege as an "awkwardness of language" (CP 185) and, moreover, as Diamond reads him, one he believes will not be encountered in a logically perfect notation like his *Begriffsschrift*. Thus, in the *Begriffsschrift*, statements about concepts and objects of the sort represented by Kerry's example—indeed, the very claim that there *is* a fundamental distinction between concepts (or functions) and objects—will not be formulatable. Instead, *that* there is such a division will come out in the distinctive use of the signs of the notation.

Diamond then terms remarks like the one expressing the difference between concepts and objects "transitional";¹⁷ their purpose is solely to lead us *into* the *Begriffsschrift*, to begin operating within its parameters. But once we have effected this transition, these remarks are recognized as completely without sense and are in fact inexpressible. Here we begin to see how Diamond draws the connection with Wittgenstein. For her, Wittgenstein is fundamentally concerned to extend to the whole philosophical vocabulary Frege's way of excluding notions like "function" and "concept". Toward that end, he is understood as having formulated a number of transitional statements—namely, remarks 1–6.522 of the *Tractatus*. All these claims, as transitional, will then have to be given up by the close of the *Tractatus*. After we have read—and understood—the text, we cannot suppose ourselves to be left hinting at some important truth with a statement like "The world is the totality of facts, not things," any more than we would suppose this about an attempt to state something about functions and objects from within the *Begriffsschrift*. Instead, 1.1, like every other remark of the *Tractatus*, is now seen as it really is—that is, as a claim completely on a par with "Socrates is frabble"¹⁸ or "T was brillig, and the slithy toves did gyre and gimble in the wabe."¹⁹ To attempt to ascribe any further content to Wittgenstein's claims is, for her, to "chicken out."

A quite natural first response to this approach focuses on the extraordinary expressive power it attributes to the supposed gibberish of the *Tractatus*. For clearly it is not at once obvious that this text's propositions are utter nonsense,

any more than it is obvious that the traditional claims of metaphysics have such a character; if it were obvious, if the *Tractatus*, Russell's *Principles of Mathematics*, Aristotle's *Metaphysics*, and so forth were plainly indistinguishable from Lewis Carroll's "Jabberwocky", none of these works could ever have the power to mislead. (Why haven't any books been written claiming to have established the nonsensicality of the Carroll poem?) Wittgenstein's claims are then assumed to be capable of *themselves* effecting the "transition" Diamond describes, of somehow *bringing us* to recognize the fact that they are, contrary to all appearances, absolutely devoid of sense. Indeed, in "What Nonsense Might Be,"²⁰ Diamond sketches in some detail Wittgenstein's account of the precise way in which philosophical nonsense is to be viewed, suggesting, in particular, that he rejects (what Diamond takes to be) Carnap's view that it consists of category errors.²¹ But this is as much as to acknowledge the special character of the Tractarian propositions, their dissimilarity to pseudosentences like "Socrates is frabble."²² The latter expression, after all, would seem to have none of the capacity for self-illumination that is thought to belong to the remarks of the *Tractatus*. We might say that it "shows us" that it does not make sense, but this is presumably a result of our understanding the syntax of the English language; if we did not already know that syntax, the expression surely could not itself *teach us* that (let alone why) it is meaningless. The point, in other words, is that if Wittgenstein's claims—which do not obviously thwart English (German) syntax—are assimilated to ordinary nonsense sentences, then it becomes difficult to explain how we could ever come to *recognize* them as such.

How would Diamond reply to this objection? I suspect she would suggest that it fastens on what is for her only a kind of rhetorical move in a polemic against a confused reading of the *Tractatus*. Her assimilation of metaphysical claims to "plain nonsense" is a means of denying the coherence of the notion of an ineffable content, but it should not be viewed as saying anything *more* than that; Diamond's aim here is not, in other words, to provide a genuine characterization of Wittgenstein's remarks. To demand from her an explanation of precisely how the plain nonsense of the *Tractatus* is illuminating could thus be said to miss the point: rather than seeking to provide an account of the mechanism of the text, Diamond's purpose is simply to steer us away from imagining any role for its propositions—*after* that "mechanism" has (somehow) performed its function.

D. What Does the *Tractatus* Bring Us To See?

We now can begin to see the real question that is opened up by "Throwing Away the Ladder" and its central idea of the *Tractatus* as a series of "transitional remarks". That question can be brought into full view by here asking ourselves, transitional to *what*? I certainly agree with Cora Diamond's premise that much of the original motivation for both the show/say distinction and the idea of "throwing away the ladder" comes from Frege's concept "horse" problem (as well as the related difficulty inherent in Russell's Theory of Types). But it would seem to be of paramount importance at this point not to push the parallel too far, to realize that Wittgenstein is shifting quite fundamentally the Fregean perspective. For let us grant for the moment that Frege has a full awareness of the implications of the idea of extra-Begriffsschrift "elucidations". Still, it must be acknowledged that

there exists for him a concrete means of avoiding the utterance of such statements—namely, by always working within the confines of his formal language. In other words, Frege's (supposed) contention that certain prose judgments (the “elucidations”) can ultimately be transcended gains its force from the fact that one can operate perfectly well with his *Begriffsschrift* without ever making such judgments. So, for example, a statement like “There are functions and objects” cannot even be formulated within his “concept script”—“ $\exists x \ & \ \exists f$ ” is not a well-formed formula—but the language nonetheless allows us to *use* these notions in the formalization of logical inferences. But what is the domain in which Wittgenstein would have us operate once we have dispensed with the elucidations that constitute the *Tractatus*? There is of course a long tradition of Tractarian interpretation, going back to Russell's introduction to the book (TLP, p. 8), which views Wittgenstein as concerned with laying down conditions for an ideal language. However, while it is unquestionable that the notion of a canonical *Begriffsschrift* plays an important (if extremely unclear) role in the *Tractatus*, it is equally certain that Wittgenstein has not actually *provided* us with any such language. We cannot confuse what are, at best, indications of some of the elements of a proposed formalism—such as, e.g., the absence of a sign for identity—with Frege's systematic specifications in the *Begriffsschrift* and the *Grundgesetze*. The point, then, is that despite Wittgenstein's talk of employing a symbolism that “excludes” the “errors” of traditional philosophy (see TLP 3.325), at the end of the *Tractatus* we remain very much within the context of our “ordinary” language, the same language in which the nonsensical propositions of metaphysics were originally formulated.²³

The whole idea of an adequate notation can therefore only be part of Wittgenstein's way of leading us to a new *perspective* on logic, as opposed to the adoption of an actual new language. One might then describe the central problem that Diamond's work points us toward as one of becoming clear on the nature of this perspective, once we understand that it is not embodied, as it were, in a formal language, in a specifiable *method* for eliminating the metaphysical pseudo sentences. How are we to characterize what the *Tractatus* brings us, in the end, to see? Given the difficulties that we saw above in the attempt to describe that insight in terms of the *literal* unintelligibility of the language of metaphysics, it may be tempting at this point to reach for a notion of “deep nonsense”. The propositions of the *Tractatus* really are nonsense, one will now maintain, except not in the plain, garden-variety sense. They violate not ordinary syntax, but a deeper underlying structure—what the text refers to as *logical syntax* (see TLP 3.325, 3.33, 3.334, 6.124). We can then hold that it is just toward the recognition of the claims of all metaphysics as nonsense in this special sense that the text aims to bring us.

But this strategy is less promising than it may initially seem, as the appearance of the term “logical syntax” in the above purported exposition of the text's central purpose should indicate. For the necessity of here bringing in the notions of the *Tractatus* itself—the very notions we have presumably “thrown away” at the book's close—indicates the hollowness of supposing that we have, as yet, proffered any sort of explanation. Indeed, one begins to wonder about the coherence of even *asking* for an explanation in this context. The problem now appears to lie not merely with how to characterize the text's point—whether to describe it as the exposing of deep nonsense or plain nonsense—but with the very notion that we might “characterize” that point at all.

The difficulty, then, is that we are from the start assuming that the statements proclaiming the nonsensicality of the *Tractatus*'s remarks could be *true*. What we are beginning to see, however, is that Wittgenstein is concerned precisely to deny the possibility of a general assessment of the text's propositions, and indeed of the nature of any metaphysical claim. What we are beginning to see, in other words, is that for Wittgenstein, a sentence like “The world is everything that is the case” is nonsense” is *itself* nonsense.

This may seem to leave the would-be reader of the *Tractatus* in a difficult, if not impossible, position. To some, the above claim will appear as a kind of *reductio* of the whole attempt to read this text.²⁴ Still, while I by no means wish to downplay the peculiarity of the position in which we find ourselves, I would urge that the situation is perhaps not quite so dire. Let us then consider these three sentences:

1. “The world is everything that is the case” is nonsense.
2. “The world is everything that is the case” is nonsense.
3. The world is everything that is the case.

The appearance of paradox in (1) would seem to stem from the assumption that the terms that compose this sentence are all used in their ordinary senses—as if we were here committed to asserting the “plain nonsensicality” of the attempt to say anything whatsoever about Wittgenstein’s remarks. However, it is important to emphasize that the term “nonsense” must be interpreted in the same sense in its second occurrence in (1) as in its first occurrence within that sentence (i.e., as it occurs in (2)). That is, we understand what it means to ascribe this property to attempts to characterize statements of the *Tractatus* only to the extent that we understand the meaning of the predicate in the text itself. But to deny, as we have, the possibility of a general characterization of the text’s propositions is just to call into question the possibility of making straightforward assertions about “the meaning” of this predicate in the *Tractatus*; it is to claim that an understanding of the term “nonsense” can only be attained through viewing in what Wittgenstein would regard as the appropriate way the (Tractarian) sentences to which it is appended. Precisely *this* point is then expressed by (1). This sentence serves to reflect the reducibility *within* the *Tractatus* of (2) to (3). The sentence (1) is therefore not itself to be construed as nonsense, but rather as a meaningful and—I would claim—true statement about Wittgenstein’s use of the string “nonsense”.

At the same time, however, we must recognize the very limited nature of this claim; we must recognize how our capacity to make accurate statements about the *Tractatus* comes at the price of a restriction on their un informativeness. For while the above description of the role of certain signs in this text may be correct, again we must emphasize that this as yet tells us nothing about what those signs mean. Surely, though, the meaning of Wittgenstein’s signs is precisely what any commentary on the text is ultimately concerned to elucidate! In an interesting way, then, the Fregean concept “horse” dilemma can be seen to extend not only to the *Tractatus*, but also to any interpretation of the *Tractatus*: the commentator now finds himself in the position of, like Frege, ultimately having to ask for “a pinch of salt”; he must appeal to his reader to, as it were, jump into Wittgenstein’s text, to begin *using* its language (or, at least, his own interpretation thereof). But this is as much as to admit that our fundamental question about what the *Tractatus*

brings us to see can permit of no answer. Or, better, it is to admit that whatever is proposed as an answer cannot take the form that one will almost instinctively require of it. For while we can offer restatements, in putatively clearer terms, of what we will claim to be Wittgenstein's point, the force of these considerations is to deprive such restatements of any privileged status, to lead us to see that they must stand on the same level as the propositions of the *Tractatus* itself.

E. A Dialectical Approach

It is now possible to describe in a new way Wittgenstein's declaration of the nonsensicality of his own propositions at 6.54. Rather than a neutral summing up of the real purpose of the *Tractatus*, this remark would seem to function as a way of orienting us toward the text as a whole, of indicating how we are to read it. Nonsense, we might say, forms the lens through which all Wittgenstein's propositions are to be viewed: we grasp his point (we understand *him*, as he puts it in 6.54) just when we are inclined to understand these remarks *as nonsense*. That, of course, is not to explain the meaning of the term "nonsense" in the *Tractatus*, but to face us back toward the text. It is to suggest that the nonsensicality of Wittgenstein's propositions only emerges through a detailed consideration of those propositions themselves, that it cannot be understood apart from such a consideration.²⁵ My contention, in other words, is that the Wittgensteinian view of the nature of his own claims, of philosophy generally, is not expressible in some self-standing formula, but is rather given entirely in and through the recognition of an intrinsic instability in a particular kind of utterance; it is, as one could put it, contained in seeing *how* our philosophical assertions change their character, *how* they undermine their own initial presentation as straightforward truth claims.²⁶

In different terms, what this discussion helps to make evident is the fundamentally *dialectical* nature of Wittgenstein's thought in the *Tractatus*.²⁷ It brings to the fore the extent to which we are, at every juncture of the book, engaged with the very metaphysics that is apparently being disparaged. We now see that the text's remarks have no purpose except for one who is genuinely tempted by the illusions the *Tractatus* aims to expose; Wittgenstein's "elucidations" depend for their effect on a prior yearning for the deepest—and therefore, we may come to say, most empty—philosophical *Gedanken*.

With this idea in mind, we can begin to better understand the nature of the liberation I have claimed the *Tractatus* aims to bring about. Given the emphasis of so much recent literature on 6.54, one might well suppose that this remark was in fact the text's final statement, that Wittgenstein leaves us with his pronouncement of the nonsensicality of everything philosophical. In fact, though, the *Tractatus* ends with proposition 7's call for silence: "Whereof one cannot speak, thereof one must be silent." Wittgenstein's claims, it would seem, find their real fulfillment not in what we say, but in what we *do*. But this is precisely what is required by the text's stance, as we have characterized it. Wittgenstein's final remark brings out how we are in a sense violating the spirit of the text every bit as much by proclaiming the complete and utter nonsensicality of metaphysics, as by proclaiming, for example, that the number 1 is, really is, an *object*. It makes clear that the charge of "nonsense" against philosophy is not a claim alongside the claims of science, but another move in the Wittgensteinian dialectic. That is

not to deny that the move is one of particular importance. But if that dialectic's purpose is to be achieved, if it is to lead us to "see the world rightly" (TLP 6.54), it must ultimately culminate in its own *cessation*. Liberation, for Wittgenstein, is nothing other than the end of philosophy.

It might then seem that we would best make the *Tractatus*'s point by stopping our commentary right here. Of course, Wittgenstein's own continued preoccupation with the problems of philosophy indicates how difficult it can be simply to remain silent. But it is not merely psychological compulsion that might lead us to continue—or, more important, that led Wittgenstein himself to go on with philosophy. For assuming that our aim really was to cease speaking metaphysics, what is it exactly that we are not to say? Consider my own earlier reference to Wittgenstein's desire to preclude a "particular kind of utterance." What *kind* of utterance, then? We, of course, want to say "philosophical" or "metaphysical"—but do we really know in advance the extension of those concepts? We are faced here with the essential difficulty of Wittgenstein's dialectical enterprise, the dilemma inherent in the attempt to "draw a limit . . . to the *expression* of thoughts" (TLP, p. 27, my emphasis). That is: we do not know beforehand exactly what is to count as an illicit, metaphysical claim; we in fact *cannot* know this (here we have what the later Wittgenstein would have called a "grammatical point"), since to do so would be to think what the *Tractatus* aims to reveal as not really thinkable.²⁸

We could say, then, that the central task of the *Tractatus* is one of somehow delineating the class of those utterances it seeks to eviscerate. For the young Wittgenstein at least, this aim is achieved by becoming clear on what he takes to be these utterances' *essence*—the fundamental impulse that leads us to make them, the single question that he imagines to lie at their heart. Just this view of a unitary core to the problems of philosophy is expressed in Wittgenstein's audacious preface claim to have found "on all essential points, thei[r] final solution" (TLP, p. 29). The same idea also appears in the *Notebooks*, where Wittgenstein twice speaks of the sense that his seemingly multifarious investigations are all manifestations of a "single great problem" (see NB 23, 40). The *Tractatus*, I claim, is then really nothing but Wittgenstein's extended attempt to characterize that single great problem, the root of the drive toward metaphysics.

Now this whole line of thought may sound suspiciously reminiscent of the uncharitable reading of Cora Diamond's notion of transitional remarks—as if I were suggesting that Wittgenstein's remarks first make some sort of sense and then subsequently become nonsensical. But my claim is that, for Wittgenstein, the revealing of the essence of metaphysics and the "demonstration" of the nonsensicality of metaphysics are, in fact, two sides of the same coin. To read the *Tractatus* dialectically, in my sense of the term, is to recognize that the successful characterization of philosophy *is* its dismissal as *Unsinn*. Wittgenstein's enterprise is an attempt to lead us to a view of metaphysics so complete that it dissolves itself.

This notion of a kind of double aspect to Wittgenstein's remarks provides, I suggest, the key to an understanding of the show/say distinction at the heart of the *Tractatus*. I would like now to develop this point—and, moreover, to instantiate my own principle regarding the importance of returning to the details of the text—through a close consideration of one of that text's central apparent doctrines, the so-called picture theory. It is to this that we now turn.

II

A. Motivations for the Picture Theory

The picture theory is typically regarded as central to the teaching of the *Tractatus*. It has generated a volume of literature at least as great as any other portion of the text, although, predictably, little consensus has emerged as to its central point. To be sure, nearly everyone agrees that the initial discussion of picturing at 2.1–2.225 is meant to shed light on the proposition—propositions are explicitly described as pictures at 4.01—but how this end is achieved is by no means readily apparent. What does it really mean to bring to the fore the supposed pictorial character of the proposition? What sort of insight is the comparison between propositions and pictures supposed to yield? Now one standard interpretive strategy is to propose at the start some philosophical theory that is of primary interest to the *Tractatus*, and then show how the picture theory either undermines it or is somehow better equipped to deal with its shortcomings.²⁹ Still, while it is of course important to relate the discussion here to the views of Wittgenstein's philosophical influences—and I will in fact be concerned to suggest ways in which he is directing himself against the thought of Frege and Russell—I think it is necessary first to see how it emerges as a response to certain tensions within the text.

I want to begin, then, by noting the apparently shifting sense of the term “world” in the remarks leading up to the picture theory. In 2.04 “the world” is identified with the totality of *existent* atomic facts (*die Gesamtheit der bestehenden Sachverhalte*; my emphasis). Given the text's initial association of the world with all that is the case (1), of all that is the case with the totality of facts (1.1), and of these facts with the existence of atomic facts (2), we might have assumed that the totality of existent atomic facts constitutes the limit of the world. The subsequent introduction of the term “reality”, a notion with an apparent wider extension than “the world”, then comes as a surprise: “The existence *and non-existence* of atomic facts is the reality” (TLP 2.06; my emphasis). Is Wittgenstein imagining facts that lie outside of the world? The idea is in itself strange enough. But it would also seem to blur the *Tractatus*'s fundamental distinction between the fact as what happens to obtain in the world and the object as its logical condition. For it is very tempting to view the nonexistent atomic fact—the fact that is not but could be—as in some sense *another* condition of the possibility of the (existent) atomic fact. Or, equally complicating matters, we might be tempted to maintain the distinction between a fact and its condition, but then feel driven to posit possible *objects* (possible possibilities, as it were) as the logical bases of these nonexistent, merely possible facts.

Wittgenstein's position becomes still more puzzling when we look at the supposed further clarification of 2.06 provided by 2.063: “The total reality is the world.” The just-introduced distinction between “reality” and “the world” is now apparently denied or in some sense overcome; the *Tractatus*, we might say, both proposes the existence of facts outside of the world and implies that such an idea does not make any sense. Why this equivocation? What is the status of the “negative fact”, as Wittgenstein refers to the nonexistent atomic fact at 2.06? It is, it would seem, some version of the ancient problem of the nature of “what is not” which confronts us at the close of the 2.06s.

A second tension in the remarks ending the 2.0s centers around the question of the relation between atomic facts. At 2.062, in keeping with the 1.21's claim of the logical independence of facts, Wittgenstein claims: "From the existence or non-existence of an atomic fact we cannot infer the existence or non-existence of another" (TLP 2.062). This assertion, however, would seem to run counter to what is maintained just three remarks previously: "The totality of existent atomic facts also determines which atomic facts do not exist" (TLP 2.05). One wonders how one set of facts can "determine" a second set if these facts are entirely independent of each other. It might be tempting to suppose that Wittgenstein is introducing the possibility of a relation of determination distinct from one of logical *inference*. But that would fly in the face of 6.37: "A necessity for one thing to happen because another has happened does not exist. There is only *logical* necessity" (TLP 6.37). The solution, then, is not to view positive and negative atomic facts as separate or independent notions: we can infer the nonexistence of $\sim A$ from the existence of A (A "determines" $\sim A$ and vice versa), but only because, from the perspective of A , $\sim A$ does not count as *another* fact; which of these is "positive" and which "negative" cannot be answered, for the notions are wholly correlative. For Wittgenstein we thus have no way of meaningfully *saying* that there is no negative fact corresponding to " $\sim A$ ".

Once more, then, we run up against the intrinsically ambiguous status of "what is not." And now we see that the notions of logical relations between facts, of a "reality" that extends beyond the limits of the world, and of nonexistent *Sachverhalten* are all closely linked. It is through the picture theory that we begin to see how this cluster of notions relates to the show/say distinction at the heart of the text.

B. Clarifying the Problem

The move to a discussion of pictures is initiated abruptly, without any explanation: "We make to ourselves pictures of facts" (TLP 2.1). It is important to note at once the emphasis here on picturing as an activity: we *make* pictures of facts and to *ourselves*, for our own purposes. From the beginning, it would seem, Wittgenstein is viewing the picture not as an autonomous, self-interpreting entity, but rather as something to be understood within the context of its use.³⁰ This emphasis is indeed made even more explicit by Wittgenstein in a later conversation about the *Tractatus* with Waismann. After suggesting that the *Tractatus*'s notion of a picture was used to highlight certain important features of a proposition, Wittgenstein goes on: "I could also use a measuring-rod as a symbol, that is, insert a measuring-rod into a description and use it in the same way as a proposition. You may even say, In many respects a proposition behaves just like a measuring-rod, and therefore I might just as well have called propositions measuring-rods" (VC 185). To speak of a measuring rod is of course to speak of something that cannot be understood apart from its connection with human purposefulness—presumably no one will suppose that a ruler might apply itself to the object to be measured. We note, then, that the *Tractatus* does in fact at one point draw the analogy between a picture and a measuring rod (*Masstab*) (2.1512). The above passage would appear to suggest—and our later discussion will make clear—that the comparison is to be taken quite seriously, that it gives us the aspect of the picture that is crucial for Wittgenstein's whole account.

The significance of this emphasis on the picture as it is used is not yet apparent. But we shall see how some such idea will already be essential in making sense of the next several remarks. First, then, 2.11: "The picture presents [*vorstellt*]³¹ the states of affairs [*Sachlage*] in logical space, the existence and non-existence of atomic facts [*Sachverhalten*.]" The reference here to nonexistent atomic facts calls us back to 2.06 and the apparent problem of the negative fact. Taken as it stands, however, this remark seems to render the status of nonexistent facts still more mysterious. How could the picture considered in itself *present* a negative fact? Is what is not present in the picture imagined also to be part of what it presents? The elements of the picture are said to stand for (*vertreten*) things (2.131), but this seems to say nothing about the possibility of depicting things that do not exist, or at least that do not exist in the configuration presented by the picture.

It is 2.15 that allows us to begin to understand the fundamental direction of Wittgenstein's account: "That the elements of the picture are combined [*verhalten*] with one another in a definite way presents [*vorstellt*] that the things [*Sachen*] are so combined with one another." The picture, as 2.141 states, is a fact (*Tatsache*), and it would seem to be just this "facticity" that allows it to portray. Now Wittgenstein is here implicitly challenging Frege's assimilation of a proposition to a name, as is often noted:³² propositions as pictorial *facts* must be sharply distinguished from names, which, as analogues to the pictorial elements, serve only as proxies for objects. But it is essential to understand the real purpose of Wittgenstein's attack on the Fregean conception. For it is tempting to suggest, even while pointing out Wittgenstein's difference with Frege, that the *Tractatus* is here ultimately concerned with the problem of how some particular picture fact can be connected with the appropriate world fact. Thus Black,³³ for example, sees the picture theory as an attempt to give an account of how the relational proposition "aRb" could mean that some *specific* state of affairs cSd obtains. To state the problem in this way, however, is just to reiterate the Fregean construal of the proposition (picture) as a kind of name (i.e., pictorial element). It is to view the sense of the picture as something to which the picture *corresponds*, a kind of entity to which that picture must somehow be securely fastened. And, indeed, we can now see the connection of such a Fregean view with the perplexity over the issue of negative facts. For on this conception we are naturally led to wonder about what it is that can form the reference of pictures that depict facts which do not obtain. It is then a short step—a step not actually taken by Frege himself, but one toyed with by Russell³⁴—to begin postulating a special domain of nonexistent facts, a shadowy realm of all that *is not* but *could* be.

Wittgenstein's emphasis on the facticity of the picture is in part meant to get at the confusion that would bring us to make such a move. Rather than leading us to imagine picturing as a relation between a new kind of entity—facts—we are urged to see that no real relation is at issue in the first place. Wittgenstein states that it is just *that* these proxies for objects stand to each other in the way they do that says things in the world so stand. This is meant to bring out that what is doing the expressing cannot be the set of pictorial proxies as such, but rather our having *taken* these proxies as a certain kind of fact about the world. To speak meaningfully of the picture as a "model of reality" (TLP 2.12) thus presupposes understanding the picture within the context of its application; *the* picture, we might say, is in a certain sense an abstraction from the process of picturing.

With this emphasis on the activity of picturing in place, it now becomes clear that in speaking at 2.11 of what the picture “presents”, Wittgenstein is conceiving of the picture in relation to those facts it can be *used* to depict. The picture presents the existence and nonexistence of atomic facts precisely because it is the same picture that allows us to say that some fact either is or is not the case. This idea Wittgenstein indeed came back to over and over again in his *Notebooks*.³⁵ His point is to bring out now more sharply the way in which the “negative” fact is dependent on or given entirely by means of its “positive” counterpart. In conceiving of matters in this manner we will be far less tempted to reify the negative fact, to attribute to it some special ontological status. (Recall Wittgenstein’s initially suggesting that the negative fact lies in some sense outside the world.) At the same time, we see that from the point of view of logic there is nothing sacrosanct about the existent fact either; positive and negative fact stand on the same level, a contrast between two uses of a picture. (This was the point of Wittgenstein’s identifying the total reality and the world at 2.063.) Positive and negative fact are co-equal inhabitants of logical space, introduced together at 2.11.

C. Pictorial Form and the Essence of Representation

In recognizing the interdependence of these notions, we see that our proper concern here must be to go behind the positive and negative fact, to account for the possibility of both together. Rather than attempting to explain how a particular picture can correctly designate some fact in the world, the *Tractatus* suggests that philosophy must properly inquire into the possibility of representation, true or false.³⁶ How are our propositions capable of representing *all* and *only* states of affairs in the world? How it is that these propositions are guaranteed to make sense in the first place? These questions give expression to the “single great problem” that, as I suggested above, the early Wittgenstein sees as lying at the heart of the philosophical enterprise. It is in the next several remarks of Wittgenstein’s discussion of the picture that we can see the core of his response.

In the context of picturing, then, the problem will have to do with specifying the *possibility* of the picture’s “presenting” (in Wittgenstein’s special sense of this term)³⁷ what it does. Now the possibility of the pictorial fact, of the particular determinate structure itself, the *Tractatus* calls the picture’s “pictorial form”³⁸ (*Form der Abbildung*; TLP 2.15). Remark 2.151 states that this pictorial form is at the same time the possibility of the things being related to each other in the same way. And, indeed, just *that* the pictorial elements and the things depicted have the same possibilities of combination is standardly taken to be the central contention of the picture theory.³⁹ Wittgenstein’s answer to the question of how the picture—and hence language—can always be about the world is thus supposedly to be: they share a form.

But while I certainly do not deny that Wittgenstein speaks of something in common between the picture and what is pictured—this is explicitly asserted, for example, at 2.16 and 2.161—the important question concerns his attitude toward this claim. In that regard, I insist that, as an *explanation* of how the picture is always capable of depicting the world, the strategy of taking recourse in talk of identity of form is empty; it amounts to nothing more than the claim that depicting the world is possible because the world has the possibility of being depicted. Our

proper aim, then, must be to understand how we are driven into taking this apparent assertion as informative, and why it is that, despite appearances, we cannot here draw a meaningful distinction between the picture and what it depicts.⁴⁰

Toward that end, let us first seek to become clearer about what it would mean to specify the picture's pictorial form (*Form der Abbildung*). What is wanted is an account of the coordination of pictorial elements and objects referred to at 2.13 and 2.131, the coordination that 2.154 terms the picture's "pictorial relationship"⁴¹ (*die abbildende Beziehung*). Clearly one dimension of this correlation, the choice of particular pictorial representatives, is arbitrary: if my aim is to represent a book lying on a table, it is entirely up to me whether to use rectangles, squares, color patches, or what have you to stand for the objects composing that fact. At 2.131 Wittgenstein uses the term *vertreten* to designate this connection between pictorial element (and, later, name) and thing. But setting up this sort of arbitrary correlation would not by itself suffice to ensure that any picture I construct will portray a possible state of affairs. What is to stop me from, for example, placing a pictorial representative of an event into a pictorial representative of a *hole*,⁴² or picturing a situation in which red is *louder* than green? Evidently, the assuming of a particular outward appearance is not enough to guarantee that a set of correlated pictorial elements is a genuine picture in the *Tractatus'* sense of the term. Only if the pictorial representatives have all the same possibilities of combination as their real-world counterparts will we say that they are *really* representatives of the latter. The legitimacy of the arbitrary correlations we set up would appear to depend in some sense upon a deeper coordination of form.

The pictorial form has to do with this idea of an inner, nonarbitrary connection of the picture and reality. As the possibilities of combination common to the pictorial elements and the objects they stand for, it constitutes the ultimate ground of our ability to picture the world. In laying bare the pictorial form, then, it seems that we would come to see the *essence* of representation; that is, the a priori core both of our means of representing and of what is represented. And that is to say that the specification of the pictorial form would constitute nothing other than the fulfillment of one of the most fundamental tasks of philosophy, as traditionally conceived.

D. The Disintegration of the Question

Now Wittgenstein's way of attempting to drain this whole inquiry of its philosophical allure begins to become apparent when we closely consider his remarks:

- 2.151 The pictorial form is the possibility that the things are so combined with one another as are the elements of the picture.
- 2.1511 *That* is how a picture is attached to reality; it reaches right out to it.
- 2.1512 It is laid against reality like a measure [*ein Massstab*—i.e., a ruler].

We must ask ourselves what it means in this context to compare a picture with a ruler. A useful way of approaching this question is suggested by remarks 43 and 44 of the *Philosophical Remarks*. Here, after again stressing the importance of the picture-ruler comparison, Wittgenstein points out that the possibility of measuring in general does not presuppose a particular length for the object to be measured. All that is necessary is for me to have a way of using the ruler, of applying it to the

world.⁴³ The *Tractatus* similarly suggests that the possibility of *depicting* in general does not assume the existence of some fact or other in the world, but only a way of picturing—a way of projecting our pictures. And since, as we have seen, our hold on the picture is parasitic upon a notion of the picture in use, the method of projection is already given, with the pictorial fact. It follows that a picture, simply in virtue of *being* a picture in the *Tractatus*'s sense, *must* always present some possible state of affairs. This, indeed, is just what Wittgenstein suggests at 2.1513: "According to this view the pictorial relationship which makes it a picture also belongs to the picture." What makes something into a picture of the *world*—the pictorial elements being correlated with the things they stand for—is also what makes it into a picture in the first place. Conceived in this way, a picture is seen to carry within it its inner coordinations with reality and thus cannot fail to depict.⁴⁴

To say that its inner coordinations with reality are part of the picture, however, is to bring out the insubstantiality of the pictorial form, to make evident how that form is built into the notion of picturing from the start. (As always in the *Tractatus*, the appearance of substantial necessity—in this case the necessity of the picture's attachment to the world—is a mark of one's failure to have made a genuine claim.) What Wittgenstein's account suggests is that the essential possibilities of combination common to the pictorial elements and the objects are given after the fact, *through* the projection of the picture on to reality. Rather than having an *a priori*, normative status, the pictorial form is parasitic on our way of picturing with a picture.

Wittgenstein's point can be brought into still sharper focus if we reflect on the notion of space, one of the pictorial forms referred to at 2.171.⁴⁵ The above discussion is meant to get us to see the incoherence of supposing that in order to construct a spatial picture we must "have" beforehand a notion of space to function as a kind of constraint. It is not, in other words, as if in constructing a picture of, say, a book lying on a table, I must take care to have chosen proxies not only for these objects, but also for their capacity to assume spatial relations. Instead, that these objects are in space—their spatial form—is revealed through my being able to construct a picture in the first place: the fact that these two shapes can be correlated with that book and that table in such a way as to make a genuine picture *gives us* part of what we mean by "space". The limits of my ability to make a picture of this kind then constitute the limits of my notion of spatiality—which is to say that this pictorial form is not a constituent of a given picture, but part of its way of being related to the world, part of what this picture *is*. That, however, would seem to be just another way of stating the point of 2.172: "A picture, however, cannot depict its pictorial form: it shows it forth [*weist sie auf*]."

The Tractarian notion of showing is thus introduced at the crucial moment in Wittgenstein's discussion of the picture. Viewed in isolation, this notion naturally leads us to imagine the existence of necessary, but ineffable, features of reality. With the above considerations in mind, however, we see that the real aim of the *Tractatus* is to turn such an idea on its head: the show/say distinction functions as part of a dialectical attempt to dissipate the urge to *look for* any such "necessary features." In suggesting that the pictorial form can only be shown, Wittgenstein is trying to make evident how the sort of ground for meaning sought by philosophy is ultimately inseparable from our ongoing methods of representing the world. The real point is then to express the emptiness of the question motivating our whole inquiry; it is to bring us to recognize that everything we *would* want to say about

the essence of representation is already given in the right way of viewing the application of the picture.

But notice that, without the initial question, the idea that there is any contrast with “what can be said” has no role. That is, to put it somewhat crudely, it means nothing to state as a general, self-standing claim either that something is or that something is not shown, for example, by this particular picture of a book lying on a table. Instead, the possibility of introducing this language is dependent on the prior existence of one for whom the depicting ability of the picture could constitute a great question, a gap in our understanding of the world; we take recourse to showing only as a kind of answer to *him*. To read the *Tractatus* dialectically is in this way to recognize that the notion of showing has its life entirely in relation to the metaphysical temptations which the book aims to eradicate.

E. The Liberating Word?

At the same time, however, we must come to recognize the troubling consequences of this idea for the sort of intellectual liberation I have said the text aims to effect. For if the Wittgensteinian attempt at undermining metaphysics can only be understood in relation to the attempt to carry out still more rigorously philosophy’s traditional goals, if, that is, the insight into emptiness really is, for Wittgenstein, the flip side of the insight into essence, it begins to seem as if our grasp on emptiness must be as precarious, as tenuous, as our grasp on essence. The “framing remarks” of the preface and 6.54 notwithstanding, it would appear that nothing *compels* us to maintain the stance of silence advocated at the close of the text, to cease the dialectical activity it begins. And that is to say that what I am describing as an attempt to *free* us from metaphysical illusion can just as easily serve to lead us more resoundingly into its depths. Nor is this a mere idle possibility, as might be suggested by our brief overview of the almost exclusively metaphysical responses the *Tractatus* has occasioned. What does this mean for our understanding of the ultimate point of the book, of the dialectical approach it exemplifies? How stable is the state of ultimate clarity with which this text tantalizes us? If the *Tractatus* wants us to understand philosophy as, at its heart, nothing but illusion, it also teaches that philosophical reflection is itself our means of genuine escape. For Wittgenstein, it would seem, liberation comes only by way of the most uncertain of paths.

Notes

I would like to thank Juliet Floyd and the late Burton Dreben for their help with the many drafts of this essay. I am deeply indebted to both of them.

1. Cf. NB 39, 54; PG 193; PO 165; and CV 33.

2. This translation is from the original Ogden version of the *Tractatus*. I will for the most part rely on Ogden but will occasionally make use of the Pears-McGuinness translation when it seems clearer.

3. Ramsey, 1965.

4. Black, 1964.

5. Stenius, 1960.

6. Hintikka and Hintikka, 1986.

7. Pears, 1987.

8. Although I believe that it is a mistake to downplay the significance of 6.54 in this way, I do not wish to suggest that such approaches to the *Tractatus* result from mere oversights on the part of commentators. Instead, some justification for this strategy can be offered: one might, e.g., suggest (as Pears did in a 1998 meeting of the Boston Colloquium for the Philosophy of Science) that this remark was inserted by Wittgenstein simply as a means of protecting himself from criticism. Of course, it remains for such an interpretation to account for 4.1272 at the center of the *Tractatus*, whose implication would seem to be much the same as 6.54; also, it must explain the remarks in the *Notebooks* on pages 45 and 50, where Wittgenstein is already found to be asserting that claims about the existence of simple objects—the sort of seemingly meaningful claim that dominates the opening section of the *Tractatus*—are nonsensical. Nonetheless, I think it is important to acknowledge that the invocation of such passages is unlikely to be conclusive; their significance, too, can be downplayed. And that is to suggest that what is ultimately at issue in all interpretive disputes of this sort are really questions of what *kind* of thinker we understand Wittgenstein to be.

9. See, e.g., Carnap's *Philosophy and Logical Syntax* (1935) for a statement of this way of responding to Wittgenstein's text. We must of course bear in mind that Carnap is writing always as a philosopher himself, and not simply as an expositor of Wittgenstein. He in fact explicitly acknowledges that his own position may differ from that of the *Tractatus* (see pp. 37–38 of *Philosophy and Logical Syntax*).

10. In the language of the *Tractatus*, we could say that Carnap views such formal assertions as *sinnlos* rather than *unsinnig*. Still, the real force of characterizing logical proposition as tautologies is itself, like almost everything in the *Tractatus*, very delicate and open to a variety of interpretations. See Dreben and Floyd's "Tautology: How Not to Use a Word" (1991) for an excellent discussion of the shifting senses of the notion of tautology.

11. Anscombe, 1965.

12. Geach, 1976.

13. Hintikka and Hintikka, 1986.

14. See Hacker's classic work *Insight and Illusion* (1986). Hacker presents an even more forceful argument for this view in his recent response (2000) to Cora Diamond's way of reading the *Tractatus*.

15. Diamond (1991). James Conant (1989, 1991, 1993, 1998, 2000) has further elaborated Diamond's interpretation. Warren Goldfarb independently argued for a related position in an unpublished but frequently noted 1979 paper. Dreben (unpublished), and Floyd (1998, 2000) have also more or less independently articulated a similar interpretation of the text.

16. Ramsey, 1965.

17. Diamond, 1991, p. 185.

18. Diamond (1991, p. 197) introduces this expression as a way of describing "a is an object," the sort of sentence that Wittgenstein explicitly dismisses as a "nonsense pseudo-sentence" (TLP 4.1272). But clearly her contention is that his own seemingly positive assertions have exactly the same status.

19. In "What Nonsense Might Be", "Jabberwocky" is put forth by Diamond (1991, p. 96) as an example of the kind of thing Wittgenstein has in mind whenever he speaks of philosophical nonsense.

20. Diamond, 1991, pp. 95–114.

21. Conant (1991, pp. 341–42; 1998, pp. 244–50; 2000, pp. 194–95) tends to emphasize this point quite strongly as well, arguing that, for Wittgenstein, philosophical nonsense resists logical segmentation and thus cannot be understood as resulting from the illegitimate combination of intrinsically intelligible components. Aside from my concerns about leaning too heavily on this particular formulation, I also worry that it can lead to attributing to Wittgenstein just the sort of theoretical doctrine that Conant most wants to avoid—as if Wittgenstein's central aim were now understood as one of giving an elaborate *account* of the nature of nonsense. Instead, it seems that we should say that the distinction, implicit especially in 5.4733, between what

Conant calls the “substantial” and “austere” conceptions of nonsense (2000, p. 176) belongs to the internal apparatus of the *Tractatus*, and as such should be seen as having the same status as the show/say distinction, the conception of logic as tautologous, and so forth.

22. Hacker (2000, p. 361) makes a similar point (although in a rather more contentious manner), suggesting that Diamond surreptitiously introduces a distinction between “plain nonsense” and “transitional nonsense”.

23. Goldfarb 1997, pp. 70–72 argues in much the same way in his discussion of Diamond’s *The Realistic Spirit*.

24. Indeed, Hacker (2000, p. 362) tries to use something very much like this claim as a way of dismissing the Conant/Diamond reading of the *Tractatus*.

25. Goldfarb (1997) again makes something like this point with his remark that, for Wittgenstein, “‘nonsense’ cannot really be a general term of criticism” (p. 71). Floyd (1998, p. 44) makes a similar assertion. Diamond, in her response to Goldfarb (1997, p. 80), appears to acknowledge the problem, understanding it as the question of whether Wittgenstein has “a general approach to the issue of irresoluteness.” She does not, however, attempt to provide a determinate answer.

26. Conant (1989, p. 266) presumably is saying something of this sort when he holds “I, over and over again, want to say something like this: ‘Wittgenstein’s (or Kierkegaard’s) teaching cannot be stated, it can only be shown’.”

27. In viewing this most dogmatic-seeming of texts as intrinsically dialectical, I am to some extent following Dreben (unpublished); Floyd (1998); and Goldfarb (unpublished manuscript, 1997). Needless to say, the idea of dialectic is rather slippery—I do not attempt to define this term—and I make no claim to be using it in exactly the same sense as any of these authors (indeed, it is not clear that they all understand “dialectical” in just the same way either). I think it is safe to say, however, if I may be forgiven the employment of a piece of Wittgenstein jargon, that there is at least a family resemblance among all these various uses of the notion.

28. Again this point is essentially made in the preface, when Wittgenstein asserts: “In order to be able to draw a limit to thought, we should have to find both sides of the limit thinkable (i.e., we should have to be able to think what cannot be thought)” (TLP, p. 3).

29. One recent example of this strategy is found in Ricketts’s “Pictures, Logic, and the Limits of Sense in Wittgenstein’s *Tractatus*” (1996). Ricketts sees the picture theory as motivated by a very specific problem with Russell’s multiple relation theory.

30. Schwyzer (1966, pp. 277–78) and Friedlander (1992, p. 96) also focus on this aspect of the picture theory.

31. Here I follow Friedlander (1992) in translating *vorstellen*, a term which Ogden here renders as “represent”, always as “present”. This is done to mark a systematic distinction from Wittgenstein’s use of the term *darstellen*, which, again like Friedlander, I translate as “represent”.

32. See, e.g., Mounce, 1981, pp. 23–25.

33. See Black, 1964, pp. 77–78.

34. See, e.g., *Principles of Mathematics*, p. 427.

35. Cf., e.g., these remarks from November 1914:

That two people are not fighting can be represented by representing them as not fighting and also by representing them as fighting and saying that the picture shows how things are not. We could represent by means of negative facts just as much as by means of positive ones. (NB 23)

In order for it to be possible for a negative atomic fact [*Sachverhalt*] to be given, the picture of the positive atomic fact must be given. (NB 24)

Negation refers to the finished sense of the negated proposition and not to its way of representing. If a picture represents what-is-not-the-case in the

aforementioned way, this only happens through its representing that which is not the case. For the picture says, as it were: "This is how it is not," and to the question "How is it not?" just the positive proposition is the answer. (NB 25)

36. The question which the picture theory begins to bring into relief is then the question Wittgenstein poses early on in the *Notebooks*: "What is the ground of our—certainly well founded—confidence that we shall be able to express any sense we like in our two-dimensional script?" (NB 6).

37. Recall that I always use this word as a translation of Wittgenstein's *vorstellen*.

38. I am here using the Pears-McGuinness translation of this phrase.

39. The *Tractatus* is in this way sometimes assumed to rest on the postulation of a fundamental "isomorphism" between language and the world. Hacker (1986, p. 61) and Stenius (1960, pp. 91–96), among others, put forward this idea. Black (1964, pp. 69, 90–91) prefers the term "homomorphism", but holds to the same basic notion of the centrality of this "hypothesis" to the *Tractatus* view.

40. This difficulty in assessing Wittgenstein's real orientation toward the question at hand calls to mind his remark in *Zettel*: "Here we come up against a remarkable and characteristic phenomenon in philosophical investigation: the difficulty—I might say—is not that of finding the solution but rather that of recognizing as the solution something that looks as if it were only a preliminary to it. 'We have already said everything.—Not anything that follows from this, no, this itself is the solution!' This is connected, I believe, with our wrongly expecting an explanation, whereas the solution of the difficulty is a description, if we give it the right place in our considerations. If we dwell upon it and do not try to get beyond it. The difficulty here is: to stop" (Z 314).

41. Again, I am here using the Pears-McGuinness translation of the term.

42. Cf. NB 108.

43. He also seems to suggest this idea in the *Notebooks*, p. 37.

44. By contrast, the mathematical notion of an isomorphism conceives of a one-to-one function mapping one *independently* given domain onto another.

45. We may leave it an open question as to whether the *Tractatus* will ultimately count space as one of the logical forms—i.e., as one of the fundamental categories of thought or language that will be made manifest in the elementary propositions. It is thus not yet decided whether "space" is put forward as an example of the results of logical analysis or as a mere analogy.

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Abbreviations used:

CP	"Concept and Object," in Frege's <i>Collected Papers on Mathematics, Logic and Philosophy</i>
CV	<i>Culture and Value</i>
NB	<i>Notebooks 1914–16</i>
PG	<i>Philosophical Grammar</i>
PO	<i>Philosophical Occasions</i>
PI	<i>Philosophical Investigations</i>
TLP	<i>Tractatus Logico-Philosophicus</i>
VC	<i>Ludwig Wittgenstein and the Vienna Circle</i>
Z	<i>Zettel</i>

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The Method of the *Tractatus*

James Conant

I've written a book called "Logisch-Philosophische Abhandlung" Nobody will understand it; although I believe it's all as clear as crystal.

Wittgenstein, *letter to Russell*, 13.3. 1919

How could Wittgenstein have believed that he had written a book that was as clear as crystal *and* that nobody would understand? One criterion of the clarity of a book is that most readers *are* able to understand it. The clarity to which Wittgenstein alludes is one that he anticipates will elude most readers. In this essay, I hope to shed some light on the sort of clarity Wittgenstein believed he had attained (and sought to impart) in the *Tractatus*. This requires shedding some light on the criterion that Wittgenstein himself furnishes, at the end of his book, for what it would be to understand the book: he says that anyone who understands him eventually recognizes his propositions as *nonsensical*.

Two standard readings of Wittgenstein's *Tractatus* have shaped the ways in which contemporary philosophers think about nonsense—both as a term of philosophical criticism generally and as a term that figures specifically in the pages of the *Tractatus*. These readings might be termed the *positivist reading* and the *ineffability reading*, respectively. In this essay, I argue (1) that these two mutually opposed readings of the *Tractatus* presuppose a common conception of nonsense, one which the *Tractatus* itself sought to undermine; (2) that the sources of Wittgenstein's own conception are to be found in the work of Frege; (3) that, once we trace this conception back to its sources, we will be in a better position to see what sort of criticism (of Frege and others) the *Tractatus* seeks to advance; and (4) that, once we see this, we will be in a better position to understand the relation between Wittgenstein's early and later work.

I. Two Readings of the *Tractatus*

How do I arrive at the concept of a kind of description that
I cannot possibly give?

Wittgenstein, *Remarks on the Philosophy of Psychology*

What I am calling “the positivist reading”¹ of the *Tractatus* came first historically and has allegedly been discredited. But the reports of its demise are greatly exaggerated. It continues to shape most interpretations of Wittgenstein’s early work and, in particular, of what Wittgenstein means by the term “nonsense.” According to the positivist reading, the Wittgenstein of the *Tractatus* holds that much of what we say in ordinary language is at best misleading or vague and at worst empty. The logically imperfect character of ordinary language renders such defects in our ordinary modes of expression inevitable and makes possible the confusions of the metaphysicians. The defects in the metaphysician’s utterances are disguised by the logically imperfect character of ordinary language, causing the metaphysician to be attracted to forms of words that purport to describe various superempirical states of affairs. In such cases, the utterances of the metaphysician can be demonstrated to be nonsense by exposing them as logically (or conceptually) flawed, where these flaws are to be traced to specifiable infringements upon the conditions of meaningful discourse. Particular versions of the positivist reading differ over matters of detail—some will say these infringements arise through violations of “the principles of logical syntax,” others will appeal to “criteria of cognitive significance,” yet others to “rules of grammar.” What all such readings have in common is the idea that Wittgenstein seeks a theory that would enable him (a) to furnish a method for exposing the sentences of metaphysicians as intrinsically nonsensical, and (b) through the application of such a method to demarcate meaningful from meaningless discourse.

What I am calling “the ineffability reading”² arose in opposition to the positivist reading. According to the ineffability reading, the *Tractatus* seeks to distinguish between *kinds* of nonsense—in particular, between *misleading* nonsense and *illuminating* nonsense. According to this reading, the former sort of nonsense is to be committed to the flames, but the latter is to be cherished for a special sort of insight it is able to impart. According to the ineffability reading, the error of the positivist reading is to be traced to its twofold assumption that (1) for the *Tractatus* all nonsense is misleading nonsense, and (2) someone would traffic in nonsense only if he did so unknowingly.³

In recent decades, the positivist reading has fallen out of favor, and the ineffability reading has become the standard reading. This development can be traced to the brute fact that the positivist reading of the *Tractatus* faces—as a *reading* of Wittgenstein’s text—a number of exegetical embarrassments. The most notorious of these is what the book has to say concerning the status of its own propositions: that they are nonsense. The work openly declares that it traffics in nonsense knowingly. The most notorious of these declarations is to be found at the climax of the work, where the reader is presented with the trope of the ladder: the reader is told

that she has been offered a series of nonsensical propositions that form a ladder—a ladder that she is apparently meant (somehow) to ascend, and the ascension of which apparently (somehow) enables the attainment of a (presumably otherwise unattainable) sort of insight. Any attempt to take this climactic trope seriously as a description of the literary strategy of the work would seem to commit one to ascribing to the work a conception of nonsense that is sufficiently broad to allow for the possibility of illuminating nonsense. Most recent commentary has assumed that some version of the ineffability interpretation is the only viable alternative to the positivist interpretation; and thus, if one wants to take what the work has to say about the status and purpose of its own propositions seriously, the only alternative.

According to the ineffability interpretation, illuminating nonsense illuminates by “showing” what cannot be said.⁴ The enlightened philosopher intentionally avails himself of this kind of nonsense in order to convey certain truths that exceed the reach of (the meaningful employment of) language. This special kind of nonsense flouts the rules of logic (syntax, grammar, or whatever) in its own special kind of way and thereby succeeds in conveying a special kind of thought.⁵ The philosopher who avails himself of these logically self-defeating forms of expression does so, not (pace the positivists) because he is unaware of their logically flawed character, but because he seeks to make manifest features of reality that well-formed sentences of language are unable to represent. Whereas the positivist reading takes the *Tractatus* to be (at least in intention) a resolutely antimetaphysical work, the ineffability reading takes the work to be resolutely opposed only to the possibility of the (meaningful) *expression* of metaphysical insight. The ineffability reading thus ascribes to the *Tractatus* not only a distinction between kinds of nonsense (misleading nonsense and illuminating nonsense), but also a parallel distinction between kinds of thought: sayable and unsayable ones. The latter is a kind of thought that outruns the limits of language: a kind of thought that we must not try to—because we cannot—state, but that we should nonetheless endeavor to think.⁶ So this reading agrees with the positivist reading that Wittgenstein (i) draws limits to what can be said, and (ii) urges us to relegate what cannot be said to silence—but, on this reading, (a) the limits are drawn to “show” what cannot be said, (b) the silence in question is a pregnant silence in which *something* (unsayable) is “passed over,” and (c) that which is “passed over” is disqualified only as a candidate for talk—it remains an eligible candidate for thought. This places two additional demands on Wittgenstein’s theory (over and above those ascribed to it by the positivists): (iii) to explain how the logical structure of language obstructs the possibility of expressing certain insights in well-formed propositions, and (iv) to show how that obstacle can be circumnavigated so as to allow such insights to be nonetheless communicated.

In this essay, I will argue that neither the positivist reading nor the ineffability reading is simply untrue to the text. Each of these readings advances a conception of the task of philosophy (and a correlative conception of the target of philosophy) that figures centrally in the *Tractatus*—only not as its doctrine, but rather as a candidate for (what the *Tractatus* calls) *elucidation*. The *Tractatus*, according to the reading set forth in this essay, aims to demonstrate how each of these conceptions is the mirror image of the other, each feeding on and sustaining the other. There is (what the *Tractatus* calls) “an understanding of the logic of our

language" that is common to these two readings of the *Tractatus*—one that the *Tractatus* itself seeks to expose as a misunderstanding. It is this (mis)understanding of the logic of our language that allows the positivist to imagine that he can furnish a method for exposing the sentences of the metaphysician as intrinsically meaningless (and thereby to imagine that he has found the key to doing away with metaphysics once and for all), and that allows the antipositivist to imagine that he can grasp a kind of a thought which cannot be accommodated by the logical structure of language (and thereby to imagine that, albeit at the cost of banishing it to wordlessness, he has found the key to rescuing metaphysics once and for all).

The correlative conceptions of language (i.e., of sense and nonsense) and philosophical illumination (i.e., of how a reader of the work is to be brought to insight) presupposed respectively by the positivist and ineffability readings are the expressions of philosophical impulses that the *Tractatus* seeks to engage. The reader is to feel the force of each of these mutually antagonistic impulses, but not to remain in the thrall of either. The conceptions to which these impulses give rise figure in the work as dialectical way stations that are to be successively recognized as the antepenultimate and penultimate rungs on the ladder that the reader is invited to ascend and—once having ascended—called upon to throw away. To throw away the ladder is to throw away the allied conceptions of language and philosophical elucidation upon which these two (apparently) mutually opposed readings rest: to recognize them as only apparently sustainable conceptions.

II. Elucidation and Nonsense

There are authors in whose work form matches content not merely as clothes do the body but as the soul does the body.

Karl Kraus, *Aphorismen*

One subsidiary aim of this essay is to furnish some materials for answering the following question: Why does the *Tractatus* have the *form* that it does? (Why does this funny-looking book look like *this*?) Is the form of the work (as philosophers are wont to say) "merely a matter of style"—an outer layer of literary ornamentation of which the work can (and perhaps should) be divested without violence to the content it harbors?⁷ I shall approach Wittgenstein's early work (and hope to provide some materials that might encourage one to approach his later work) with the contrary assumption—namely, that the work's mode of presentation bears a profoundly intimate relation to its philosophical ambitions. To understand why the text comes in the shape it does, we need to understand what those ambitions are; and, to understand how those ambitions are pursued within the body of the work, we need to understand why the text comes in the shape that it does.

This essay aspires, in particular, to supply two of the pieces of the puzzle that need to be in place before we can make out the point of the famous penultimate section of the *Tractatus*: "My propositions serve as *elucidations* in the following way: anyone who understands me eventually recognizes them as *nonsensical*, when he has used them—as steps—to climb out through them, on them, over them. (He must, so to speak, throw away the ladder after he has climbed up it.)"⁸ This

passage tells a reader of the work what he must “eventually recognize” in order to understand its author. No understanding of the *Tractatus* is possible apart from an understanding of what this passage asks of its reader—apart, that is, from an understanding of what the authorial strategy of the work as a whole is. Wittgenstein says of Carnap that he failed to understand this passage and *therefore* failed to understand “the fundamental conception of the whole book.”⁹ What did Carnap fail to understand, and how did that failure lead him to misunderstand the fundamental conception of the whole book?

Two important terms occur in this passage. Not only Carnap, but also several subsequent generations of commentators have paid insufficient heed to what the *Tractatus* itself has to say about how these terms (as deployed within the work) are to be understood. The two terms in question are:

1. to elucidate (*erläutern*)
2. nonsense (*Unsinn*)

This essay is about how to understand these two words in the *Tractatus*. Only once we understand the specific valence these terms have in this work will we be in a position to understand what the *Tractatus* says (in §6.54) about its method.

The aim of the *Tractatus* as a whole is twofold: (i) to elucidate the logic of our language (i.e., to render perspicuous the difference between sense and nonsense), and (ii) to elucidate certain philosophical matters (i.e., to show how the apparent sustainability of certain philosophical conceptions rests upon a misunderstanding of the logic of our language). We learn, at the end of the work, that the sentences of the work have succeeded as elucidations when we recognize them as *Unsinn*. But in order to recognize them as such, we must first come to terms with the question of wherein such recognition consists and hence with the understanding of *Unsinn* that the work as a whole seeks to impart. Thus our task is one of trying to break into a circle. In order to understand the *Tractatus*, one needs to understand what each of these two terms in this book—“elucidation” and “nonsense”—means. But an understanding of these terms as deployed within the work requires an appreciation of the structure and method of the work as a whole. So these two forms of understanding—an understanding of the point of the work as a whole, and an understanding of what each of these terms means within the work—must come together or not at all. The obscurity of the work is a function of the peculiarity of its method, one of trafficking in nonsense—which, in turn, is a function of the peculiarity of its aim: (which we are told is) elucidation.

The first translator of the *Tractatus*, C. K. Ogden, thought the work *unnecessarily* obscure. Since he knew that Wittgenstein had been working on the book for years, Ogden made bold to inquire (in a letter to Wittgenstein) whether there might not be, lying about the house somewhere, a collection of material with which the work might be supplemented—passages that had not been included in the book but that further develop some of the lines of thought contained in it. Wittgenstein writes back: Yes. Ogden is delighted. He asks Wittgenstein if it might not be possible to incorporate some of this supplementary material into the English edition of the work. Surely, it could only be a great help to readers if material were added to the book. Wittgenstein demurs, and one of the reasons he gives is of particular interest for our purposes: “I am very sorry indeed I cannot send you the supplements. There can be no thought of printing them. . . . The supplements are exactly

what must *not* be printed. . . . THEY REALLY CONTAIN NO ELUCIDATIONS AT ALL . . . As to the shortness of the book I am *awfully sorry for it, but what can I do?*! If you were to squeeze me like a lemon you would get nothing more out of me” (Wittgenstein, 1973, p. 46).¹⁰ These supplementary passages (*Ergänzungen*) “are exactly what must *not* be printed,” for they contain no elucidations. What is wrong with adding passages that contain no elucidations? Here is a preview of the answer to this question, which this essay will defend: to add passages to the work that do not subserve its elucidatory aim would be to compromise its fundamental conception.

In §4.1212 of the *Tractatus*, we are told that a work of philosophy “consists essentially of elucidations.” “Philosophy” here means: philosophy as practiced by the author of the *Tractatus*. The notion of elucidation is tied in §4.1212 to the idea of philosophy being a certain kind of *activity*: “Philosophy is not a theory [*Lehre*] but an activity. A philosophical work consists essentially of elucidations” (§4.112). The word ‘*Lehre*’—which Ogden translates as ‘theory’—is rendered as ‘body of doctrine’ by Pears and McGuinness. Wittgenstein claims that the work of philosophy, as he pursues it, does not consist in putting forward a doctrine but rather in offering elucidations. This provides a criterion of adequacy that must be met by any textually faithful account of what Wittgenstein means by “elucidation”: it must be able to illuminate how Wittgenstein could intelligibly have thought that the philosophical work accomplished by the *Tractatus* “consists essentially of elucidations”—where “elucidation” is the name of an activity that contrasts with the (conventional philosophical) activity of presenting the reader with a doctrine.¹¹ When Wittgenstein says (in §4.112) that a philosophical work consists essentially of elucidations, the term “elucidation” is a rendering of the same German word (*Erläuterung*) that occurs in §6.54 and that also, as we shall see in a moment, figures pertinently in Frege’s writings.¹²

When Ogden first attempts to translate 6.54, he renders the opening line of the passage as follows: “My propositions are explained in that he who understands me.” Wittgenstein corrects this translation in two important respects. He changes “explain” to “elucidate,” and he changes the verb from an intransitive to a transitive one: “My propositions elucidate—whatever they do elucidate—in this way.” What object does the transitive verb take? (*What* does an elucidation elucidate?) Wittgenstein tells Ogden that what requires elucidation are “philosophic matters.”¹³ How does elucidation differ from explanation? (*How* does an elucidation elucidate?) We are told in §6.54: his propositions serve as elucidations by *our*—that is, the reader—coming to *recognize* them as nonsensical. But how can the recognition that a proposition is *nonsense* ever elucidate—ever shed light on—anything?¹⁴ Evidently we need a better understanding of how this work thinks about nonsense. We need to look closely at those passages in which the work tells us what it takes *Unsinn* to be and, in particular, what it tells us it takes it *not* to be.

This is what the *Tractatus* has to say about what is distinctive about its own conception of nonsense: “Frege says: Every legitimately constructed proposition must have a sense; and I say: Every possible proposition is legitimately constructed, and if it has no sense this can only be because we have given no meaning to some of its constituent parts” (§5.4733). Wittgenstein here contrasts a formulation of Frege’s¹⁵ with one of his own. At first blush, it is hard to see how they differ. The critical difference between Frege’s formulation and the one the *Tractatus* endorses

is that the former implicitly distinguishes between those propositions that are legitimately constructed and those that are not, while the latter rejects the idea that there is such a thing as a logically illegitimately constructed proposition: "Every *possible* proposition is legitimately constructed." It is this difference (that Wittgenstein sees between his own view and Frege's) that we need to understand. As this passage suggests—and as the preface of the *Tractatus* makes clear—a good place to seek further understanding is "the great works of Frege."¹⁶

III. The Neglect of Frege?

Wittgenstein's *Tractatus* has captured the interest and excited the admiration of many, yet almost all that has been published about it has been wildly irrelevant. If this has had any one cause, that cause has been the neglect of Frege . . . In the *Tractatus* Wittgenstein assumes, and does not try to stimulate, an interest in the kind of questions that Frege wrote about.

G. E. M. Anscombe, *An Introduciton to
Wittgenstein's Tractatus*

How can the neglect of Frege be the reason why much of the commentary on Wittgenstein's *Tractatus* is wildly irrelevant to a proper understanding of that work? What more widely accepted platitude about the book could there be, than that it develops and responds to ideas put forward by Frege and Russell? But Anscombe's point presumably is not that Frege is seldom mentioned in discussions of Wittgenstein's *Tractatus*. Her point must rather be that we do not know who Frege is for the author of the *Tractatus*—an appreciation of that work presupposes an immersion in a certain philosophical background ("an interest in the kind of questions that Frege wrote about"), which most of the commentary on that work has lost sight of. It is not that we are unfamiliar with Frege's or Wittgenstein's texts, but that we have failed to see what it is that is at issue in them. We fail to get hold of the questions that figure most centrally in these texts and of *the kind of questions* these questions are for Frege and for Wittgenstein. One aim of this essay is to draw attention to two aspects of that background of which we have lost sight: Frege's thought about the character of philosophical nonsense and Frege's conception of elucidation.

The central claim of this essay can be summarized as follows: Wittgenstein saw a tension in Frege's thought between two different conceptions of nonsense, which I shall call the *substantial conception*, and the *austere conception*, respectively. The substantial conception distinguishes between two different kinds of nonsense: mere nonsense and substantial nonsense. Mere nonsense is simply unintelligible—it expresses no thought. Substantial nonsense is composed of intelligible ingredients combined in an illegitimate way—it expresses a logically incoherent thought. According to the substantial conception, these two kinds of nonsense are logically distinct: the former is mere gibberish, whereas the latter involves (what commentators on the *Tractatus* are fond of calling) a "violation of logical syntax."¹⁷

The austere conception, on the other hand, holds that mere nonsense is, from a logical point of view, the only kind of nonsense there is. Along with these two different conceptions of nonsense go two different conceptions of elucidation: according to the substantial conception, the task of elucidation is to “show” something that cannot be said; according to the austere conception, it is to show that we are prone to an illusion of meaning something when we mean nothing. The *Tractatus* is standardly read as championing the substantial conception. This is to mistake the bait for the hook—to mistake the target of the work for its doctrine. On the reading of the *Tractatus* I shall try to sketch here, the *Tractatus* is to be seen as resolving the tension in Frege’s thought between these two conceptions of nonsense in favor of the austere view.¹⁸ The strategy of the *Tractatus* is to short-circuit Frege’s view from within by bringing these two halves of Frege’s thought in immediate proximity with each other.

The substantial conception of nonsense represents the (previously indicated) common ground between the positivist and ineffability interpretations of the *Tractatus*. (It is in opting for this conception, according to the *Tractatus*, that the crucial move in the philosophical conjuring trick is made, and it is the one that we are apt to think most innocent.) This tiny patch of common ground can seem insignificant in comparison with the vehemence with which the ineffability interpretation laments the obtuseness of the positivist interpretation (epitomized by its failure to allow for the possibility of illuminating nonsense) and the equal vehemence with which the positivist interpretation rejects the mysticism of the ineffability interpretation (epitomized by its hankering after ineffable forms of insight).¹⁹ In seeking to emphasize their differences from one another, proponents of the two interpretations tend to articulate the details of the substantial conception in apparently distinct ways. It will therefore help to distinguish between two (apparently distinct) variants of the substantial conception. I shall term these the *positivist variant* and the *ineffability variant* (after the readings of the *Tractatus* in which they respectively figure).²⁰ According to the former variant, violations of logical syntax are a kind of *linguistic* phenomenon: identifying a violation of logical syntax is a matter of isolating a certain kind of (logically ill-formed) linguistic string. According to the latter variant, a violation of logical syntax is a kind of phenomenon that can only transpire in the medium of thought and necessarily eludes the medium of language. Though proponents of the ineffability variant hold that language is powerless to express such thoughts, they nonetheless deem language an indispensable tool for “conveying” such thoughts. They hold that language can “hint” at what it cannot say.

Before we turn to how the *Tractatus* seeks to resolve the tension in Frege’s thought between the substantial and austere conceptions of nonsense, it will help first to see that Frege can be read as a champion, not only of the substantial conception per se, but specifically of the ineffability variant.²¹ To see this requires that we see how what is typically taken to be the central and most original doctrine of the *Tractatus*—the doctrine that there are certain insights that can only be “shown” and cannot be said—can be discerned (by some readers of Frege) to be a central doctrine of Frege’s philosophy. That such a doctrine already figures in Frege’s thought has been argued particularly forcefully by Peter Geach; and, indeed, Geach attributes the occurrence of such a doctrine in the *Tractatus* to the influence of Frege: “Reflection upon ‘the great works of Frege’ . . . can never be

out of place for anybody who seriously wants to understand Wittgenstein. . . . The influence of Frege on Wittgenstein was pervasive and life-long, and it is not of course just confined to places where Frege is mentioned by name or overtly referred to. . . . Fundamental aspects of the Wittgensteinian saying/showing contrast are already to be discerned in Frege's writings" (Geach, 1976, p. 55). I think Geach is right to think that Wittgenstein found in Frege a conception of what cannot be said but only "shown"—and that the *Tractatus* has therefore been credited in putting forward such a conception with an originality to which it cannot justly lay claim. Geach continues: "Paradoxical as is the doctrine of aspects of reality that come out but cannot be propositionally expressed, it is hard to see any viable alternative to it so long as we confine ourselves to philosophy of logic: and in this domain Wittgenstein revised Frege's views without unfaithfulness to Frege's spirit" (p. 68). Geach here attributes a certain doctrine to both Frege and the *Tractatus*: the doctrine that there are *certain aspects of reality* that cannot be expressed in language but can nonetheless be conveyed through certain sorts of employment of language. I think Geach is mistaken in supposing that the *Tractatus* seeks simply to incorporate this Fregean doctrine into its own teaching. That is to say, I think Geach is right to find this doctrine propounded where most commentators have failed to look for it (namely, in Frege), and wrong to find it propounded where most commentators assume they are supposed to look for it (namely, in the *Tractatus*). I shall therefore be concerned to argue that the *Tractatus*, in its criticism of Fregean doctrines, seeks to mount a criticism of the very doctrines that are standardly attributed to it.

In order to see this, we first need to refrain from speaking about the distinction between saying and showing in the usual loose fashion. Where most commentators on the *Tractatus* discern only one distinction, we need to see that there are two different distinctions at work. A version of each of these distinctions is already at play in Frege's work. But these distinctions are drawn in Frege's work in such a way as to be deeply entangled in one another, whereas they are refashioned in the *Tractatus* in a manner that allows them to become disentangled. The first distinction is drawn *within* the body of meaningful propositions. (Thus, according to this first sense, only meaningful propositions can *show*.) The second distinction marks off, from various ways of employing language, a particular way of employing (apparently meaningful) sentencelike structures—an employment that "takes as its object" (what Wittgenstein calls in his letter to Ogden) "philosophic matters" (Wittgenstein, 1973, p. 51). (Thus, according to this second sense of 'show', nonsense can show.)²² The first of these distinctions is (at least terminologically) the more familiar and notorious of the two: it is (the one that gets called in the *Tractatus*) the distinction between saying and showing (or, more precisely, in Tractarian jargon, the distinction between what a *proposition* says and what it shows).²³ The second distinction is relatively neglected and is the one with which the rest of this essay is concerned. It is a distinction between two different kinds of use of language:²⁴ constative uses,²⁵ in which a proposition states what is the case (or, in Tractarian jargon, represents a state of affairs), and elucidatory uses, in which an apparently constative use of language (one that offers an appearance of representing a state of affairs) is revealed as illusory.²⁶ It is primarily through the manner in which the *Tractatus* reshapes the second of these Fregean distinctions

that the criticism of Frege is mounted. Only once we understand how the *Tractatus* seeks to modify Frege's conception of elucidation (*Erläuterung*) will we be in a position to understand what the *Tractatus* means to say about itself when it declares that it is a work which "consists essentially of elucidations."

IV. Frege on Concept and Object

The style of my sentences is extraordinarily strongly influenced by Frege. And if I wanted to, I could establish this influence where at first sight no one would see it.

Wittgenstein, *Zettel*

Here is how Geach summarizes the region of Frege's thought that is "revised without unfaithfulness" in (what he takes to be) the Tractarian distinction between saying and "showing": "Frege . . . held . . . that there are logical category-distinctions which will clearly show themselves in a well-constructed formalized language, but which cannot properly be asserted in language: the sentences in which we seek to convey them in the vernacular are logically improper and admit of no translation into well-formed formulas of symbolic logic" (Geach, 1976, p. 55).²⁷ Frege's favorite example of a logical-category distinction that clearly shows itself in a well-constructed formalized language (but that "cannot properly be asserted in language") is the distinction between concept and object—and it is an example that continued to exercise Wittgenstein throughout his life.²⁸ For something to be an object (or a concept), for Frege, is not for it to possess certain metaphysical or psychological characteristics, but rather for it to belong to a particular logical category. Frege takes it to be "a sure sign" of confusion if logic seems to stand in "need of metaphysics or psychology" (Frege, 1967a, p. 18).²⁹

Frege's most famous discussion of the distinction between concept and object is his article entitled "On Concept and Object"—an article that is structured around his reply to an objection put forward by Benno Kerry.³⁰ Kerry objects to Frege's claim that concepts cannot be objects and objects cannot be concepts. Kerry proposes as a counterexample to Frege's claim the statement "the concept *horse* is a concept easily attained." This statement seems to assert that something—the concept *horse*—falls under a concept (namely, that of being a concept easily attained). Now anything that falls under a (first-level) concept must—on Frege's conception of an object—be an object. That is what it is to be an object for Frege—to be the kind of a thing of which concepts can be predicated. So, for Frege, the grammatical subject of Kerry's statement—the concept *horse*—(since it falls under a concept) must be an object. But, if what the statement says is true, then it is a concept easily attained; and if it is a concept easily attained, then it is a kind of a concept. The two prongs of Kerry's argument, based on his putative counterexample, can thus be summarized as follows: (a) given Frege's conception of what it is to be an object, we have reason (by virtue of its logical role in the statement) to conclude that "the concept *horse*" is an object; and (b) given the (apparent) truth of what the statement itself asserts, we have reason to conclude that it is a concept. So

Kerry concludes that his statement furnishes us with an example of something—the concept *horse*—that is both an object and a concept.

Frege's article responding to Kerry begins with the following remark:

The word 'concept' is used in various ways; its sense is sometimes psychological, sometimes logical, and perhaps sometimes a confused mixture of both. Since this license exists, it is natural to restrict it by requiring that when once a usage is adopted it shall be maintained. What I decided was to keep to the strictly logical use. . . . It seems to me that Kerry's misunderstanding results from his unintentionally confusing his own usage of the word 'concept' with mine. This readily gives rise to contradictions, for which my usage is not to blame. (Frege, 1984, p. 182)

Frege insists here that he uses the word 'concept' in "a strictly logical sense" and that Kerry's misunderstanding of his view is due to his failure to appreciate this. In particular, Frege will charge that Kerry's apparent counterexample is generated by equivocating between "a strictly logical" and (what Frege will call) a "psychological" sense of the term 'concept'.³¹ But what is it to use the word 'concept' in a strictly logical sense? This question is best approached through a consideration of Frege's three principles (which he presents at the beginning of his *Die Grundlagen der Arithmetik*):

In the enquiry that follows, I have kept to three fundamental principles:

- (1) always to separate sharply the psychological from the logical, the subjective from the objective;
- (2) never to ask for the meaning of a word in isolation, but only in the context of a proposition;
- (3) never to lose sight of the distinction between concept and object.

(Frege, 1968, p. x)

Each of these principles is reworked and plays a central role in the *Tractatus*.³² These three principles are closely linked: to deny any one of them is to deny each of the other two. Frege himself immediately goes on to explicate how a denial of the first principle leads to a denial of the second: "In compliance with the first principle, I have used the word 'idea' always in the psychological sense, and have distinguished ideas from concepts and from objects. If the second principle is not observed, one is almost forced to take as the meanings of words mental pictures or acts of the individual mind, and so to offend against the first principle as well" (p. x). If we disobey the second principle and ask for the meaning of a word in isolation, we shall look for an answer in the realm of the psychological—we shall explain what it is for a term to have a meaning in terms of mental accompaniments (such as the psychological associations the word carries with it), or in terms of mental acts (such as the linguistic intention with which we utter it); and *that* will constitute a violation of the first principle.

Underlying these principles is a doctrine of the primacy of judgment. Frege writes: "I do not begin with concepts and put them together to form a thought or judgment; I come by the parts of a thought by analyzing the thought" (Frege, 1979, p. 253). Frege here opposes an extremely intuitive view of how we come by a

thought: namely, by taking hold of its independently thinkable components and putting them together so as to form a coherent whole.³³ The sort of “parts”³⁴ that are at issue here are only to be identified by comparing and contrasting the logical structure of whole propositions and seeing how the respective “parts” resemble and differ from one another in the contributions they make to the respective wholes.³⁵ Here is one of Frege’s many exhortations to the reader not to lose sight of the primacy of the propositional whole over its parts: “[W]e ought always to keep before our eyes a complete proposition. Only in a proposition have the words really a meaning. It may be that mental pictures float before us all the while, but these need not correspond to the logical elements in the judgement. It is enough if the proposition taken as a whole has a sense; it is this that confers on the parts also their content” (Frege, 1968, p. 71).³⁶ To determine the meaning of a word, according to Frege, we need to discover what contribution it makes to the sense of a proposition in which it figures. We need to know what logical role it plays in the context of a judgment.³⁷ What we want to discover is thus not to be seen at all, if we look at the mere isolated word rather than at the working parts of the proposition in action.³⁸ Thus, for example, the mere fact that the words at the beginning of Kerry’s sentence purport to refer to (something called) “the concept *horse*” hardly suffices, by Frege’s lights, to ensure that they indeed successfully refer to a concept. When Frege insists that he is going to keep to a strictly logical use of the word ‘concept’, he is declaring his interest in how a certain kind of working part of a judgment—what he calls the unsaturated or predicative part—contributes to the sense of a judgment as a whole.³⁹

There are no symbols for terms such as ‘function’, ‘concept’, and ‘object’ in Frege’s *Begriffsschrift*. Nevertheless, these terms play an ineliminable role in his explanations of his symbolism. He thinks that an understanding of these terms is required if one is to master the notation of the symbolism and properly understand its significance. Yet he also insists that what he thus wishes to draw our attention to—when he employs, for example, the word ‘concept’ in its strictly logical sense—is not something that can be properly defined. It can only be exhibited through (what Frege calls) an elucidation.⁴⁰ Such elucidations, in turn, play only a transitional role: once they have successfully conveyed the logical distinctions that form the basis of Frege’s *Begriffsschrift*, we are to see that there is no way to express the thoughts which they (appear to be attempting to) convey in a *Begriffsschrift*.⁴¹ Yet if we appreciate the logically fundamental character of the distinctions upon which Frege’s *Begriffsschrift* is based, then we will see that anything that can be thought can be expressed in *Begriffsschrift*. In grasping the distinction between that which can and that which cannot be expressed in a *Begriffsschrift*, we furnish ourselves with a logically precise articulation of the distinction between that which (“in a strictly logical sense”) is, and that which is not, a *thought*. Thus Frege’s elucidations are meant to play the role of a ladder that we are to climb up and then throw away.⁴² Frege might have said about his own elucidatory remarks, echoing §6.54 of the *Tractatus*: “My propositions serve as elucidations in the following way: he who understands me recognizes that my propositions cannot be expressed in my *Begriffsschrift*, once he has used them—as steps—to climb up beyond them. He must, so to speak, throw away the ladder after he has used it to climb up to my *Begriffsschrift*.”

V. Fregean Elucidation

God can do everything, it is true, but there is one thing He cannot do, and that is speak nonsense.

Leo Tolstoy, *The Gospel According to Tolstoy*

The distinction between elucidation and definition in Frege rests upon a prior distinction between what is primitive and what is defined in a theory. Any theoretical term that is not susceptible of a formal definition requires elucidation. Every science must employ some primitive terms whose meanings must be presupposed from the outset. Even in a logically perfect language there will be some terms that are not (and cannot) be introduced by definition and that must remain undefinable. The purpose of elucidations is to convey the meanings of such terms:

Definitions proper must be distinguished from elucidations. In the first stages of any discipline we cannot avoid the use of ordinary words. But these words are, for the most part, not really appropriate for scientific purposes, because they are not precise enough and fluctuate in their use. Science needs technical terms that have precise and fixed meanings, and in order to come to an understanding about these meanings and exclude possible misunderstanding, we give elucidations [*Erläuterungen*] of their use. (Frege, 1979, p. 207, translation emended)

In “On Concept and Object,” Frege is concerned with only one species of the genus elucidation, namely, the activity of elucidating what is *logically* primitive.⁴³ When one is engaged in this particular species of elucidation, Frege thinks one is compelled to come out with sentences that cannot be translated into a proper *Begriffsschrift*.

One might ask: Doesn’t Frege furnish us with examples of statements that define what a concept or an object is? Frege will answer that nothing his own sentences (appear to) assert about the nature of concepts or objects can ever, without entering into a confusion, be taken as (a contribution to) a definition of what kind of a thing a concept or an object is.⁴⁴ For something to count as a definition, for Frege, it must be possible to invoke it in proofs. Wherever the definiendum occurs in a sentence, it must be possible to replace it with the definiens. Nothing of the sort is possible, Frege maintains, for those terms occurring in his elucidatory remarks that refer to logically primitive categories.⁴⁵ Their meaning must be presupposed from the outset. The most one can do is to lead the reader to what is meant by such terms—what it is one’s words are trying to gesture at—by means of a series of *hints*.⁴⁶ Early on in his reply to Kerry, Frege insists upon the ineliminable role of hints in offering an elucidation of that which is logically fundamental and hence indefinable:

Kerry contests what he calls my definition of ‘concept’. I would remark, in the first place, that my explanation is not meant as a proper definition. One cannot require that everything shall be defined, any more than one can require that a chemist shall decompose every substance. What is simple cannot be decomposed, and what is logically simple cannot have a proper definition. Now something logically simple is no more given to us at the outset than most of the chemical elements are; it is reached only

by means of scientific work. If something has been discovered that is simple, or at least must count as simple for the time being, we shall have to coin a term for it, since language will not originally contain an expression that exactly answers. On the introduction of a name for what is logically simple, a definition is not possible; there is nothing for it but to lead the reader or hearer, by means of hints, to understand the word as it is intended. (Frege, 1984, pp. 182–83)

Yet only a few lines further on, Frege offers something that has the appearance of offering a specification of the meaning of the term ‘concept’: “A concept (as I understand the word) is predicative. On the other hand, a name of an object, a proper name, is quite incapable of being used as a grammatical predicate.” Frege immediately goes on to say: “This admittedly needs elucidation, otherwise it might appear false” (p. 183). The term ‘elucidation’ here stands for the activity of leading the reader by means of hints to what is intended by a term that denotes something logically primitive. This requires not only that we count on the patience and goodwill of our audience while we encourage them to guess at our intended meaning, but also that—here in the antechamber to that most precise of all sciences: the science of logic—we resort to figurative modes of expression (e.g., to talk about objects being “saturated” and concepts being “unsaturated”⁴⁷). Worse still, Frege thinks that in the elucidation of logically primitive notions (such as that of *concept* or *object*) there is an ineliminable role to be played by (the artful employment of) nonsense.⁴⁸ According to Frege, in elucidating the meaning of terms such as ‘object’ and ‘concept’, we attempt to help our audience to latch on to the intended meaning of a term for something logically fundamental by coming out with forms of expression that misfire, and then helping our audience to see how and why they misfire.

It is of crucial importance when offering such an elucidation, Frege goes on to say, that the originator of the elucidation himself understand the transitional character of the talk that he engages in, and that he know at every point what he means by a particular term and remain throughout in agreement with himself:

Since definitions are not possible for primitive elements, something else must enter in. I call it elucidation [*Erläuterung*]. . . . Someone who pursued research only by himself would not need it. The purpose of elucidations [*Erläuterungen*] is a pragmatic one; and once it is achieved, we must be satisfied with them. And here we must be able to count on a little goodwill and cooperative understanding, even guessing; for frequently we cannot do without a figurative mode of expression. But for all that, we can demand from the originator of an elucidation [*Erläuterung*] that he himself know for certain what he means; that he remain in agreement with himself; and that he be ready to complete and emend his elucidation [*Erläuterung*] whenever, given even the best of intentions, the possibility of a misunderstanding arises. (Frege, 1984, pp. 300–301, translation emended)⁴⁹

Frege frankly concedes that such a process of offering hints and relying on guess-work might, in principle, never culminate in the desired meeting of minds between the elucidator and the audience of an elucidation. He hastens to reassure us, however, that it turns out that, in practice, we are quite good at guessing what another person means even when all we are offered is a series of such hints: “Theo-

retically, one might never achieve one's goal this way. In practice, however, we do manage to come to an understanding about the meanings of words. Of course we have to be able to count on a meeting of minds, on others guessing what we have in mind. But all this precedes the construction of a system and does not belong within a system" (Frege, 1979, p. 207, translation emended). This last sentence alludes to a point touched on earlier: once the elucidation is successful, the recourse to figurative modes of speech and bits of nonsense can be dispensed with; the elucidations will have served their transitional pragmatic purpose and are to be thrown away. The activity of elucidation "has no place in the system of a science." Its role is entirely that of a propaedeutic.⁵⁰

Frege's procedure in "On Concept and Object" relies on an understanding of the logical structure of language implicit in his reader's everyday command of ordinary language. Frege's purpose—when he introduces terms such as 'concept' and 'object'—is to isolate and coin terms for the logically discrete functioning parts of a judgment: parts that can be seen to play logically distinct roles in the antecedently understood content of the sentences of everyday language. In aiming to communicate the meaning of these terms he has coined, Frege (since he cannot resort to definition) appeals to "the general feeling" for our common language (our shared sense of the contribution which the parts of a proposition of ordinary language make to the sense of the whole).⁵¹ It is through our general feeling for our common language that we achieve agreement on what is a proper logical segmentation of a sentence of our language and hence what is (and what is not) a concept or an object.

The elucidatory strategy of the essay "On Concept and Object" can (according to this reading of Frege in the spirit of Geach) be seen as proceeding in five steps: (1) to make explicit a logical distinction implicit in our everyday linguistic practices, (2) to demonstrate that Kerry's employment of the terminology of 'object' and 'concept' fails to track the distinction in question, (3) to furnish statements (employing the terminology of 'object' and 'concept') that aim to track the distinction in question, (4) to elicit an appreciation of what is defective about such statements, and (5) to indicate how a recognition of the defective character of such statements enables one to attain an insight (into, e.g., what a concept is) that could not have been communicated in any other way. Thus Frege might have said: he who *recognizes* my elucidatory remarks in "On Concept and Object" as *defective* understands me. Such a reading of Frege (in the spirit of Geach)—according to which Fregean elucidation is to be understood as a strategy for conveying insights into ineffable features of reality—as we shall see, closely parallels the reading of the *Tractatus* favored by proponents of the ineffability interpretation.

VI. Elucidatory Nonsense

Don't, for heaven's sake, be afraid of talking nonsense! But
you must pay attention to your nonsense.

Wittgenstein, *Culture and Value*

By way of further response to Kerry's counterexample, Frege goes on in "On Concept and Object" to make a remark that is likely to cause even an inattentive reader

to pause. He says: "The concept horse is not a concept." This remark is evidently intended to be paradoxical. The self-defeating character of Frege's counterthesis, which he opposes to Kerry's thesis, is meant to draw attention to what is already self-defeating (though less self-evidently so) in the form of words that Kerry calls upon to express his claim. If one is partial to a reading of Frege that aligns him with the standard reading of the *Tractatus* (as Geach is), then one will think that part of Frege's point here is to draw our attention to how Kerry's words represent an attempt to say something that cannot be said. Such a reading of Frege attributes to Frege a commitment to the ineffability variant of the substantial conception of nonsense. In a passage such as the following, Frege can be heard as pressing a claim (concerning how a primitive feature of the logical structure of language can never itself figure as the subject of a logically well-formed judgment) of a sort that many have taken to be a Tractarian claim: "What is . . . asserted about a concept can never be asserted about an object. . . . I do not want to say it is false to assert about an object what is here asserted about a concept; I want to say it is impossible, senseless, to do so" (Frege, 1984, p. 189).⁵² The idea that what such an attempt (to assert of a concept what can only be asserted of an object) ends up saying is not merely false, but senseless, is one that runs throughout Frege's writings.⁵³ But what are we to make of such an admission? In claiming that what Kerry says is nonsensical, Frege commits himself to the conclusion that what he himself wants to say about concepts (both in response to Kerry and elsewhere) is also nonsensical.⁵⁴ Indeed, Frege seems at various junctures to be disarmingly ready to embrace such a conclusion about the status of many of his own remarks:

In the case of a concept we can also call the unsaturatedness its predicative nature. But in this connection it is necessary to point out an imprecision forced on us by language, which, if we are not conscious of it, will prevent us from recognizing the heart of the matter: i.e. we can scarcely avoid using such expressions as 'the concept *prime*'. Here there is not a trace left of unsaturatedness, of the predicative nature. Rather, the expression is constructed in a way which precisely parallels 'the poet Schiller'. So language brands a concept as an object, since the only way it can fit the designation for a concept into its grammatical structure is as a proper name. But in so doing, strictly speaking, it falsifies matters. In the same way, the word 'concept' itself is, taken strictly, already defective, since the phrase 'is a concept' requires a proper name as grammatical subject; and so, strictly speaking, it requires something contradictory, since no proper name can designate a concept; or perhaps better still [would be to say that], it requires something nonsensical. (Frege, 1979, pp. 177–78, translation emended)⁵⁵

Frege's discussion here turns on the idea that we know *what* it is that we are trying to say (when we employ an expression such as 'the concept *prime*'), but when we try to say "it," we realize that what we are trying to say requires that what we actually say be something nonsensical. We have in passages such as this the idea that we can discern what a piece of nonsense is *trying* (but failing) to say. When we use such expressions as 'the concept *X*', we are trying to refer to a concept, but in the mode of expression with which we end up—when we try to express our thought—there is not a trace of unsaturatedness left.⁵⁶ We are left

with something that does not have a predicative nature, and, Frege therefore concludes, we have failed to refer to a concept. An attempt to treat a concept as an object is an attempt to do something *impossible*, an attempt to do something we *cannot* do:

If I want to speak of a concept, language, with an almost irresistible force, compels me to use an inappropriate expression which obscures—I might almost say falsifies—the thought. One would assume, on the basis of its analogy with other expressions, that if I say ‘the concept *equilateral triangle*’ I am designating a concept, just as I am of course naming a planet if I say ‘the planet Neptune’. But this is not the case; for we do not have anything with a predicative nature. Hence the meaning of the expression ‘the concept *equilateral triangle*’ (if there is one in this case) is an object. We cannot avoid words like ‘the concept’, but where we use them we must always bear their inappropriateness in mind. From what we have said it follows that objects and concepts are fundamentally different and cannot stand in for one another. And the same goes for the corresponding words or signs. Proper names cannot really be used as predicates. Where they might seem to, we find on looking more closely that the sense is such that they only form part of the predicate: *concepts cannot stand in the same relations as objects*. It would not be false, but impossible to think of them as doing so. (pp. 119–20)⁵⁷

This passage (and many others, in Frege’s work, like it) make reference to there being a *thought* underlying the nonsense we come out with (when we attempt to assert of a concept what can only be asserted of an object). Language itself obstructs us from expressing the thought we are after: “language, with an almost irresistible force, compels me to use an inappropriate expression which obscures—I might almost say falsifies—*the thought*.” The nonsense we come out with represents an unsuccessful attempt to put that (unsayable) thought into words. Thus, for example, in responding to Kerry, Frege says certain things that by his own lights are nonsense, and what we (his readers) are to do is attend not simply to what he says (since it is, after all, nonsense) but to “the thought” that his words fail to express but attempt to gesture at.⁵⁸

One such example of Fregean elucidation occurs in a letter to Russell:

In the proposition ‘Something is an object’, the word ‘something’ . . . stands for a proper name. Thus whatever we put in place of ‘something’, we always get a true proposition; for a function name cannot take the place of ‘something’. Here we find ourselves in a situation where the nature of language forces us to make use of imprecise expressions. The proposition ‘A is a function’ is such an expression: it is always imprecise; for A stands for a proper name. . . . While I am writing this, I am well aware of having again expressed myself imprecisely. Sometimes this is just unavoidable. *All that matters is that we know we are doing it and how it happens.* (Frege, 1980, p. 136, my emphasis)⁵⁹

The proposition ‘A is a function’ is here invoked as an example of the paradoxical character inevitably attaching to the sort of utterances one comes out with when one attempts to elucidate what a function is. Consider the following four propositions:

- (1) A is an object.
- (2) Everything is an object.
- (3) A is a function.
- (4) Nothing is a function.

In (1), the word ‘A’ stands for a proper name; and so, by Frege’s lights, whatever we plug in for ‘A’ will occupy the argument place for an object, and thus (according to Frege’s second principle) will be an object. Thus it would appear that no matter what we plug in for ‘A’, (1) will be true. But if (1) is true no matter what we plug in for ‘A’, it would seem to follow that (2) is true! Similarly, in (3), as in (1), the word ‘A’ stands for a proper name; and so, once again, whatever occupies this argument place will be an object. Thus it would appear in this case that no matter what we plug in for ‘A’, (3) will be false. But if (3) is false no matter what we plug in for ‘A’, it would seem to follow that (4) is true! The point of this elucidation is not to secure the truth of the paradoxical claim that “Nothing is a function” (or “There are no functions”). On the contrary: it is to offer a *reductio ad absurdum* of the idea that the proposition ‘A is a function’ can just straightforwardly say what Russell (for the sake of his argument with Frege) wants it to. The point is to show that sentences in which the expression ‘function’ occurs misfire, and to show that—as long as we know what we are doing with such sentences—such self-defeating sentences can nonetheless be put to use to communicate an insight into what a function is. What matters when we employ such sentences, as Frege’s final sentence (in the passage quoted above) indicates, is that we know what we are doing (i.e., uttering nonsense) when we come out with them, and that we know how it has come to pass that we find ourselves doing it.

The point of the paradoxical assertions that constitute the preceding elucidation is to show us (a) that we end up speaking nonsense when we try to say what a function is, (b) that we here “find ourselves in a situation where the nature of language itself” makes it impossible for us to say that which we want to say, (c) that to grasp how it is that the nature of language itself thus stands in the way of saying what we want to say (when we want to say what a function is) is to grasp what a function is. The point is thus not *merely* to expose what we end up saying when we employ such a term as nonsense (in order to debar us from engaging in such ways of speaking) but, rather, to teach us how *self-consciously* to cultivate such ways of speaking (in order to allow us to attain insight into the nature of functions). The point of cultivating such ways of speaking is to enable us to recognize *why* it is that we end up with nonsense when we try to say such things. The attainment of such a recognition constitutes the sign that we have grasped an elucidation of the meaning of a term (such as ‘function’) that denotes something logically primitive.⁶⁰

Frege repeatedly says, when offering such elucidatory examples, that he is *forced* or *compelled* to express himself in an infelicitous manner: he is attempting to struggle against “an imprecision forced on us by the nature of language,” one that “compels” him “to use an inappropriate expression which obscures—falsifies—the thought.” Now, as I have already indicated, what is significant about such remarks for our purposes is that they reveal a parallel between a possible reading of Frege and a standard reading of the *Tractatus*. On this reading, Frege (1) takes himself in such cases to be trying to say something that, properly speak-

ing, *cannot* be said; and (2) speaks in such cases of there being a *thought* that his words struggle but fail adequately to express. In a famous passage in “On Concept and Object” he writes: “I admit that there is a quite peculiar obstacle in the way of an understanding with the reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my *thought*; I mention an object when what I *intend* is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me half-way—who does not begrudge a pinch of salt” (Frege, 1984, p. 193, my emphases).⁶¹ His words miss his thought (and end up being nonsense); so there is a thought they are aiming at: an understanding of what his words intend to say depends upon his reader latching onto the thought his words fail properly to express. This failure is due, according to Frege, to “a kind of necessity of language.” If he is to convey, the thought he here seeks to convey, he has no alternative but to have recourse to (elucidatory) nonsense.

A reading of Frege (on the impossibility of asserting of a concept what can only be asserted of an object) such as the one sketched above involves attributing to Frege a commitment to the substantial conception of nonsense; that is, it involves attributing to Frege the very conception of philosophically illuminating nonsense that is standardly thought to be the innovation of the *Tractatus*. Once one sees how this conception is at odds with other aspects of Frege’s philosophy—those aspects of his philosophy that the *Tractatus* is most concerned to inherit—one is in a better position to see where the philosophical innovation of the *Tractatus* truly lies.⁶² But before we turn to a discussion of those aspects of Frege’s philosophy, it will help to have before us a slightly fuller sketch of where the philosophical innovation of the *Tractatus* has standardly been thought to lie.

VII. The Standard Reading of the *Tractatus*

What causes hesitation is the fact that, after all, Mr. Wittgenstein manages to say a good deal about what cannot be said.

Bertrand Russell, introduction to the *Tractatus*

According to most readings of the *Tractatus*, Wittgenstein is the preeminent champion of the substantial conception of nonsense. But commentators in their expositions of Tractarian doctrines tend to waver between ascribing to him the positivist and the ineffability variants of the substantial conception of nonsense. The positivist and ineffability variants of the substantial conception differ over where the violation transpires when a transgression of logic occurs—and hence apparently over what the transgression itself really consists in. The positivist variant takes the locus of transgression to be the medium of language and takes the transgression itself to consist in an illegitimate combination of linguistic signs; the ineffability variant takes the locus to be the medium of thought and the transgression to consist in an illegitimate combination of logical categories: in an ill-fated attempt to express in language that which inherently resists linguistic expression. These two variants of the substantial conception lean toward opposite metaphysical doctrines. The former fits comfortably with the doctrine that the limits of

thought cannot outrun the limits of language. The latter presupposes the doctrine that thought not only can but (as putatively evidenced by our capacity to frame in thought such transgressions of logical syntax) demonstrably does outrun these limits.

Most commentators on the *Tractatus* do not distinguish between these two variants of the substantial conception. Proponents of the ineffability interpretation, however, do seek to distinguish, in some way or other, between what counts for the *Tractatus* as *misleading nonsense* and what counts as *illuminating nonsense*.⁶³ The tendency among commentators who do so distinguish is to characterize misleading nonsense in terms that accord more comfortably with the positivist variant of the substantial conception and to characterize illuminating nonsense in terms that presuppose the ineffability variant.⁶⁴ Thus misleading nonsense is characterized as a strictly linguistic affair, while illuminating nonsense is characterized as a vehicle for grasping that which cannot be said. In the case of misleading nonsense, our task is merely to recognize that an abuse of language has been committed and that nothing has been said. But in the case of illuminating nonsense, we are to recognize that the abuse of language was committed as a means to a further end. Our task as a reader of the *Tractatus* is to grasp (not what the nonsense says—because it says nothing, but rather) that which the linguistic performance “intends.” In both cases, these commentators will say, there is nothing the words *say*—there is no thought that corresponds to the words—in the one case, because there is no thought; in the other, because the thought in question eludes the words. In the case of misleading nonsense, there is no thought available to be grasped because there is no thought which is the thought those words intend. When a metaphysician commits such an abuse of language, he is to be brought to see that he has failed to say anything.⁶⁵ In the case of illuminating nonsense, there is a thought available to be grasped—a thought which is the thought those words intend. But the thought our words here intend cannot be said: our audience must be made to look beyond what our words merely say.

Here is Peter Hacker's description of how illuminating nonsense is supposed to illuminate:

Within the range of philosophical . . . nonsense we can distinguish . . . between . . . illuminating nonsense and misleading nonsense. Illuminating nonsense will guide the attentive reader to *apprehend what is shown* by other propositions which do not purport to be philosophical; moreover it will intimate, to those who *grasp what is meant*, its own illegitimacy . . .

The *Tractatus* does indeed consist largely of pseudo-propositions. Of course, what Wittgenstein *meant* by these remarks is, in his view, quite correct, only it cannot be said. Apparently what someone *means* or *intends* by a remark can be *grasped* even though the sentence uttered is strictly speaking nonsense. (Hacker, 1986, pp. 18–19, 26, my emphasis)

Hacker here attributes to the *Tractatus* the idea that there is a kind of thought (a kind of “grasping” or “apprehending” what is “meant” or “intended”) that outruns the limits of language.⁶⁶ One might ask: If nonsense is nonsense in virtue of its failure to make sense, then how are we to “grasp” what is “meant”? How are we to discern the presence of meaning in the absence of meaning?⁶⁷ Well, it is not

what the words say that we are after, but—to echo Frege—what they *hint at*. But, ordinarily, we grasp what someone's words hint at by first grasping what his words actually say, and then going on to read between the lines. But how do we grasp what nonsense hints at? The ineffability reading of the *Tractatus* invokes the idea of a violation of logical syntax to solve this problem (of how nonsense can so much as hint at something). According to the ineffability variant of the substantial conception, these violations arise through attempts to try to express fundamental features of the logical structure of language *in* language.⁶⁸ These attempts, as Peter Hacker puts it, “unavoidably violate the bounds of sense, misuse language, and produce nonsense” (Hacker, 1986, p. 21). The rules of logic render the “it” (which such nonsense is attempting to express) unsayable. The logical structure of language keeps us from being able to say “it”—because of the logical structure of our thought there is something we *cannot* do: we cannot think against the grain of logic. When we try, we come out with bits of nonsense. But these bits of nonsense are, nonetheless, useful; they can convey the unsayable thing our words were after but could not reach.

Hacker, like most commentators, takes Wittgenstein's conception of “showing”⁶⁹ in the *Tractatus* to represent a departure from the views of Frege and Russell. Nevertheless, it should be evident by now that this conception of “showing” parallels (according to the reading of Frege sketched above) Frege's conception of elucidation.⁷⁰ Peter Geach, as we saw earlier, is sensitive to this affinity between Frege's views and those standardly attributed to the *Tractatus*. Here is how Geach summarizes what he takes Wittgenstein to have learned from Frege:

Wittgenstein holds that *various features of reality* come out . . . in our language, but we cannot use this language to say, assert, that reality has these features: if we try to frame propositions ascribing these features to reality, then it will be possible to show that strictly speaking these are not propositions, only sentence-like structures which violate the principles of logical syntax and are thus devoid of any sense, true or false. All the same, these nonsensical . . . structures may be useful; they may serve to *convey from speaker to hearer an insight* that cannot be put into proper propositions. (Geach, 1976, p. 54, my emphases)

This reading—in order to make sense of the idea that nonsense is able to provide metaphysical illumination—attributes to the *Tractatus* the idea that there are features of reality that cannot be expressed because of the logical structure of our thought. But they can nevertheless be conveyed by language. According to this reading of the *Tractatus*, these features of reality can be made manifest by language because they correspond to features of language: they are reflected in the mirror of the logical structure of language. The relevant features of language taken together comprise the logical form of language. According to the standard reading of the *Tractatus*, we cannot express “it”—the logical form of language—in language; but we can gesture at it.

VIII. A Closer Look at the Substantial Conception

[C]ertain kinds of verbal association, certain grammatical expectations fulfilled, stand for a good part of our impres-

sion that a sentence has a meaning and is dominated by the Unity of one Thought. Nonsense in grammatical form sounds half rational . . . Each word, in such a sentence, is felt, not only as a word, but as having a meaning.

William James, *The Principles of Psychology*

The ineffability interpretation of the *Tractatus* relies not only on the distinction between kinds of *philosophical* nonsense (illuminating and misleading), but also on the further distinction (which it shares with the positivist interpretation) between philosophical nonsense and mere nonsense. Both kinds of philosophical nonsense are taken by proponents of the ineffability interpretation to be more “substantial” than mere nonsense. This more substantial kind of nonsense is a kind in which we can recognize the place in the syntax of a sentence for an item of a certain logical category, but something of the wrong category has been put in that place. Mere nonsense is a kind of nonsense in which we are unable to discern sufficient syntactic structure to identify any part of a linguistic string as being the place for an item of a certain logical category. Mere nonsense is not, as it were, even trying to play by the rules of logic. Substantial philosophical nonsense, on the other hand, involves (what Carnap—one of the first and most lucid proponents of the positivist variant of the substantial conception—calls) countersyntactic formation: it plays by the rules up to a point and then breaks them. By breaking the rules of logic, substantial nonsense brings these rules out into open view. Let us take a closer look at how this is supposed to work.

Following Geach’s lead, we have sketched a reading of Frege that ascribes the ineffability variant of the substantial conception to him. One commentator who attributes to Frege (something that might appear at first glance to be) a version of the positivist variant is Michael Dummett. There is never any reference in Dummett’s exposition of Frege to thoughts that can only be gestured at or to that which Frege’s elucidations might be attempting—but failing—to express.⁷¹ In this respect, Dummett’s exposition diverges markedly from that of Geach. Yet, in other respects, Dummett’s account of substantial nonsense in Frege parallels the account offered by most commentators on the *Tractatus* of what “a violation of logical syntax” is. Here is Dummett on Frege’s theory of how such violations arise:

[It] is a theory of what expressions can be accepted as significant: only certain functions—those of the appropriate type—can “occur significantly” as arguments of other functions; expressions which violate the theory of types are simply meaningless. . . .

We, therefore, have to have some conception of logical valency, of different categories of expression, governed by rules determining that expressions of certain categories will fit together to form a sentence, while expressions of certain other categories will not. (Dummett, 1983, pp. 50, 62)

Dummett employs here the chemical metaphor of valency: just as certain elements can be combined so as to form a compound while others cannot be so combined, so items of certain logical categories can be combined so as to form a proposition and others cannot be so combined. Underlying this conception of logical valency

is the idea that we get a very special kind of nonsense when we abortively attempt to combine incompatible logical items—that is, when we attempt to combine logical items from logical categories that do not *fit* together. Dummett is certainly right that Frege often speaks in ways which encourage the attribution to him of the view that there are instances of this sort of nonsense. The following three passages furnish some examples of Frege's willingness to talk in these ways:

For not all the parts of a thought can be complete; at least one must be “unsaturated”, or predicative; otherwise *they would not hold together*. For example, the sense of the phrase ‘the number 2’ *does not hold together* with that of the expression ‘the concept prime number’ without a link. . . . They hold aloof from one another . . . ; however we put them together, we get no sentence. (Frege, 1984, p. 193, my emphasis)

Take the proposition ‘Two is a prime number’ The two parts of the proposition are . . . essentially different; and it is important to realize that this difference cuts very deep and must not be blurred. The first constituent ‘two’, is a proper name of a certain number; it designates an object, a whole that no longer requires completion. The predicative constituent ‘is a prime number’, on the other hand, does require completion and does not designate an object An object, e.g. the number 2, *cannot logically adhere* to another object, e.g. Julius Caesar, without some means of connection. This, in turn, cannot be an object but rather must be unsaturated. A logical connection into a whole can come about only through this, that an unsaturated part is saturated or completed by one or more parts. . . . Now it follows from the fundamental difference of objects from concepts that *an object can never occur predicatively* or unsatisfactorily; and that logically, *a concept can never stand in for an object*. One could express it metaphorically like this: There are different logical places; in some only objects can stand and not concepts, in others only concepts and not objects. (Frege, 1984, pp. 281–92, my emphasis)

We can analyze the proposition ‘3 is a prime number’ into ‘3’ and ‘is a prime number’. These parts are essentially different: the former complete in itself, the latter in need of completion. Likewise, we can analyze the proposition ‘4 is a square number’ into ‘4’ and ‘is a square number’. Now it makes sense to fit together the complete part of the first proposition with that part of the second proposition which is in need of completion (that the proposition is false is a different matter); *but it makes no sense to fit together the two complete parts; they will not hold together*; and it makes just as little sense to put ‘is a square number’ in the place of ‘3’ in the first proposition. (Frege, 1980, pp. 141–42, my emphasis)

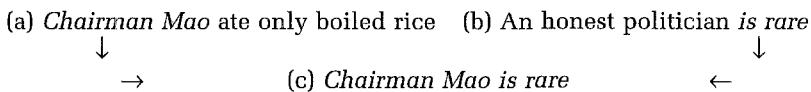
In passages of Frege's such as these there is the idea of a kind of nonsense that arises from an impermissible combination of logical categories—a kind of nonsense that results because “it makes no sense to fit together” the parts that we are attempting to combine.

Frege in these passages seeks to draw attention to examples of this kind of nonsense in order to elucidate the distinction between object-expressions and concept-expressions. Such an elucidation can only be conducted in ordinary language. In a proper *Begriffsschrift* such illegitimate combinations cannot be con-

structed. Here is Dummett's account of the kind of nonsense that ordinary language permits but a proper *Begriffsschrift* excludes:

Precisely because natural language violates the principle that each expression incomplete in sense must carry with it its argument-place(s), it does become possible within natural language to form meaningless but grammatically correct sentences which violate the distinctions of type and in the symbolic language could not be constructed at all. For instance, the sentence 'Chairman Mao is rare', while perfectly grammatical, is meaningless because 'rare', though in appearance just like a first-level predicative adjective, has the sense of a second-level predicate. The diagnosis and explanation of such failures of significance in natural language can easily be accomplished by reference to the impossibility of constructing a corresponding sentence in the symbolic language. (Dummett, 1983, p. 51)

Let us consider Dummett's example here.⁷² 'Chairman Mao is rare', Dummett says, is meaningless because 'rare' ("although in appearance just like a first-level predicative adjective") here actually expresses a second-level function (a function that takes first-level functions as its arguments). Sentences that involve "such failures of significance" can be constructed in natural language, thus sometimes leading us to mistake sense for nonsense. We are able to see clearly and to explain precisely how a sentence such as the one that figures in Dummett's example involves the particular sort of "failure of significance" it does by reflecting on "the *impossibility* of constructing" such a sentence (i.e., one that involves the "corresponding" failure of significance) in a proper *Begriffsschrift*. Dummett's picture of why this sentence is nonsense can be illustrated through the following diagram:



The proposal is to combine the underlined portions of propositions (a) and (b) so as to form a third proposition that, if there could be such a proposition, would be expressed by (c). We attempt to combine the 'Chairman Mao' of (a) [the 'Chairman Mao' that denotes *that* individual] and the '___ is rare' of (b) [the '___ is rare' that denotes *that* second-level function], and we thus arrive at (c), which, according to Dummett's Frege, is a concrete instance of a special type of meaningless sentence—one that involves a violation of logical category: we have tried to put a proper name into an argument place into which only a first-level function fits. Moreover, what we have here is (alleged to be) a case of *fully determinate* nonsense: (a) it is *logically distinct* from other fully determinate cases of substantial nonsense; (b) each of the "parts" of this proposition has a *fully determinate sense*; and (c) though the sense of the resulting whole is flawed, it is flawed in a *determinately specifiable respect*—it involves a determinate kind of "failure of significance" (whereas other cases of substantial nonsense each involve some other equally determinate "violation" of logical principles). That we have here to do with a logically determinate example of nonsense can be seen from the fact that other natural languages, unlike a proper *Begriffsschrift*, permit the construction of substantially nonsensical sentences that "correspond" (in the sort of flawed

sense they each possess) to this one. The determinately specifiable respect in which Dummett's case of substantial nonsense possesses a flawed sense is the following: it represents "an attempt" to put *that* proper name into *that* argument place for a first-level function. But it will not fit—(in Frege's words) "the parts cannot logically adhere," "it makes no sense to fit them together," "they will not hold together"—thus we get nonsense; but not mere nonsense, but a special variety of nonsense that arises from attempting to do something logically impossible. Wittgenstein's critique of Frege turns on his critique of this idea—an idea that is common to both the positivist and ineffability variants of the substantial conception: the idea that we can so much as try to put a logical item into an argument place in which it does not fit—the idea that we can have a proposition that has a fully determinate kind of sense but the kind of sense that it has is nonsense.

IX. The Tractarian Critique of the Substantial Conception

The great difficulty here is not to represent the matter as if there were something one couldn't do.

Wittgenstein, *Philosophical Investigations*

Is it possible to identify an expression as being of a particular logical category if it occurs in the wrong place? It is here, in its response to this question, that the *Tractatus* sees a tension in Frege's view. A number of Frege's doctrines and a great deal of his own methodological practice suggest that the answer to this question should be: No! It is reflection on these aspects of Frege's thought and practice that leads Wittgenstein to embrace the austere conception of nonsense. If one takes Frege's three principles to heart—as the author of the *Tractatus* does—then you will say to Dummett: if you want to know whether a particular word in a proposition is an object-expression or a concept-expression, you cannot just rely on your previous commerce with that word; you have to analyze the logical structure of the judgment and see what logical role is played by that segment of the proposition—how it contributes to the sense of the whole.

Frege warns in "On Concept and Object" (and elsewhere) that the same word in ordinary language can be used in some contexts as a proper name and in others as a concept word. Frege's favorite example of such a word is 'moon'.⁷³ It can also happen in ordinary language that an object-expression that has never been previously used to express a concept can suddenly be used, for the first time, as a concept-expression; and that we can understand what is meant by such an unprecedented usage. A famous example of a proper name suddenly being used as a concept-expression is Lloyd Bentson (in the 1988 vice presidential debate) saying to Dan Quayle: "You're no Jack Kennedy." Bentson's point was not that two individuals (Quayle and Kennedy) are not identical, but rather that there is a concept (of, say, exemplary statesmanship) that Quayle does not fall under. Frege offers as an example of this sort of creative use of language the lovely sentence "Trieste is no Vienna": "We must not let ourselves be deceived because

language often uses the same word now as a proper name, now as a concept word; in our example ['There is only one Vienna'], the numeral indicates that we have the latter; 'Vienna' is here a concept-word, like 'metropolis'. Using it in this sense, we may say: 'Trieste is no Vienna'" (Frege, 1984, p. 189). In this example, Frege says, we encounter a word that usually functions as a proper name playing the role of a concept-expression. Frege's reading of this sentence is arrived at through reflection upon what possible use this combination of words might have; that is, by asking himself: in what context would one utter such words, and what thought would one then be expressing? If we reflect on when we would utter such a sentence and what we might mean by it, Frege suggests, we will see that 'Vienna' here could mean something like 'metropolis' (or perhaps even beautiful or majestic metropolis)—and thus the sign 'Vienna' used in this way should be expressed in a proper logical symbolism by a completely different kind of symbol than that which we would use to express the occurrence of the word 'Vienna' in the sentence "Vienna is the capital of Austria." Notice that Frege does not conclude that what we have here in his lovely sentence about Trieste is a piece of nonsense—one that results from trying to put a proper name where a concept-expression should go. He concludes instead that what appears in the guise of a concept-expression here *is* a concept-expression—and then makes a suggestion about what the sentence as a whole might mean (and hence about *which* concept might be meant). Thus Frege's methodology here is to begin with our *understanding* of the proposition as a whole and to use that as a basis for segmenting it into its logically discrete components.⁷⁴ One can see Frege's methodological practice here as illustrating the close relationship between his three principles. If we disobey the second principle in our approach to this example, we end up violating the third: when we consider the word in isolation we take 'Vienna' for an object-expression, yet in this context it does not denote an individual; so if we fail to attend to the logical role of the word in this context, we mistake a concept for an object. What fuels such a mistake is one's tendency to think that one already knows what 'Vienna' means taken all by itself outside the context of that proposition—it means one presumes roughly what it means in a sentence like "Vienna is the capital of Austria." Although we do not realize it, Frege thinks that what is really going on when we think in this way is that we succumb to the all but irresistible urge to transgress against his first principle. When we ask for the meaning of the word in isolation, we unwittingly end up looking for the meaning in what Frege wants to teach us to recognize as the realm of the psychological. It may well be true that when I utter the word 'Vienna' in saying the sentence "Trieste is no Vienna," I intend to mean the same thing as when I utter the word 'Vienna' in saying "The capital of Austria is Vienna"—the same mental image of the spires of the *Stefansdom* rising up over the skyline of the city of Vienna may float before my mind's eye—but that, Frege thinks, does not bear on whether the word has the same meaning in these two sentences.

The methodological import of Frege's three principles is developed in the *Tractatus* through the claim that in ordinary language it is often the case that the same sign symbolizes in different ways. The distinction between sign (*Zeichen*) and symbol (*Symbol*) which this claim presupposes can be summarized as follows:

sign	an orthographic unit, that which the perceptible expressions for propositions have in common (a sign design, inscription, icon, grapheme, etc.) ⁷⁵
symbol	a logical unit, that which meaningful propositions have in common (i.e., an item belonging to a given logical category: proper name, first-level function, etc.)

Armed with the Tractarian distinction between sign and symbol, we can formulate the contrast between the two conceptions of nonsense (which Wittgenstein sees Frege as torn between) in a more precise manner. To recall, the two conceptions of nonsense were:

the substantial conception	which holds that there are two logically distinct kinds of nonsense: <i>substantial nonsense</i> and <i>mere nonsense</i>
the austere conception	which holds that there is, from a logical point of view, only one kind of nonsense: <i>mere nonsense</i>

The italicized terms in the above formulations can now be defined as follows:

substantial nonsense	a proposition composed of signs that symbolize, but which has a logically flawed syntax due to a clash in the logical category of its symbols
mere nonsense	a string composed of signs in which no symbol can be perceived, and which hence has no discernible logical syntax

I have, until now, pretended to be able to distinguish between the positivist and ineffability variants of the substantial conception. But, armed with the distinction between symbol and sign, we can start to see why the distinction between these two variants is an inherently unstable one.⁷⁶

Any attempt to clearly articulate the positivist variant will lead to its collapse either into the ineffability variant or into the austere conception. Either the proponent of the positivist variant holds that a violation of logical syntax involves an impermissible combination of symbols, or he holds that it involves an impermissible combination of signs. If he holds the former, then the positivist variant collapses into the ineffability variant; if the latter, then he abandons the substantial conception altogether. To take an example of the former case, Dummett's account of "Chairman Mao is rare" teeters throughout on the brink of collapse into a version of the ineffability variant. The items combined in Dummett's example—items which (in Frege's words) "cannot logically adhere," which "it makes no sense to fit together," which "will not hold together"—can not be mere *signs*. For the four signs 'Chairman', 'Mao', 'is', and 'rare' can be combined (as can any four signs). What cannot be combined, says Frege, is that which the signs *symbolize*: items belonging to incompatible logical categories. The expressions of which the example is composed are taken by Dummett to be incompatible (not because of their typographic properties, but) because of what he takes these expressions to symbolize: an object and a second-level function, respectively. But if the flaw lies in what is symbolized by the resulting combination, then, it would seem, there is *something* which these words, so combined, symbolize—an "it" which logic de-

bars but which Dummett is nonetheless able to frame in thought and identify as involving a violation of logic. If, on the other hand, the proponent of the positivist variant holds that a violation of logical syntax involves an impermissible combination of (mere) signs, then he teeters on the brink of abandoning the substantial conception altogether (in favor of the austere conception). For if his account of the impermissibility fails to turn on any logical feature(s) of the allegedly impermissible string, then he has deprived himself of the resources requisite for claiming that there are two logically distinct kinds of nonsense.

In order to begin to see why this is so, it will help to look more closely at the distinction between sign and symbol as it is drawn in the *Tractatus*. It is introduced as part of the commentary on §3.3, which is the *Tractatus*'s reformulation of Frege's second principle.⁷⁷ Section 3.3 runs as follows: "Only the proposition has sense; only in the context of a proposition has a name meaning." Then, beginning immediately thereafter (with §3.31), comes the following commentary:

Every part of a proposition which characterizes its sense I call an expression (a symbol).

(The proposition itself is an expression.)

Everything essential to their sense that propositions can have in common with one another is an expression.

An expression is the mark of a form and a content.

An expression presupposes the forms of all propositions in which it can occur. It is the common characteristic mark of a class of propositions (§§3.31–3.311)

An expression has meaning only in a proposition (§3.314)

I conceive the proposition—like Frege and Russell—as a function of the expressions contained in it. (§3.318)

The sign is that in the symbol which is perceptible by the senses. (§3.32)

Two different symbols can therefore have the sign (the written sign or the sound sign) in common—they then signify in different ways. (§3.321)

It can never indicate the common characteristic of two objects that we symbolize them with the same signs but by different *methods of symbolizing*. For the sign is arbitrary.

We could therefore equally well choose two different signs [to symbolize the two different objects] and where then would remain that which the signs shared in common? (§3.322)

The point of the commentary is in part to clarify the notion of "proposition" which figures in the context principle (only the *proposition* has sense; only in the context of a *proposition* has a name meaning⁷⁸). The relevant notion is one of a certain kind of a symbol—not a certain kind of a sign—something that only has life in language.⁷⁹ The sign, Wittgenstein says, "is that in the symbol which is perceptible by the senses" (what is now sometimes called the sign design). The symbol is a logical unit; it expresses something that propositions—as opposed to propositional signs—have in common.⁸⁰ Thus the sentences "Trieste is no Vienna" and "Vienna is the capital of Austria" have the sign 'Vienna' in common. These two sentences taken together offer an instance of what Wittgenstein means when he

says (in §3.321) "two different symbols can have the sign (the written sign or the sound sign) in common—they then signify in different ways." The sentences "Trieste is no Vienna" and "Vienna is the capital of Austria" have no symbol in common—all they have in common are the signs 'Vienna' and 'is'. In (what Wittgenstein calls) a proper logical grammar, each sign would wear its mode of symbolizing on its sleeve. We can, somewhat anachronistically, use modern logical notation to illustrate this point:

(a) Vienna is the capital of Austria	$v = c$
(b) Trieste is not the capital of Austria	$t \neq c$
(c) Trieste is not (identical to) Vienna	$t \neq v$
(a') Trieste is no Vienna	$\sim Vt$
(b') Trieste is no metropolis	$\sim Mt$
(c') Trieste is a Vienna	Vt

When written in ordinary language, sentences (a) and (a') have two signs ('Vienna', 'is') in common; when expressed in a proper logical notation, they are inscribed in such a way that their lack of a common symbol is reflected in the absence of a common sign. When written in ordinary language, sentences (c) and (a') have three signs ('Trieste', 'Vienna', "is") in common; when expressed in a proper logical notation, it is rendered perspicuous that they have only a single symbol in common. Once transposed into a proper logical notation, it would also be manifest which of the following three propositions have a propositional symbol in common:

- (a) Socrates was bald.
- (b) Socrates, who taught Plato, was bald.
- (c) A philosopher whose teacher was Socrates was bald.

It would become clear, from the manner in which these three propositional symbols were expressed in the notation, that (a) and (b) have a propositional symbol in common (though they have no sequence of words in common), and that (a) and (c) have no propositional symbol in common (despite their having the sequence of words 'Socrates was bald' in common). Taken together, (a) and (b) furnish an example of how in ordinary language different sequences of signs can have the symbol in common; and, taken together, (a) and (c) furnish an example of how in ordinary language the same sequence of signs can have no symbol in common, and thus how the same signs can belong to different symbols. Wittgenstein comments on these features of ordinary language:

In the language of everyday life it very often happens that the same word signifies in two different ways—and therefore belongs to two different symbols—or that two words, which signify in different ways, are apparently applied in the same way in the proposition.

Thus the word "is" appears as the copula, as the sign of equality, and as the expression of existence; "to exist" as an intransitive verb like "to go"; "identical" as an adjective; we speak of *something* but also of the fact of *something* happening.

(In the proposition "Green is green"—where the first word is a proper name and the last an adjective—these words have not merely different meanings but they are *different symbols*.) (§3.323)

It is worth elaborating how Wittgenstein's example in the last paragraph illustrates the point of the first paragraph of §3.323. The propositional sign 'Green is green' can be naturally taken as symbolizing in any of three different ways⁸¹—and hence can be understood as an expression for any one of three different thoughts:

(a) Mr. Green is green.	Gg
(b) Mr. Green is Mr. Green.	$g = g$
(c) The color green is the color green.	$(\forall x) (Gx \leftrightarrow Gx)$

One way of noticing how the same sign symbolizes differently in each of these three cases is to focus on the word 'is'. In each of the propositions expressing each of these three different thoughts, the sign 'is' symbolizes a different logical relation. In (a), the sign 'is' symbolizes the copula (a relation between a concept and an object); in (b) we have the 'is' of identity (a relation between objects); in (c), we have the 'is' of coextensionality (a relation between concepts).⁸² In the ordinary language version of (a)—“where the first word is a proper name and the last an adjective”—‘green’ can be seen to be not merely ambiguous with respect to its meaning (the way ‘bank’ is in “The bank is on the left bank”), but ambiguous with respect to its logical type: “these words have not merely different meanings but they are *different symbols*.” The point of the example is to show us that we cannot gather from the notation of ordinary language how a given sign (e.g., ‘green’, or ‘is’) symbolizes in a given instance. Wittgenstein suddenly follows this example with the observation: “Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full)” (§3.324). In a proper Begriffsschrift, a different sign would express each of these “different methods of symbolizing,” thus enabling us to identify the sources of certain confusions. In §3.325, Wittgenstein immediately goes on to say that in order “to avoid such errors” we require a symbolism that obeys the rules of *logical grammar*. How can such a Begriffsschrift enable us to avoid “the most fundamental confusions (of which the whole of philosophy is full)?” In order to answer this question, we need first to explore what sorts of “confusions” these are and what role a Begriffsschrift plays in their elucidation.

In the *Tractatus*, Wittgenstein argues that once we appreciate how Frege’s three principles work in conjunction with one another, we will see that there will always be room for a question as to whether a given sign, when it occurs in two different sentences of ordinary language, is symbolizing the same way in each of those occurrences. And this question cannot be settled simply by appealing to the fact that the same word (sign) ordinarily occurs (symbolizes) as a name⁸³ (e.g., as a name of the capital of Austria); nor by appealing to the fact that if I were asked what I meant when I uttered one of those sentences, I would reply that I meant the word in the same sense as I have on other occasions; nor by appealing to the fact that I, on this occasion of utterance, exert a special effort to mean the word in the same way as before. How can this question be settled? Wittgenstein says: “In order to recognize the symbol in the sign we must consider the context of significant use” (§3.326). We must ask ourselves on what occasion we would utter this sentence and what, in that context of use, we would then mean by it. (This is what we saw Frege do in his handling of the example “Trieste is no Vienna.”) In asking ourselves this, we still rely upon our familiarity with the way words (signs) ordinarily occur (symbolize) in propositions to fashion a segmentation of the propo-

sitional sign in question.⁸⁴ The point of §3.326 can be brought out by returning to Dummett's example. If, for example, we attempt to provide a context of significant use for "Chairman Mao is rare," it becomes possible to see the symbol in the sign in ways which Dummett does not consider. There are two equally *natural* ways to segment this string: (1) to construe 'Chairman Mao' as symbolizing a first-level function (on the model of "You're no Jack Kennedy"),⁸⁵ (2) to construe 'rare' as symbolizing a first-level function.⁸⁶ These are "natural" ways of "reading" the string because each reading segments the string along lines dictated by an established usage (i.e., an established method of symbolizing by means) of signs.⁸⁷ The expression '____ is rare' has an established use in the language (in sentences such as "An honest politician is rare") as a second level function; the expression 'Chairman Mao' has an established use in the language (in sentences such as "Chairman Mao ate only boiled rice") as a proper name. Each of these established uses dictates a possible segmentation of the string—each of which excludes the other.⁸⁸ There is not anything that is simultaneously segmenting the string along both lines at once.⁸⁹ Segmenting it either way, we supply a possible context of significant use and thus confer upon the string 'Chairman Mao is rare' a sense. According to the *Tractatus*, until we have done this, we have yet to confer any method of symbolizing on any of the signs that make up the string.

In §3.326, "the context of significant use" translates *sinnvollen Gebrauch*; "recognize" translates *erkennen*, which might also be translated "perceive." The latter is the same word that occurs in §6.54: "My propositions serve as elucidations in the following way: anyone who understands me eventually *perceives* them as nonsensical." To perceive a Satz as *sinnvoll* is to be able to perceive the propositional symbol in the propositional sign. To perceive a Satz as *Unsinn* is to be unable to recognize the symbol in the sign. For the *Tractatus*, these two forms of recognition eclipse one another.⁹⁰ Building on Frege's three principles, the *Tractatus* argues that in the case of a piece of nonsense—that is, in the absence of the provision of a context of *sinnvollen Gebrauch* determining a possible logical segmentation of the *Satz*—we have no basis upon which to isolate the logical roles played by the working parts of a propositional sign; for, *ex hypothesi*, there are no working parts of "the proposition." One can identify the contribution the senses of the parts of a proposition make to the sense of the whole only if the whole has a sense—if it stands in some identifiable location with respect to the other occupants of logical space. According to the *Tractatus*, there are no examples of a proposition's failing to make sense because its parts do not "fit" together. Thus there are no examples of the sort Dummett was looking for—examples of putting a proper name where a concept word belongs—for if one can properly make out that what belongs in that place is a concept word, then that is a sufficient condition for treating whatever is in that place as a concept word. There is not anything, on the conception of *Unsinn* which the *Tractatus* advances, that corresponds to a proposition's failing to make sense because of the meaning that the parts already have taken in isolation.⁹¹ On the Tractarian conception, the *only* way a sentence can be *Unsinn* is by its failing to symbolize.⁹² This conception does not rule out the possibility of Sätze (such as tautologies and contradictions) that have logical structure and yet are devoid of Sinn. (To think that it did would be to lose sight of the distinction between that which is *Unsinn* and that which is *sinnlos*.⁹³) It only rules out a sentence's having a fully determinate yet logically impossible sense—a sense that

it *cannot* have because of the logically determinate but logically mutually incompatible senses that its parts already have.

X. Carnapian Elucidation

It was Wittgenstein who first exhibited the close connection between the logic of science (or “philosophy,” as he calls it) and syntax . . . He has shown that all the so-called sentences of metaphysics are nonsense.

Rudolf Carnap, *The Logical Syntax of Language*

The contemporary understanding of the *Tractatus*'s conception of nonsense has been shaped in large part by the enthusiastic reception of the *Tractatus* by members of the Vienna Circle and the subsequent role that their understanding of its doctrines played in the development of their own highly influential doctrines. Carnap, in his essay “The Elimination of Metaphysics through the Logical Analysis of Language” (Carnap, 1959, pp. 59–81),⁹⁴ puts forward a conception of logical syntax that seeks to articulate a version of the positivist variant of the substantial conception.⁹⁵ The word “elimination” in the title of Carnap's essay is a translation of the German word *Überwindung*, which might be better translated “overcoming”, or even “subjugation”. In the final sentence of §6.54, it is said of the reader of the *Tractatus* that *er muß diese Sätze überwinden*: he must overcome (or defeat) these sentences⁹⁶—the ones that serve as elucidations and are, eventually, to be recognized by the reader as nonsense. We can get clearer about how Wittgenstein's *Sätze* are meant to serve as elucidations by pressing the following question: How faithful an inheritance of Wittgenstein's project (to teach his reader to “overcome” the sentences of the *Tractatus*) is Carnap's project of “overcoming” metaphysics?⁹⁷

Carnap took his understanding of the sources of metaphysical nonsense (and the role of the tools of logical syntax in effecting its cure) to be enormously indebted to the *Tractatus*; and he borrowed heavily on (what he took to be) the ideas of the *Tractatus* in elaborating his own (successive versions of an) *Überwindung der Metaphysik*.⁹⁸ Due to the considerable influence of Carnap's own ideas on several generations of analytic philosophers, subsequent commentators on the *Tractatus* have, often quite unknowingly, read Wittgenstein's work through Carnap's spectacles, construing the *Tractatus*'s often quite distinctive notions along more familiar Carnapian lines and importing additional Carnapian terminology to fill in the gaps in Wittgenstein's original exposition.⁹⁹ In §4.003 of the *Tractatus*, Wittgenstein writes: “Most propositions and questions, that have been written about philosophical matters, are not false, but nonsensical. . . . Most questions and propositions of the philosophers result from the fact that we do not understand the logic of our language.” On what has become the standard interpretation of the *Tractatus*, this passage is interpreted to mean (1) that “the nonsensical pseudo-propositions of the philosophers” are nonsensical because they “violate the rules of logical syntax,” (2) that this is what philosophers need to be brought to see about their pseudopropositions, (3) that this requires that they be

instructed in logical syntax (so as to be able to identify such violations), and hence (4) that “the misunderstanding of the logic of our language” that is the source of the confusions of philosophers is to be traced to their present inability to identify such violations. This (standard) interpretation of the *Tractatus* is broadly Carnapian: it takes the Tractarian project of uncovering *Unsinn* to be a project of uncovering instances of substantial nonsense, it takes Tractarian *logical syntax* to be a formal combinatorial theory governing the legitimate employment of signs or symbols, and it takes Tractarian elucidation to consist in the specification of ill-formed sequences of signs or symbols. In the following pages we will be concerned to recover the original reference of each of these three pieces of Tractarian terminology—nonsense, logical syntax, elucidation—each of which has, due to the Carnapian inflection it has acquired, become all but inaudible to the ears of contemporary commentary. We will proceed by examining the senses of each of the terms as they respectively figure in the Carnapian and Tractarian projects of *Überwindung der Metaphysik*.

Let us begin with Carnap on *Unsinn*. Carnap distinguishes two kinds of *unsinnige* pseudo-propositions: those that contain a meaningless word or words and those that contain only meaningful words, but put together in such a way that no meaning results.¹⁰⁰ I will refer to these as type (i) nonsense and type (ii) nonsense, respectively. Metaphysical nonsense, Carnap thinks, can occasionally be traced to an unwitting attraction to type (i) nonsense. He speculates that some stretches of metaphysical discourse about “God” are of this sort. It involves a simple failure to settle on any specific meaning for the term ‘God’. In such cases, the metaphysician, in point of fact, simply does not know what he means by ‘God’ but nonetheless continues to employ the term under the impression that it does have a definite and familiar meaning.¹⁰¹ The tools of logical syntax only play an indirect role in the exposure of type (i) nonsense. Such an employment of the term ‘God’ can be seen to be nonsense from the fact that it fails to satisfy “the first requirement of logic”: the requirement that one be able to specify how it occurs meaningfully in elementary statements of the form “*x* is a God.” The diagnosis and cure of type (i) nonsense does not require any detailed attention to the logical structure of the speaker’s propositions; and, indeed, strictly speaking, type (i) nonsense has no (fully) determinate logical syntax. All that is required to “overcome metaphysics” in such a case is to bring the speaker to realize that she is unable to provide a specification of the meaning of the word in question.

Carnap is of the view that an unwitting attraction to type (i) nonsense accounts for a certain portion of the pseudostatements of metaphysicians. But, more often, a metaphysician does know what she means by each of her words. When a speaker is able to specify what each of her words mean (i.e., how it occurs in elementary propositions), and yet sense fails to result from the combination of her words, then the source of the failure is to be traced (not to an absence of meaning on the part of one of the constituents of her propositions, but rather) to the illicit character of the combination—to its being a case of type (ii) nonsense. Type (i) nonsense is literally unintelligible: it contains (at a point where something with meaning should be) a void. Type (ii) nonsense is not literally unintelligible: we know what each of the parts of the proposition mean—the trouble lies with the composite which they form. It is often not evident to speakers of a natural language that such type (ii) sequences are meaningless, because the sequences in question do not

violate the excessively permissive combinatorial rules of ordinary grammar. Their accord with the rules of ordinary grammar *masks from view* their true underlying character. The point of translating a type (ii) sequence of words into logical notation is to bring to the surface what natural-language syntax obscures from view.

This brings us to Carnap on logical syntax. The syntax of a language, for Carnap, specifies which combinations of words are admissible and which are not. The syntax of a natural language allows for the formation of type (ii) nonsense—sequences of words that are meaningless because of the incompatible meanings of the words involved. In the case of type (ii) nonsense, the meaninglessness of the combination is to be traced to what Carnap calls “a violation of *logical syntax*” or, alternatively, “*logically* counter-syntactic formation.” Such formations can be demonstrated to be *irremediably* flawed as vehicles for the expression of thought. Let us consider the example that Carnap himself offers of a violation of logical syntax: “Caesar is a prime number.”¹⁰² Carnap wants to say that in ordinary language it is possible to form the nonsensical sentence (c) by combining the portions of the (meaningful) propositions (a) and (b) italicized below:

$$\begin{array}{ccc} \text{(a)} & \text{Caesar crossed the Rubicon} & \text{(b)} \\ & \downarrow & \downarrow \\ & \rightarrow & \leftarrow \\ & \text{(c)} & \text{Caesar is a prime number} \end{array}$$

To make sense of Carnap’s example, it would appear that we must (as we earlier had to in the case of Dummett’s example) take (c) to be the result of concatenating (logically incompatible) symbols, and not mere signs. About this example Carnap wants to say that it is nonsense, but not that it is type (i) nonsense.¹⁰³ The resulting nonsense is not due to the absence of meaning on the part of some word or words, but rather to precisely the meanings that the words already have: meanings that clash with one another when imported into this context. We therefore have here an analogue of the sort of example Dummett wanted to employ in order to illustrate Frege’s view: it is supposed to be an example of a kind of nonsense that is due to the way in which the meanings of the parts of the sentence fail to fit together so as to make sense.

Let us now turn to Carnapian elucidation. Carnap furnishes a detailed example of how the elucidation and elimination of metaphysical nonsense are supposed to proceed. Carnap takes a passage from Heidegger as his illustration. Heidegger’s text allegedly furnishes a particularly vivid case of type (ii) nonsense. It is adduced as a *typical* case of type (ii) metaphysical nonsense—one ripe for the application of Carnap’s method. Here is the text:

What is to be investigated is Being only and—*nothing* else; Being alone and further—*nothing*; solely Being, and beyond Being *nothing* . . . Does the Nothing exist only because the Not, i.e. Negation, exists? Or is it the other way around? Does Negation and the Not exist only because the Nothing exists? . . . We assert: the Nothing is prior to the Not and the Negation. . . . Where do we seek the Nothing? How do we find the Nothing. . . . We know the Nothing. . . . Anxiety reveals the Nothing. . . . That for which and because of which we were anxious, was ‘really’—nothing. Indeed, the Nothing itself—as such—was present. . . . What about this Nothing?—The Nothing itself nothings. (Carnap, 1959, p. 69, emphases in the original)¹⁰⁴

What basis does Carnap have for suspecting these statements of Heidegger's of being nonsense? One suspects that what initially brought them under a cloud of suspicion is that they are not obviously even *grammatically* well formed. The same word ('nothing') that ordinarily signifies a logical particle (used to form negative existential statements) appears in this text sometimes in the grammatical role of a substantive, sometimes in that of a verb. Carnap furnishes the reader with an elaborate chart that purports to demonstrate how someone might be misled by features of surface grammar into thinking that he was employing the word 'nothing' in a grammatically unobjectionable manner when, in reality, employing it in the logically illicit manner of a Heidegger. But it is hard to see how Heidegger's text is supposed to furnish an illustration of Carnap's theory: *This* is supposed to be an example of how metaphysical nonsense remains *undetected* until brought to the surface through the application of the principles of logical syntax? Carnap's elaborate analysis is scarcely credible as an account of how Heidegger is led to employ the word 'nothing' as he does here. It will not do to say of Heidegger's sentences that "the fact that the rules of the grammatical syntax of ordinary language are not violated [is what] seduces one into the erroneous opinion that one still has to do with a statement" (p. 67). Such a diagnosis would be blind to the stunningly virtuosic character of Heidegger's employment of the word, even when judged by the allegedly comparatively permissive lights of ordinary grammatical syntax. This virtuosity renders Heidegger's text utterly unsuitable as an example of that which it was allegedly introduced as an example of: the surreptitious misuse of language. It is hard to credit the hypothesis that the author of this text has been led astray by the surface grammar of ordinary language; for precisely what puzzles and challenges us in Heidegger's assertions is their peculiar surface grammar. The disclosure that language is under some extraordinary pressure in this text does not wait on the application of the principles of logical syntax. Heidegger is evidently speaking here in an unusual way: openly forcing his reader to reflect on how his words are meant (on what symbol we are to see in the sign).

Carnap's analysis rests on the supposition that Heidegger's words are employed by him in nothing other than their usual senses. But the problem with the example is that it is hard to see how this author *could* imagine that he was employing the words in their usual senses. Carnap sees this problem. Here is his first line of response:

In view of the gross logical errors which we find [in Heidegger's text] . . . , we might be led to conjecture that perhaps the word "nothing" has in Heidegger's treatise a meaning entirely different from the customary one. And this presumption is further strengthened as we go on to read there that anxiety reveals the Nothing, that the Nothing itself is present as such in anxiety. For here the word "nothing" seems to refer to a certain emotional constitution, possibly of a religious sort, or something or other that underlies such emotions. If such were the case, then the mentioned logical errors . . . would not pertain. But the first sentence of the quotation at the beginning of this section proves that this interpretation is not possible. The combination of "only" and "nothing else" shows unmistakably that the word "nothing" here has the usual meaning of a logical particle that serves for the formulation of a negative existential statement. (Carnap, 1959, p. 71)

If we adopted the assumption that “The word ‘nothing’ has in Heidegger’s treatise a meaning entirely different from the customary one,” then we would have to know what Heidegger means by the word ‘nothing’ before we could conclude that its occurrence here violated the principles of logical syntax. On what basis does Carnap rule out the possibility that its meaning here might be different from the customary one? How does Carnap know that Heidegger means the word ‘nothing’ throughout the course of his enigmatic assertions always in the same way and always only in its usual sense (and thus, in most of its occurrences, incoherently)? His evidence for this claim is that in the *first* sentence (of the sequence of sentences that Carnap has chosen to excerpt from Heidegger’s essay¹⁰⁵) we find the author using the word ‘nothing’ in the usual way: “What is to be investigated is Being only and—*nothing* else; Being alone and further—*nothing*; solely Being, and beyond Being *nothing*.” The employment of the word ‘nothing’ in this sentence is, by Carnap’s lights, grammatically and logically unobjectionable. The sentence is nonetheless included as part of Carnap’s extract from Heidegger because of the light it ostensibly sheds on the rest of the text. Its role is to show that the overall context of Heidegger’s remarks supports Carnap’s reading of them. The occurrence of the word ‘nothing’ in this first sentence “shows unmistakably,” says Carnap, that the word ‘nothing’ is used univocally in none other than its usual meaning throughout Heidegger’s text. Frege and Wittgenstein would object: to imagine that an examination of Heidegger’s first sentence suffices to establish that the word ‘nothing’ retains its usual meaning in its occurrences throughout the subsequent sentences just is to violate Frege’s second principle (and with it, *Tractatus*, §3.3). Moreover, Carnap’s basis for his conclusion (i.e., the claim that Heidegger *intends* to continue to use the word the same way in the subsequent sentences) runs afoul of Frege’s first principle. Carnap proceeds toward his conclusion in precisely the way we saw Kerry proceed: namely, first, by noticing how a sign is used in one context of use; then, second, by attempting to establish what is meant in a subsequent context of use by appealing to an intention to employ the same sign in the same way as in the original context; and then, finally, by imagining that the existence of the postulated intention can fix the meaning of the sign in the subsequent context, enabling it to continue to symbolize in the same way (regardless of its logical role within the subsequent context).

As §3.323 of the *Tractatus* says: “In the language of everyday life it very often happens that the same word [sign] signifies in two different ways—and therefore belongs to two different symbols.” It also happens, as demonstrated by Bentson’s use of ‘JFK’ and Frege’s example of ‘Vienna,’ that a word that has a conventionally established usage is suddenly, yet (potentially) intelligibly employed in an unprecedented manner. How can Carnap be sure that in Heidegger’s passage the sign ‘nothing’ always symbolizes in the same way? How can he be sure that Heidegger’s later uses of the word do not represent an attempt to employ the word ‘nothing’ in a linguistically innovative yet (potentially) intelligible manner? Carnap answers: Heidegger *self-consciously* aspires to speak nonsense. It is actually Heidegger’s *aim*, in these sentences, to (try to) jam the negative existential quantifier first into an argument place that can only accommodate an object-expression, then into an argument place that can only accommodate an expression for a first-level function, and so on.¹⁰⁶ The attribution of such an intention would be uncharitable in the absence of any evidence suggesting that Heidegger does pos-

sess such an extraordinary aim. Carnap (imagines he) possesses a way of ruling out the alternative charitable construal.¹⁰⁷ He has *evidence* which shows that Heidegger intends to speak nonsense. Heidegger elsewhere in his work, Carnap tells us, explicitly avows the intention that Carnap here attributes to him.¹⁰⁸

The presumption behind Carnap's procedures initially appeared to be that no one would intentionally speak nonsense. The original idea was supposed to be that if the nonsensical character of the metaphysician's utterances were made evident to him, he would no longer be attracted to them. It is hard to see how Carnap can attribute to the author of a purportedly typical case of metaphysical nonsense an intention to speak nonsense without abandoning his original claims concerning how to diagnose and cure metaphysical nonsense (or at least abandoning his claim that Heidegger is a representative example of the phenomenon that Carnap's essay seeks to bring to his reader's attention). The advent of the cure was originally advertised as coinciding with the metaphysician's epiphany that his employment of words involved an illegitimate combination of meanings. It is difficult to see how, by Carnap's own lights, the application of the principles of logical syntax could ever lead to a cure of the philosopher who self-consciously aspires to produce nonsensical combinations of word. Frege (as Geach and Wittgenstein read him) in "On Concept and Object," however, is just such a philosopher. It is this sort of philosopher—against whom Carnap's methods are impotent—that the *Tractatus* is, above all, concerned to wean from nonsense.

It is no accident that Carnap has fixed upon an example that has the features exhibited by this passage from Heidegger. Though it fails to accord with his own description of metaphysical nonsense, Carnap needs to avail himself of an example with these features to be able so much as to appear to provide any sort of illustration of the practical application of his theory. Heidegger's text offers the appearance of simultaneously satisfying three conditions all of which an example must satisfy if Carnap's methods are to seem to stand a chance of unmasking it as a case of metaphysical nonsense: (1) it must consist of sequences of words that a human being was actually moved to write with the intention of communicating a thought, (2) it must be possible to identify it as a case of nonsense simply by attending to the words as they stand on the page, (3) it must be possible to forestall the objection that the words have been construed in an uncharitable manner. If Carnap fixed upon an example in which the speaker did not exhibit the slightest paradoxical animus and uttered only statements that were by the lights of ordinary grammar apparently unimpeachable, the question would always arise: Is the speaker really speaking nonsense? How does the speaker mean her words? Is there a way to make sense of her words? (Is there a way to see the symbol in the sign?) Carnap does not want the application of his methods to be forestalled by such preliminary inquiries. The only contexts in which such inquiries have a legitimate place, for Carnap, are ordinary nonmetaphysical cases of obstructed comprehension—cases in which we encounter hermeneutical difficulties concerning the semantics of syntactically well-formed sentences. Carnap wants, in his application of the method of logical analysis, to be able to bypass such inquiries altogether—to eschew any consideration of the semantics of a metaphysician's utterances—identifying metaphysical statements as cases of nonsense solely through an attention to (what he calls) their syntax. He wants to apply his analytical tools directly to the metaphysician's words considered in isolation from possible contexts of use.

XI. We Cannot Give a Sign the Wrong Sense

This familiarity with wrong meanings puzzles one.

Marianne Moore, "Picking and Choosing"

Carnap's understanding of type (ii) nonsense rests on affirming precisely the possibility that the *Tractatus* is concerned to repudiate: the possibility of identifying the logical category of a term outside the context of legitimate combination—of identifying the manner in which a sign symbolizes in a context in which the reference of the parts of a sentence does not determine the reference of the whole. This repudiation is perhaps most explicit in the series of remarks that lead up to the passage in which Wittgenstein locates the difference between his own conception of nonsense and that of Frege. Here is the full context of that passage:

Logic must take care of itself.

A *possible* sign must also be able to signify. Everything which is possible in logic is also permitted. ("Socrates is identical" means nothing because there is no property which is called "identical." The proposition is nonsensical because we have not made some arbitrary determination, not because the symbol itself is impermissible.)

In a certain sense we cannot make mistakes in logic. (§5.473)

We cannot give a sign the wrong sense. (§5.4732)

Frege says: Every legitimately constructed proposition must have a sense; and I say: Every possible proposition is legitimately constructed, and if it has no sense this can only be because we have given no *meaning* to some of its constituent parts.

(Even if we believe that we have done so.)

Thus "Socrates is identical" says nothing, because we have given *no* meaning to the word "identical" as *adjective*. For when it occurs as the sign of equality it symbolizes in an entirely different way—the symbolizing relation is another—therefore the symbol is in the two cases entirely different; the two symbols have the sign in common with one another only by accident. (§5.4733)

These remarks express in an extremely compressed fashion some of the central ideas of the *Tractatus*. Let us begin by looking at the example of Unsinn ("Socrates is identical") and the commentary on it that Wittgenstein offers here. It is the sort of combination of words that Carnap would be tempted to analyze as an instance of type (ii) nonsense—as an attempt to employ the identity sign (i.e., an expression that symbolizes the relation of identity between objects) as if it were a concept-expression. Wittgenstein says in this passage that the nonsensicality of the string is due not to an impermissible employment of a symbol, but rather to our failing to make a determination of meaning. Wittgenstein's refusal to accept a Carnapian analysis of the matter here is not due to some peculiarity of the example.¹⁰⁹ Wittgenstein says: "If it has no sense this can *only* be because we have given no

meaning to some of its constituent parts.” The “only” here signals that for Wittgenstein all apparent cases of type (ii) nonsense are (in the words of §6.54) “eventually to be recognized as” cases of type (i) nonsense. Carnap’s own example could be substituted for Wittgenstein’s without affecting the point of the passage. On the Tractarian view, “Caesar is a prime number” suffers, from a logical point of view, from the same deficiency as “Caesar is blick”: “if it has no sense this can only be because we have given no *meaning* to some of its constituent parts” (§5.4733), regardless of how strong our inclination may be “to believe that we have done so.”

The Wittgenstein of the *Tractatus*, faced with Heidegger’s assertions, would not have us conclude that a sign has been given a wrong use—for example, that a logical particle that serves for the formulation of a negative existential statement has been illegitimately employed. Wittgenstein would instead have us first attempt to identify alternative ways of perceiving the symbol in the sign by reflecting upon its possible contexts of significant use.¹¹⁰ Each alternative way of perceiving the symbol in the sign yields a distinct segmentation of the propositional sign into symbolic constituents. In a symbolic notation of the sort which the *Tractatus* recommends (one “founded on the principles of logical grammar,” designed expressly to serve the purposes of philosophical elucidation), there will correspond to each possible segmentation of the string a *unique* rendering of it in the notation. In Tractarian philosophical elucidation, the role of logical symbolism is to furnish a perspicuous means of representing alternative segmentations, thus perspicuously displaying to the speaker the range of available possibilities for meaning his words. Let us consider four possible outcomes such an elucidatory employment of logical notation might have. Let us begin with the two most straightforward possible outcomes. Faced with a perspicuous representation in logical symbolism of the possibilities for meaning his words, a speaker might:

- (a) accept a particular rendition of his sentence into the symbolism or
- (b) not accept any proposed rendition of his sentence.

If the outcome is (a), then we have learned what the logical form of the speaker’s statement is—we are furnished with a means for seeing the symbol in the sign. If (b), then it remains open what (if anything) he means—it remains open whether we are faced with a case of nonsense or have simply failed to discern his meaning. In both cases (a) and (b), a *Begriffsschrift* (i.e., a symbolic notation founded on the principles of logical grammar) serves a hermeneutic role. It helps us to see better what someone means by her words or what we mean by our own words. Let us now consider a third possible outcome. Faced with a perspicuous representation in logical symbolism of the possibilities for meaning his words, a speaker might:

- (c) discover that he means nothing at all by his words, but rather has been unwittingly hovering between alternative possibilities of meaning his words, without determinately settling on any one.

Prior to a perspicuous overview of the available possibilities for meaning his words, the speaker in case (c) is under the impression of having conferred a method of symbolizing on each of his signs. But, confronted with the perspicuous overview which the symbolism furnishes, the speaker discovers that he has been wavering

between alternative possible methods of symbolizing.¹¹¹ The task of working through the options for how he *can* mean his words undermines his impression that there was something determinate that he did mean by them. His original conviction that there was such a “something” dissolves on him. (We will return to case (c).) We need to consider yet a fourth kind of case before we can see how a *Begriffsschrift* discharges the whole of its appointed task as an instrument of Tractarian elucidation. In this case, the speaker:

- (d) refuses to countenance the possibility that the full meaning of his words could correspond to anything expressible in the symbolism.

Such a response signals that an interlocutor has placed his foot on the penultimate rung of the Tractarian ladder. Outcome (d) resembles each of the first two outcomes in a certain respect. It resembles (a) in that the speaker accepts *parts* of thoughts that can be expressed in the symbolism as corresponding to his own thought—but only parts: alternative rendition(s) of his words into the symbolism are, in each case, at most partially expressive of that which he wants to mean by his words. Outcome (d) resembles (b) in that the speaker refuses to accept any single rendition as definitively capturing his meaning. “Alternative renditions can express a constituent aspect of the whole which I want to mean,” the speaker in case (d) responds, “but no single rendition can express the whole of what I want to mean; because what I want to mean requires the conjunction of logical features that the symbolism does not permit me to conjoin.”

The speaker in case (d) feels that that which he wants to mean by his words could never be expressed in a *Begriffsschrift*, for the very features of a *Begriffsschrift* that render it capable of perspicuously reflecting the logical structure of language simultaneously render it incapable of expressing that which he wants to mean by his words. It is here that Tractarian elucidation encounters its final hurdle—the case of a speaker who not only, as in case (b), rejects all of the alternative possible ways of meaning his words expressible in a *Begriffsschrift*, but one who rejects any *possible* rendition of what he wants to mean by his words into a *Begriffsschrift* on a priori grounds—on the grounds that what he wants to mean cannot be accommodated by the logical structure of language. (It is against just such a speaker, as we have seen, that Carnap’s methods are powerless.) Such a speaker is perfectly willing to concede (as is Frege, according to Geach, in “On Concept and Object”) that that which he wants to mean by his words is fully nonsensical. Only he will insist that his nonsense is unlike the nonsense which figures in outcome (c): for his nonsense is substantial nonsense, and it is his aim to produce just this sort of nonsense. The task, when faced with such an interlocutor, for the *Tractatus*, is *not* one of demonstrating to the speaker that “the proposition is nonsensical because the symbol itself is impermissible” (§5.473). (This would hardly come as news; for this is just what the sort of interlocutor that is here in question will himself maintain about his own nonsense. “Logically impermissible” nonsense is just what he aims to produce, and nothing other than such a sort of nonsense would serve his purpose.) The task for the Tractarian elucidator is rather “to demonstrate to [the interlocutor] that he has given no meaning to certain signs in his propositions” (§6.53), that the “proposition” is only apparently substantially nonsensical. The elucidation is only at an end when the interlocutor arrives at the point at which he is able of his own accord to acknowledge this. Thus it is only at an end

when the interlocutor “recognizes” his propositions as *Unsinn*—in the sense of *Unsinn* specified in §5.4733—that is, in the only way, according to the *Tractatus*, anything can be *Unsinn*. The activity of elucidation which the *Tractatus* seeks to practice on its reader is only at an end when the reader of the work is able to “recognize” the propositions that figure in the work as *Unsinn*, not for the reason that the interlocutor in case (d) imagines (because of incompatible determinations of meaning he has already made), but rather because the reader now sees that *no* determination of meaning has yet been made.¹¹² The aim is to bring the reader to the point at which he himself is able to acknowledge that, in wanting to mean these forms of words (which make up the body of the work) in the apparently determinate way in which he originally imagined he was able to “mean” them, he failed to mean anything (determinate) at all by those forms of words.

XII. Logical Syntax in the *Tractatus*

You cannot prescribe to a symbol what it *may* be used to express. All that a symbol *CAN* express, it *MAY* express.

Wittgenstein, *letter to Russell*, 19.8. 1919

Logical syntax, in the *Tractatus*, is concerned neither with what Carnap calls “logical syntax” nor with what Russell calls “a theory of types.” To express the same point in the idiom of the *Tractatus*: logical syntax is concerned neither with the proscription of combinations of signs nor with the proscription of combinations of symbols. It is not concerned with the proscription of combinations of signs, because Tractarian logical syntax does not treat of (mere) signs; it treats of symbols—and a symbol only has life in the context of a significant proposition. It is not concerned with the proscription of combinations of symbols, because there is nothing to proscribe¹¹³—“Every possible proposition is legitimately constructed” (§5.4733). Tractarian logical syntax treats of the categorially distinct kinds of logically significant components into which *sinnvolle Sätze* can be segmented—such components being the sorts of components they are only in virtue of their participation in possible propositions.

Two years after his essay “Elimination of Metaphysics,” in his book *The Logical Syntax of Language*, Carnap writes: logical syntax “should have no reference to the meaning of signs” (Carnap, 1937, p. 282 n).¹¹⁴ This means: logical syntax is concerned with strings of uninterpreted signs—that is, strings of (mere) marks on paper. In Carnap’s work, from *The Logical Syntax of Language* on, “logical syntax” treats of a class of formal structures—combinatorial structures generated by sequences of signs—where “formal” means formal in the Hilbertian sense: void of semantic content or structure.¹¹⁵ “Formal” for Wittgenstein means pertaining to that structure common to language and world (within which all semantic content has its life) considered in abstraction from any particular (true or false) content. Every state of affairs has a Tractarian logical form. The only parts of the *world* that can be said to have “formal” properties, for the Carnap of *The Logical Syntax of Language*, are mere marks on paper, spoken words, etc. The author of *The Logical Syntax of Language*, if he mistook Wittgenstein’s notion of “formal” for his

own, would be obliged to regard the *Tractatus*'s employment of the notion of "formal" or "logical" properties that are equally "properties of language" and "of the world" (§6.12) as an example of type (ii) nonsense. Wittgenstein's remark in the *Tractatus* that "in logical syntax the Bedeutung of a sign ought never to play a role" (§3.33) sounds just like Carnap's remark that logical syntax "should have no reference to the Bedeutung of signs." But Wittgenstein is not saying what Carnap is saying. Mere marks on paper have no Tractarian logical syntax. Only symbols—"the parts of a proposition which characterize its *Sinn*"—have logical syntax. In Tractarian logical syntax, the particular Bedeutungen of signs "never play a role" (not because logical syntax is concerned with *mere* signs, but) because logical syntax is concerned only with *how* signs symbolize—with what the *Tractatus* calls their *methods of symbolizing* (§3.322)—while abstracting from *what* (i.e., which particular object, property, or relation) they denote. Logical syntax thus prescinds from all content and considers only of the bare form of significant thought.

Though Wittgenstein never speaks in the *Tractatus* of "violations of logical syntax," he does remark on the ways in which a proper logical grammar would enable us to see more clearly the logical structure of ordinary language—and thus the ways in which ordinary language itself fails to reflect its own logical structure in a perspicuous manner. These remarks occur in the context of his discussion of how ordinary language allows the same sign to symbolize in different ways and the same symbol to be expressed by different signs. He goes on to say:

Thus there easily arise the most fundamental confusions (of which the whole of philosophy is full).

In order to avoid these errors, we must employ a symbolism which excludes them, by not applying the same sign in different symbols and by not applying signs in the same way which signify in different ways. A symbolism, that is to say, which obeys the rules of *logical* grammar—of logical syntax. (§§3.324–3.325)

In order to understand this passage, we need to distinguish clearly between two different things one can mean by the expression "violation of logical syntax":

(1) substantial nonsense	the result of putting an item of one logical category in the place where an item of another category belongs
(2) cross-category equivocation	the result of allowing different occurrences of the same sign to symbolize items of different logical category ¹¹⁶

Carnap's appropriation of Tractarian logical syntax, in its talk of "violations of logical syntax," conflates these two kinds of "violation," as have many commentators after him. This allows §§3.324–3.325 to appear to offer textual evidence for the claim that the *Tractatus* holds that "the most fundamental confusions (of which the whole of philosophy is full)" (§3.324) are due to "violations" of the first kind, when all that is at issue are "violations" of the second kind. The point of a proper logical symbolism for the *Tractatus* is only to exclude the latter kind of "violation," but not the former kind (because, according to the teaching of the *Tractatus*, there is no such kind). Theories of logic that seek to proscribe certain combinations of symbols seek to take care of that which must fall into place of its own

accord: “If everything in the symbolism functions as if the sign had a meaning, then it has a meaning . . . As long as one knows how each individual sign symbolizes, then the rules of logical syntax must fall into place as a matter of course” (§§3.328, 3.334). It is, Wittgenstein comes to think by the time he writes the *Tractatus*, the task of “a proper theory of symbolism” to show that all such theories are “superfluous.”¹¹⁷ (“Logic must take care itself”; §5.473.) In rejecting such theories, the *Tractatus* rejects the project standardly attributed to it: one of demarcating the bounds of sense.¹¹⁸ When in the *Tractatus*, Wittgenstein says you cannot give a sign a wrong sense, he is claiming that there is no such thing as infringing on the bounds of sense and thus no bounds of the sort that Carnap (or Wittgenstein, early or late, according to most readings of him) seeks to demarcate.

The difference between an ideal logical symbolism and ordinary language, for the *Tractatus*, is that in the former—unlike the latter—one is able to read the symbol directly off the sign. Logical syntax for the *Tractatus* is not a combinatorial theory (which demarcates legitimate from illegitimate sequences of signs or symbols) but a *tool* of elucidation (which allows us to recognize the logical contributions of the constituent parts of a *Satz*, and the absence of such a contribution on the part of the constituents of a *Scheinsatz*). The kind of cross-category equivocation exhibited by an uncontextualized sentence of ordinary language such as “Green is green” is not possible in a *Begriffsschrift*. One can, of course, if one wants, call this sort of cross-category equivocation a “violation of logical syntax” (though Wittgenstein himself never speaks in this way), but if one chooses to speak in this way, one should be clear that what is at issue in those passages where Wittgenstein alludes to the differences between ordinary language and “a logical grammar” (§3.325) are differences in notational perspicuity between various kinds of symbolism.¹¹⁹

The preceding conclusion (that the only “logical” defects of ordinary language to be corrected by “a proper logical syntax” are defects in its notational perspicuity) runs counter to the widespread assumption that the early Wittgenstein—like Frege, Russell, and Carnap—is an ideal language philosopher. This assumption is encouraged by the Pears and McGuinness translation of §4.112:

A philosophical work consists essentially of elucidations.

Philosophy does not result in “philosophical propositions,” but rather in the clarification of propositions.

Without philosophy thoughts are, as it were, cloudy and indistinct: its task is to make them clear and give them sharp boundaries.

It certainly sounds here as if the role of an elucidation is to introduce clarity into propositions that prior to elucidation lack clarity: elucidation renders what is logically cloudy and indistinct precise and sharp. The interpretative assumption underlying the standard reading of this passage is that this transformation of thoughts (that are initially cloudy and indistinct) is effected through their transposition into a medium which, unlike ordinary language, permits the expression of precise and sharp thoughts. But Wittgenstein repudiates just such an understanding of §4.112 in his correspondence with Ogden. Wittgenstein rejects “the clarification of propositions” as a translation of *das Klarwerden von Sätzen* (Wittgenstein, 1973, p. 28) and, after several exchanges, suggests instead: “the

propositions *now have become clear that they ARE clear*" (p. 49).¹²⁰ This is such terrible English that Ogden decides simply to ignore the suggestion. But the point that is obscured by the existing translations (and which the young Wittgenstein's horrendous English seeks to bring out) is that the transition from unclarity to clarity (i.e., the kind of Klarwerden) that is at issue here is not one that is effected through a transformation in the logical character of *the propositions* of ordinary language, but rather through a transformation in the view that we command of their logical character. What is cloudy and indistinct—and is rendered transparent with the assistance of a logical syntax—is our view of the logical structure that is present in the proposition all along. The aim of elucidation is not "to clarify" in the sense of making that which is said or thought intrinsically clearer (in the sense of cleaning up and, to that extent, changing the logical character of what is said), but rather "to clarify" in the sense of making that which is said or thought clear to us (in the sense of disencumbering our view of the logical character of that which we have been saying all along). It is a matter of making explicit the logical structure that had been implicit in our Sätze all along¹²¹ (and, if our Sätze are Unsinn, it is a matter of making explicit that there has, all along, been no implicit logical structure but only the appearance of such structure).

In *Tractatus*, §5.5563, we find: "All propositions of our everyday language are actually, just as they stand, logically completely in order." Commentators have found it difficult to reconcile Wittgenstein's comment in §3.325 that "we must employ a symbolism which excludes" certain possibilities which ordinary language permits with his respectful comment here in §5.5563 concerning the logical orderliness of the propositions of ordinary language.¹²² But there is no conflict. For, according to the *Tractatus*, it is the logical imperspicuity of ordinary language which leads us to believe that it is able to accommodate a kind of thought that is not, just as it is, logically completely in order. Section 3.325 recommends a notation that eliminates the sort of notational imperspicuity ordinary language tolerates in order to help us perceive how the logically imperspicuous character of ordinary language seduces us into thinking that ordinary language tolerates the expression of logically flawed thoughts. The *Tractatus* wants to show how Frege's theory of Begriffsschrift—his theory of a logically perfect language that excludes the possibility of the formation of illogical thought—is in fact the correct theory of symbolism *überhaupt*. Language itself, the *Tractatus* says, prevents the possibility of every logical mistake (§5.4731).¹²³ Ordinary language is in this respect already a kind of Begriffsschrift. What for Frege is the structure of an ideal language is for early Wittgenstein the structure of all language. In his remarks clarifying his emendations of Ogden's initial attempt to translate §5.5563, Wittgenstein explains: "By this [i.e., §5.5563] I meant to say that the propositions of our ordinary language are not in any way logically *less correct* or less exact or *more confused* than propositions written down, say, in Russell's symbolism or any other Begriffsschrift. (Only it is easier for us to gather their logical form when they are expressed in an appropriate symbolism.)" (Wittgenstein, 1973, p. 50, emphasis in the original). Already in the *Tractatus*, Wittgenstein's interest in a logical symbolism is not that of someone who seeks to overcome an imprecision in ordinary thought through recourse to a more precise medium for the expression of thought. The *Tractatus* is interested in successors to Frege's Begriffsschrift (in what the *Tractatus* calls "logical grammars") because such systems of notation exclude a

multiplicity of kinds of use for individual signs, allowing one to see in a more perspicuous manner what kind of logical work (if any) a given term in a given sentence is doing. It allows us to see how—and, most important, whether—the signs we call upon (in giving voice to the thoughts we seek to express) symbolize. The advantage of a logical symbolism, for the *Tractatus*, lies not in what it permits (or forbids) one to say, but in the perspicuity of its mode of representation: in how it allows someone who is drawn to call upon certain words to see what it is (if anything) he is saying.¹²⁴ The reason ordinary language can lead us philosophically astray is not to be traced to its (alleged) capacity to permit us to formulate illogical thoughts (i.e., to give a sign the wrong sense).¹²⁵ Rather, it is to be traced to the symbolic imperspicuity of ordinary language—our inability to read off of it what contribution, if any, the parts of a sentence make to the sense of the whole. It is this lack of perspicuity in our relation to our own words which allows us to imagine that we perceive a meaning where there is no meaning, and which brings about the need for a mode of perspicuous representation of the possibilities of meaning available to us.

XIII. The Illusion of Sense

Appearances too, like other things, must have a cause, and that which can cause anything, even an illusion, must be a reality.

J. S. Mill, "What Is Poetry?"

In his attempts to make vivid the logically flawed character of the examples of type (ii) nonsense that (allegedly) occur in Heidegger's text, Carnap occasionally comes close to saying something patently incoherent: namely, that we know what each of the parts of one of Heidegger's sentences mean, including what the word 'nothing' here means, so we know what the resulting combination *would* mean, if such a combination were an admissible combination of meanings!¹²⁶ (More briefly: we grasp what "it" *would* mean, if what "it" meant could be meant!) In §5.4733, Wittgenstein says: "if [a proposition] has no sense this can only be because we have given no *meaning* to some of its constituent parts. (Even if we believe that we have done so.)" This last parenthetical remark of Wittgenstein's gently touches on the elucidatory aim of the work as a whole: to show us that we are prone to *believe* that we have given meaning to some of the constituent parts of a proposition when we have not done so. This remark highlights an important analogy between type (ii) nonsense such as "Caesar is a prime number" (as Carnap describes it) and an innocuously meaningful sentence such as "Caesar crossed the Rubicon": in each of these cases, we *believe* that we have already given a meaning to all of the constituent parts. In such cases, we undergo the phenomenology of meaning something determinate while failing to mean anything determinate by our words. Part of what causes us to hallucinate a meaning in instances such as "Caesar is a prime number," according to Wittgenstein, is that there is more than one natural remedy for what ails the nonsensical linguistic string. (The greater the number of natural remedies that lie ready to hand for redeeming the sense of

a string, the more powerful the illusion of meaning which that string is able to engender.) We could assign a meaning to ‘Caesar’ that would allow us to treat ‘Caesar’ as the kind of logical element that symbolizes a number; or, alternatively, we could assign a meaning to ‘prime number’ that would allow us to treat it as the kind of element that symbolizes a predicate which applies to persons. So there are two natural ways of making sense of this string: it can be taken as saying something it makes sense to say of a person—in which case it contains the proper name of a person but not a numerical predicate; or it can be taken as saying of a number something which it makes sense to say of a number—in which case it contains a numerical predicate but not a proper name for a person. But, according to the *Tractatus*, there isn’t anything which is an instance of a proposition’s containing two logical elements that are incompatible. What there can be is a case in which there are two natural directions in which to seek a sense for a sentence whose sense is as yet undetermined (as is the case with Carnap’s example). But each of the available readings of this sentence eclipses the other—as each reading of a duck-rabbit figure eclipses the other. There isn’t anything which is having a part of the sentence as it is segmented on one reading illegitimately combined with a part of the sentence as segmented on the other reading—any more than one can have only the eye of the rabbit taken from one reading of a duck-rabbit figure occur in combination with the face of the duck. To see the drawing as a picture of the face of a duck *is* to see the, as it were, argument place for an eye in the picture filled by the eye of a duck—that is what it is to see the dot (that sign) *as* an eye of a duck (*as* that kind of a symbol).

If we have not made the necessary assignments of meaning to cure Carnap’s example of its emptiness, then, according to the *Tractatus*, what we have before us is simply a string of signs—a string that has a surface resemblance to propositions of two distinct logical patterns: it has a sign but no symbol in common with propositions about the great Roman general Caesar, and it also has a sign but no symbol in common with sentences such as “53 is a prime number.” Its nonsensicality is to be traced, not to the logical structure of the sentence, but to *our* failure to mean something by it: to, what the *Tractatus* calls, our failure to make certain determinations of meaning. For Wittgenstein, the source of the clash is to be located in *our relation* to the linguistic string—not in the linguistic string itself. The problem, according to the *Tractatus*, is that we often believe that we have given a meaning to all of a sentence’s constituent parts when we have failed to do so. We think nonsense results in such cases not because of a failure on our part, but because of a failure on the sentence’s part. We think the problem lies not in an absence of meaning (in our failing to mean anything by these words), but rather in a presence of meaning (in the incompatible senses the words already have—senses which the words import with them into the context of combination). We think the thought is flawed because the component senses of its parts logically repel one another. They fail to add up to a thought. So we feel our words are attempting to think a logically impossible thought—and that this involves a kind of impossibility of a higher order than ordinary impossibility. Wittgenstein’s teaching is that the problem lies not in the words, but in our confused relation to the words: in our experiencing ourselves as meaning something definite by them, yet also feeling that what we take ourselves to be meaning with the words makes no sense. “We . . . hover between regarding it as sense and regarding it as nonsense, and

hence the trouble arises.”¹²⁷ We are confused about what it is we want to say, and we project our confusion onto the linguistic string. Then we look at the linguistic string and imagine we discover what *it* is trying to say. We want to say to the string: “We know what you mean, but ‘it’ cannot be said.” The incoherence of our desires with respect to the sentence—wishing to both mean and not mean something with it—is seen by us as an incoherence in what the words want to be saying. We displace our desire onto the words and see them as *aspiring* to say something they never quite succeed in saying (because, we tell ourselves, “it” cannot be said). We account for the confusion these words engender in us by discovering in the words a hopelessly flawed sense.

XIV. The Method of the *Tractatus*

There is nothing which requires such gentle handling as an illusion—that is, if one wishes to dispel it. If anything prompts the captive of the illusion to set his will in opposition, then all is lost. . . . So one must approach him from behind. . . . This requires . . . a kind of deception in which one deceives a person for the truth’s sake. . . . “To deceive” in such a case means to begin by accepting the other man’s illusion as good money.

Søren Kierkegaard, *The Point of View for My Work as an Author* (translation emended)

Recall how Fregean elucidation is supposed to work. The aim of Fregean elucidation is to help us to understand the principles of construction that underlie Frege’s *Begriffsschrift*. The mark of our having grasped his elucidations is that we have mastered his symbolism and are able properly to use it to express thoughts. Frege’s elucidatory “propositions” cannot be expressed in *Begriffsschrift*, but the logical distinctions which they attempt to convey—such as the distinction between concept and object—show themselves through the difference in the signs of *Begriffsschrift* whose employment we have mastered. Frege, in offering his elucidations, self-consciously employs a kind of nonsense in order to bring out the confusions of people like Kerry. But—according to the interpretation of Frege that Geach favors—for Frege, that is only part of the purpose of the activity of elucidation. Frege takes his elucidations also to convey insights into necessities founded “deep in the nature of things” (Frege, 1984, p. 156). Though his expressions, through a kind of necessity of language, misfire, the insights they seek to impart can be latched onto by the reader who meets him halfway and does not begrudge him a pinch of salt. This additional positive role (of imparting a kind of inexpressible insight) that Geach ascribes to Fregean elucidation corresponds to the central purpose ascribed to Tractarian elucidation by proponents of the ineffability interpretation of the *Tractatus*.¹²⁸ The ascription of such a conception of elucidation (to either Frege or Wittgenstein) presupposes the prior ascription of the substantive conception of nonsense.

But, as we have seen, it is possible to find in certain of Frege's doctrines a ground for hostility toward the substantive conception and for hospitality toward the austere conception of nonsense. Moreover, as we have also seen, there is ample textual evidence that the *Tractatus* seeks to erect its teachings on just those doctrines of Frege's. But if one attempts to credit this textual evidence, and thus ascribe to the *Tractatus* the austere conception, what then should one take the aim of Tractarian elucidation to be? How, according to such a reading, are we to make sense of the fact that the *Tractatus* takes itself to be engaged in an activity which is properly termed one of "elucidation"—an activity, that is, which is able to achieve or confer some form of clarity, enlightenment, or insight? To understand how the *Tractatus*'s own *Unsinn* is supposed to elucidate (when that of other philosophers mostly only misleads), some distinction between misleading nonsense and illuminating nonsense is evidently required; but, on the austere reading, illuminating nonsense is no longer a vehicle for a special kind of thought. If the aim of elucidation, according to the ineffability interpretation, is to reveal (through the employment of substantial nonsense) that which cannot be said, then, according to the austere reading, the aim of Tractarian elucidation is to reveal (through the employment of mere nonsense) that what appears to be substantial nonsense is mere nonsense. While the aim of the former sort of elucidation was supposed to be the conferral of insight into inexpressible features of reality, the aim of the latter is not insight into metaphysical features of reality, but rather insight into the sources of metaphysics. The premise underlying the procedure of the *Tractatus* (and this is connected to why the point of the work is an ethical one) is that our most profound confusions of soul show themselves in—and can be revealed to us through an attention to—our confusions concerning what we mean (and, in particular, what we fail to mean) by our words.

The heart of the Tractarian conception of logic is to be found in the remark that "we cannot make mistakes in logic" (§5.473). It is one of the burdens of the elucidatory strategy of the *Tractatus* to try to show us that the idea that we can violate the logical syntax of language rests upon a conception of "the logical structure of thought" according to which the nature of logic itself debars us from being able to frame certain sorts of "thoughts." Wittgenstein says: "Everything which is possible in logic is also permitted" (§5.473). If a sentence is nonsense, this is not because *it* is trying but failing to make sense (by breaking a rule of logic), but because *we* have failed to make sense with it: "the sentence is nonsensical because *we* have failed to make an arbitrary determination of sense, not because *the symbol in itself* is impermissible" (§5.473, my emphasis). The idea that there can be such a thing as a kind of proposition that has an internal logical form of a sort which is debarred by the logical structure of our thought rests upon what Wittgenstein calls (in the preface) "a misunderstanding of the logic of our language." In ascribing to the *Tractatus* a commitment to the substantial conception of nonsense, commentators have ascribed to that work a commitment to the very misunderstanding which the elucidatory strategy of the work as a whole is centrally concerned to exorcise.

The *Tractatus* aims to show that (as Wittgenstein later puts it) "I cannot use language to get outside language" (Wittgenstein, 1975, §6).¹²⁹ It accomplishes this aim by first encouraging me to suppose that I can use language in such a

way, and then enabling me to work through the (apparent) consequences of this (pseudo)supposition, until I reach the point at which my impression of there being a determinate supposition (whose consequences I have throughout been exploring) dissolves on me. So on the reading of the *Tractatus* suggested here, what is to happen, if the book succeeds in its aim, is *not* that I (1) succeed in conceiving of an extraordinary possibility (illogical thought), (2) judge “it” to be impossible, (3) conclude that the truth of this judgment cannot be accommodated within (the logical structure of) language because it is about (the logical structure of) language, and (4) go on to communicate (under the guise of only “showing” and not “saying” “it”) what it is that cannot be said. Rather, what is to happen is that I am lured up all four of these rungs of the ladder and then: (5) throw the *entire* ladder (all four of the previous rungs) away. On this reading, first I grasp that there is something that *must* be; then I see that it cannot be said; then I grasp that if it cannot be said it cannot be thought (that the limits of language are the limits of thought); and then, finally, when I reach the top of the ladder, I grasp that there has been no “it” in my grasp all along (that that which I cannot think I cannot “grasp” either). In order for a reader to pass through the first four stages of ascent up this Tractarian ladder, he must take himself to be participating in the traditional philosophical activity of argument, to be inferring conclusions from premises (as, e.g., Frege appears to be doing when he reasons from a pair of premises concerning (a) the nature of *Begriffe* and (b) the logical structure of certain propositions—such as “The concept *horse* is not a concept”—to the conclusion that his words “miss his thought”). A reader of the *Tractatus* only ascends to the final rung of the ladder when he is able to look back upon his progress upward and “recognize” that he has only been going through the motions of “inferring” (apparent) “conclusions” from (apparent) “premises”. Thus the elucidatory strategy of the *Tractatus* depends on the reader’s provisionally taking himself to be participating in the traditional philosophical activity of establishing theses through a procedure of reasoned argument; but it only succeeds if the reader fully comes to understand what the work means to say about itself when it says that philosophy, as this work seeks to practice it, results not in doctrine but in elucidations. And the attainment of this recognition depends upon the reader’s actually undergoing a certain *experience*—the attainment of which is identified in §6.54 as the sign that the reader has understood the author of the work: the reader’s experience of having his illusion of sense (in the “premises” and “conclusions” of the “argument”) dissipate through its becoming clear to him that (what he took to be) the *philosophische Sätze* of the work are *Unsinn*.

Thus what happens to us as readers of the *Tractatus*—assuming the work succeeds in its aim—is that we are drawn into an illusion of occupying a certain sort of a perspective. From this perspective, we take ourselves to be able to survey the possibilities that undergird how we must represent things as being, fixing what is “logically” necessary and what is merely contingent. From this perspective, we contemplate the logical structure of thought as it is and imagine that we are also able to contemplate the possibility of its being otherwise. We take ourselves to be occupying a perspective from which we can view the logical structure of language “from sideways on.”¹³⁰ This illusion of perspective is engendered by the perception of a flawed sense in certain nonsensical propositions; we take these substantially nonsensical propositions to be attempting to express a state of affairs that

cannot be—and thereby to be disclosing the limits of possibility. Tractarian elucidation aims to show us that these sentences that apparently express substantially nonsensical thoughts actually express no thoughts. The “problems of philosophy” that the *Tractatus* sets itself the task of “solving” are all of a single sort: they are all occasioned by reflection on possibilities (of running up against the limits of thought, language, or reality) that appear to come into view when we imagine ourselves able to frame in thought violations of the logical structure of language. The “solution” to these problems (as §6.52 says) lies in their disappearance—in the dissolution of the appearance that we are so much as able to frame such thoughts. The mode of philosophy that this work practices (as §4.112 says) does not result in “philosophical propositions”: the “philosophical propositions” we come out with when we attempt to frame such thoughts are to be recognized as Unsinn.

This process of recognition is an inherently piecemeal one: our inclination to believe that we can perceive the symbol in the sign, when no method of symbolizing has yet been conferred on it, is not one that is to be extirpated, at a single stroke, by persuading the reader of some “theory” of meaning. As is made clear in §6.53, the aim is to demonstrate to the metaphysically inclined speaker that he has given “no meaning to certain signs in his sentences” on a case-by-case basis.¹³¹ The sign that one of the sentences of the *Tractatus* has achieved its elucidatory purpose comes when the reader’s phenomenology of having understood something determinate by the form of words in question is suddenly shattered. The reader undergoes an abrupt transition: one moment, imagining he has discovered something, the next, discovering he has not yet discovered anything to mean by the words. The transition is from a psychological experience of entertaining what appears to be a fully determinate thought—the thought apparently expressed by *that* sentence—to the experience of having that appearance (the appearance of there being any such thought) disintegrate. No “theory of meaning” could ever bring about the passage from the first of these experiences (the hallucinatory one) to the second (the experience of discovering oneself to be a victim of a hallucination). As long as we retain the relevant phenomenology of meaning (as long as it appears to us that, by golly, we do mean something determinate by our words), our conviction in such an experience of meaning will always lie deeper than our conviction in anything we are told by a theory of meaning concerning what sorts of things we are and are not able to mean by our words. (Hence the ineffectuality of Carnap’s methods.) The *Tractatus* does not aim to show us that certain sequences of words possess an intrinsically flawed sense by persuading us of the truth of some theoretical account of where to locate “the limits of sense.” Any theory that seeks to draw such “a limit to thinking” commits itself, as the preface says, to being “able to think both sides of the limit” and hence to being “able to think what cannot be thought.” The Tractarian attack on substantial nonsense—on the idea that we can discern the determinately unthinkable thoughts certain pieces of nonsense are trying to say—is an attack on the coherence of any project that thus seeks to mark the bounds of sense. The *Tractatus* seeks to bring its reader to the point where he can recognize sentences within the body of the work as nonsensical, not by means of a theory that legislates certain sentences out of the realm of sense, but rather by bringing more clearly into view for the reader the life with language he already leads—by harnessing the capacities for distinguishing sense from non-

sense (for recognizing the symbol in the sign and for recognizing when no method of symbolizing has yet been conferred upon a sign) implicit in the everyday practical mastery of language that the reader already possesses. As the preface says: "The limit . . . can only be drawn *in language* and what lies on the other side of the limit will be simply nonsense." Just as, according to the *Tractatus*, each propositional symbol—that is, each *sinnvoller Satz*—shows its sense (§4.022), so the *Tractatus* shows what it shows (i.e., what it is to make sense) by *letting language show itself*, not through "the clarification of sentences," but through allowing "sentences themselves to become clear" (through *das Klarwerden von Sätzen*, §4.112). The work seeks to do this, not by instructing us in how to identify determinate cases of nonsense, but by enabling us to see more clearly what it is we do with language when we succeed in achieving determinate forms of sense (when we succeed in projecting a symbol into the sign) and what it is we fall short of doing when we fail to achieve such forms of sense (when we fail to confer a determinate method of symbolizing on a propositional sign).

The assumption underlying Tractarian elucidation is that the only way to free oneself from such illusions is to fully enter into them and explore them from the inside. This assumption—one that underlies both Wittgenstein's early and later work—is nicely summarized in the following remark (from a 1931 manuscript of Wittgenstein's): "In philosophy we are deceived by an illusion. But this—an illusion—is also something, and I must at some time place it completely and clearly before my eyes, before I can say it is only an illusion" (Wittgenstein, 1997, quoted in Stern 1995, p. 194).¹³² The illusion that the *Tractatus* seeks to explode, above all, is that we can run up against the limits of language. The book starts with a warning about a certain kind of enterprise—one of attempting to draw a limit to thought. In the body of the text, we are offered (what appears to be) a doctrine about "the limits of thought." With the aid of this doctrine, we imagine ourselves to be able both to draw these limits and to see beyond them. We imagine ourselves able to do what the preface warns we will fall into imagining ourselves able to do (once we imagine ourselves able to draw a limit to thought): we imagine ourselves able "to think both sides of the limit" (and hence "able to think what cannot be thought").¹³³ The aim of the work is to show us that beyond "the limits of language" lies, not ineffable truth, but rather (as the preface cautions) *einfach Unsinn*.¹³⁴ At the conclusion of the book, we are told that the author's elucidations have succeeded only if we recognize what we find in the body of the text to be nonsense. In §6.54, Wittgenstein does not ask his reader here to "grasp" the "thoughts" that his nonsensical propositions seek to convey. He does not call upon the reader to understand his sentences, but rather to understand *him*, namely, the author and the kind of activity in which he is engaged—one of elucidation. He tells us in §6.54 how these sentences serve as elucidations: by enabling us to recognize them as nonsense.¹³⁵ One does not reach the end by arriving at the last page, but by arriving at a certain point in an activity—the point when the elucidation has served its purpose: when the illusion of sense is exploded from within. The sign that we have understood the author of the work is that we can throw the ladder we have climbed up away. That is to say, we have finished the work, and the work is finished with us, when we are able to *throw* the sentences in the body of the work—sentences about "the limits of language" and the unsayable things that lie beyond them—away.

Notes

This essay has been gestating for so long that it has become difficult to keep track of everyone who has helped shape it. I am indebted to conversations with Nuel Belnap, Stanley Cavell, Piergiorgio Donatelli, David Finkelstein, John Haugeland, Michael Kremer, John McDowell, Hilary Putnam, Lynette Reid, and Ed Witherspoon; to correspondence with Peter Hacker; to undergraduate and graduate courses on Frege, Russell, and/or early Wittgenstein from Burton Dreben, Warren Goldfarb, and John McNeese; to work on Frege by Robert Brandom, Peter Geach, Thomas Ricketts, and Joan Weiner; to work on Russell by Peter Hylton, Leonard Linsky, and Anthony Palmer; to a Pitt graduate seminar cotaught with Jamie Tappenden; to a session in which a portion of this essay was discussed at the University of Pittsburgh Department of Philosophy Faculty Colloquium; and to comments on earlier drafts by Alice Crary, Peter Hacker, Kelly Dean Jolley, Diego Marconi, Stephen Mulhall, Martin Stone, Michael Thompson, Lisa Van Alstyne, and Peter Winch. My most pervasive debt is to Cora Diamond, with whom—at some point or other over the past thirteen years—I have discussed every idea in this essay. This essay is dedicated to the memory of Peter Winch, at whose request it was originally written, at whose instigation it was revised, and to whom, now that it is done, it cannot be sent.

1. By this I mean to refer, above all, to a set of interpretative assumptions concerning the *Tractatus* that first came into currency through the Vienna Circle.

2. Here I have in mind what has essentially become the standard reading of the *Tractatus*. Its most lucid exponents are, in my estimation, Peter Geach and Peter Hacker.

3. On the question of the exegetical erroneousness of (1) and (2), the reading I favor is in agreement with the (now standard) ineffability interpretation. The differences between that reading and my own arise over the characterization of how and what illuminating nonsense illuminates.

4. I am using the word ‘show’ here *not* in the sense that the *Tractatus* itself reserves for this term (which, as we shall see, is not applicable to nonsense), but rather (as it is often used by proponents of the ineffability interpretation) to refer to the activity of “hinting” or “gesturing” at ineffable truths by means of nonsense. Whenever I employ the word in this latter sense I will place it in scare quotes. I am here adopting the idiom of many of the commentators with whose work I wish to take issue. But I hereby invite confusion in two ways; so let me just say for now: (1) that, in adopting this idiom, I do not take myself to be making any contact with the (actual) Tractarian notion of *zeigen*, and (2) that any commentator who holds that the sentences of the *Tractatus* aspire to hint or gesture at ineffable truths counts, by my lights, as a proponent of the ineffability interpretation, even if he or she (unlike most proponents of the ineffability interpretation) is textually scrupulous enough carefully to refrain from ever employing the term ‘showing’ to designate the activity of so hinting or gesturing.

5. The words ‘special kind of’ occur three times in this sentence. The (I am inclined to think, insuperable) challenge facing the ineffability reading is to find a way to dispense with the occurrences of these words (or equivalents thereof) without lapsing into unintelligibility: i.e., to characterize coherently what kind of nonsense this special kind of nonsense is, what sort of violation this special kind of violation (of the rules of logic) is, and what kind of thought this special kind of thought is.

6. The interpretative claim that the *Tractatus* espouses a thesis concerning the priority of thought to language—according to which our capacities for thinking, meaning, and understanding precede and outrun our capacities for the linguistic expression of thought—was first systematically defended by Norman Malcolm (1977, esp. pp. 137–41; 1986, esp. pp. 63–82). Anthony Kenny, Peter Hacker, David Stern, and Hans-Johann Glock have all subsequently defended some version of this interpretative claim. Indeed, some such claim—although often not explicitly defended—is presupposed by any interpretation that ascribes to the work the view that it is possible to *grasp* thoughts

which cannot be expressed in language. In arguing against the attribution of such a thesis to (even) early Wittgenstein, I am following the lead of Peter Winch (in "Language, Thought and World in Wittgenstein's *Tractatus*" 1987), and of Cora Diamond (1991b, chap. 6). I do not (nor, I take it, does either Winch or Diamond) mean to be arguing that one should attribute to the *Tractatus* a thesis concerning the priority of language to thought. The *Tractatus*—as, e.g., in §4 ("The thought is the significant proposition")—studiously avoids a claim of priority in either direction. The goal of the work as a whole is to show that either of these priority claims, once strictly thought through, collapses into the other.

7. Wittgenstein, in his correspondence about the *Tractatus*, repeatedly insists on the importance of the form of the book to the aim of the work as a whole. He writes Ludwig von Ficker (Wittgenstein, 1971, p. 15): "The work is strictly philosophical and at the same literary." (Indeed, Frege is quite alarmed by the stress Wittgenstein is prepared to lay on this aspect of the work: "The pleasure one is to have in reading your book can therefore not have its ground in the . . . content, but only in the form. . . . In this way the book becomes really more of an artistic than a scientific [*wissenschaftliche*] achievement; that which is said in it takes second place to how it is said." (letter to Wittgenstein, September 16, 1919, my translation; Frege, 1989, p. 21).) Commentators on Wittgenstein's work, however, generally adopt an exegetical procedure, which presupposes an affirmative answer to the question "Is the form of the work merely an optional decorative feature of the work?" Insofar, that is, as they take themselves to be able to tell us outright (in a piece of writing that has the form, say, of a journal article) what the *Tractatus* is laboring to "show" in its (putatively necessarily indirect) way, their expository practice would appear to rest on the belief that there is no great difficulty in prying the jewel loose from its setting. They seldom fail, along the way, to pay homage to the remarkable "style" of the work. Wittgenstein is praised for being "a great writer." (It is difficult, after all, simply to overlook what Ogden, in his "Translator's Note" to the *Tractatus*, calls "the peculiar literary character of the whole.") But if one looks at Wittgenstein's philosophical work and then looks at what commentators tell us his work is trying to say, it is hard to see how the usual deferential nod to Wittgenstein's literary talents amounts to more than a gesture of piety. As a literary endeavor, the work (if it is trying to say what contemporary commentary says it is trying to say) does not appear to be an interesting or subtle failure, but an abject failure. To be fully entitled to the claim that Wittgenstein is "a great writer," one must be able to make out how his laboriously crafted work serves—rather than frustrates—his philosophical ends.

8. *Tractatus Logico-Philosophicus*, §6.54, my emphases. All subsequent unspecified references to a section number are to the *Tractatus*. Quotations from the *Tractatus* are drawn from either the Pears and McGuinness translation (Wittgenstein, 1963) or the Ogden translation (Wittgenstein, 1922), or some emendation or (as in this case) combination thereof.

9. "I cannot imagine that Carnap should have so completely and utterly misunderstood the last sentences of the book—and therefore the fundamental conception of the whole book"; Wittgenstein, letter to Moritz Schlick, August 8, 1932; quoted in Nedo and Ranchetti, 1983, p. 255. For further discussion of this remark, see Conant, 1995.

10. Wittgenstein's letter continues: "To let you print the *Ergänzungen* would be no remedy. It would be just as if you had gone to a joiner and ordered a table and he had made the table too short and now would sell you the shavings and sawdust and other rubbish along with the table to make up for its shortness. (Rather than print the *Ergänzungen* to make the book fatter, leave a dozen white sheets at the end for the reader to swear into when he has purchased the book and can't understand it.)"

11. I have made a start on trying to offer a reading that meets this criterion of adequacy in Conant, 1991a. This essay is intended as a sequel to that paper.

12. That the notion of *Erläuterung* figures centrally in both Frege and the *Tractatus* is also the topic of Weiner, forthcoming; and that an understanding of the notion of

Erläuterung is central to an understanding of Wittgenstein's aim in the *Tractatus* is also the topic of McGinn, 2000.

13. In commenting on Ogden's translation of §6.54, Wittgenstein writes: "Here you *misunderstand my meaning entirely*. I didn't mean to use 'elucidate' intransitively: what I meant to say was: My propositions elucidate—whatever they do elucidate—in this way, etc. Similarly, I might have said: 'My propositions clarify in this way . . .' meaning 'My propositions clarify whatever they do clarify . . . in this way: [. . .].' Here clarify is *not* used intransitively although the object is not mentioned. You may put it thus: 'My propositions elucidate philosophic matters in this way. . . . This is something like the right meaning'" (Wittgenstein, 1973, p. 51, emphases in the original). The accompanying editor's note by G. H. von Wright reads as follows:

The original translation of this passage [6.54] had been "My propositions are explained in that he who understands me." When returning the typescript Wittgenstein changed this to "My propositions elucidate in this way that he who understands me." As seen from the Questionnaire, Ogden suggested "My propositions are elucidated in this way; he who understands me." Wittgenstein convinced Ogden that this was a misunderstanding and Ogden then changed "are elucidated" to "are elucidatory," and this is how the passage is printed. (pp. 53–54)

14. I have alluded to the standard answer to this question—the ineffability interpretation. But a word should be said about a much less promising (but surprisingly popular) alternative answer to this question—which might be called the formal (or cop-out) interpretation—an answer encouraged by a mistranslation of Ogden's (later corrected in the Pears and McGuinness translation). Ogden mistranslates *unsinnig* in §6.54 as "senseless," and indeed throughout conflates the distinction between *unsinnig* and *sinnlos*. (When I refer in this essay to Wittgenstein on nonsense, my topic throughout will be—unless otherwise stated—what is treated in the *Tractatus* under the rubric of *Unsinn*.) If the propositions of the work were only *sinnlos*, then they would have the same logical status as the propositions of logic (rather than having the same logical status as the "pseudopropositions" of the philosophers). This would (somewhat) mitigate the puzzle posed by §6.54 and allow for the sort of rescue operation one finds, for example, in Max Black: "We shall go wrong at once if we fail to remember that 'sense,' in Wittgenstein's use of that word, has two opposites (*sinnlos* and *unsinnig*), not one. . . . There remains the alternative of treating many of his remarks as formal statements, 'showing' something that *can* be shown. Then they will be in no worse case than logical and mathematical statements and there will be no theoretical barrier to their use in rational communication. A great many of Wittgenstein's remarks can be salvaged in this way" (Black, 1982, pp. 378–81). Here, briefly, are three (of the many) problems with Black's interpretation of the *Tractatus*:

(1) The claim that "there remains the alternative of treating many of his remarks as formal statements," insofar as it pretends to exploit the Tractarian notion of "formal," misses the point of that notion (as it is employed, e.g., in §§4.1212–4.1274). Wittgenstein does talk about "formal properties" and "formal concepts" but never about "formal propositions." Indeed, Wittgenstein says: "the holding of such internal properties cannot be asserted by propositions" (see, e.g., §4.122 and §4.124). (He does say that they can be *shown* through propositions, but this is of no help, for the relevant notion of *zeigen* applies only to propositions that are not *unsinnig*.) The propositions we come out with when we attempt to assert "the holding of such internal properties" are said to be *unsinnig*. The *Tractatus* therefore allows neither for "formal philosophical propositions" (in Black's sense of "formal") nor for ones that (attempt to) talk about "formal properties" (in Wittgenstein's sense of "formal") but are not *unsinnig*. (This entails that §§4.1212–4.1274 are themselves *unsinnig*—a matter we will come to.)

(2) Black's argument that the propositions of the work are only nonsensical in an innocuous sense pretends to exploit the fact (overlooked by Ogden) that 'sense,' in

Wittgenstein's employment of the term, has two opposites (*sinnlos* and *unsinnig*). But Black's interpretation can only retain a semblance of viability through systematically failing to respect the distinction between *Unsinn* and *Sinnlosigkeit* as it is actually drawn in the text. Black seeks to narrow the scope of *Unsinn* (so that it does not apply to the bulk of the propositions that comprise the book) and expands that of *Sinnlosigkeit* (so that it does). What Ogden's translation originally managed to obscure continues to be obscured by Black: namely, that *Sinnlosigkeit* is, in fact, a comparatively minor topic in the book, treated explicitly only in §§4.461, 5.132, 5.1362, and 5.5351 (thus preparing the way for §§6.1–6.111). Ogden's translation made *Sinnlosigkeit* appear a far more pervasive topic by mistranslating *unsinnig* throughout the critical discussions in §§4.003, 4.1212–4.1274, 5.473–5.4733 and elsewhere as “senseless.” Black claims to be cognizant of this oversight, but he, too, simply ignores that *unsinnig* is the term employed in the *Tractatus* to characterize the “propositions” of the work that are to serve as elucidations. Perhaps, if this were brought to his attention, he might try pleading that—at least as far as the pertinent passages of the work are concerned—Ogden got it right and Wittgenstein got it wrong: that Wittgenstein inadvertently used the incorrect word every once in a while (including in the crucial penultimate section!). But this still does not get Black very far: only tautologies and contradictions meet the specifications for *Sinnlosigkeit* set forth in the work, and the bulk of the “propositions” of the work are clearly neither tautologies nor contradictions.

(3) If Black is right that the “propositions” of the work are merely *sinnlos* (as are the propositions of logic), then the good news is that they involve no violations of logical syntax (and this is the conclusion that Black labors to secure); but the bad news is that there is then no reason to throw them away. Thus this interpretation saves one half of the text of §6.54 (the declaration that the propositions of the work are in some way lacking in sense), only at the cost of cutting loose the other half of the text: the declaration that the propositions of the work are to be “overcome” by the reader and that they form a ladder that is to be thrown away. Black (to his credit) admits that at this point he is simply giving up on the text and that, on his reading: “We need no mystifying addendum about the need to ‘overcome’ the remarks and to ‘reject’ . . . [them]—here is one ‘ladder’ that need not be thrown away” (1982, p. 381).

15. For Frege's own formulation, see Frege, 1967a, §32.

16. It is, Wittgenstein acknowledges in his preface to the *Tractatus*, “to the great works of Frege and the writings of my friend Bertrand Russell that I owe in large measure the stimulation of my thoughts.” (Anscombe comments: “Wittgenstein's relative estimate of Frege and Russell comes out in the acknowledgment he makes in the preface to the *Tractatus*” [Anscombe, 1971, p. 12]. The relative estimate comes out in the rest of the work as well.) Shortly after the *Tractatus* began to assume its final version, Wittgenstein wrote (on March 25, 1918) Frege of the great debt that he owed him; and Frege (in his reply of April 9, 1918) writes back: “Sie schreiben von einer grossen Dankesschuld, die sie mir gegenüber drückt. Ich weiss nichts von einer solchen [!].” (Frege, 1989, p. 16).

17. See Glock, 1996, pp. 259–60, for a helpfully explicit attribution of the substantial conception of nonsense to the *Tractatus*.

18. In claiming that the *Tractatus* is to be seen as resolving a tension in Frege's thought (between these two different conceptions of nonsense), I raise interpretative questions about how Frege is to be read—questions which I do not hope to resolve in this essay. I mean to take sides on this question only insofar as it bears on the claim that Wittgenstein can be fruitfully read as having read Frege in certain ways. I do not wish to deny that Frege can be fruitfully read as adhering to either one of these two conceptions of nonsense, and as having faced up to the implications of such a commitment. (Peter Geach reads Frege as an adherent of the position that there are certain truths that can be “shown” but cannot be said. Cora Diamond [1991b, chaps. 2, 4] reads Frege as having already anticipated the conception of nonsense which I attribute in this essay to the *Tractatus*.) I am inclined to think that each of these readings of Frege

has its exegetical advantages, each has moments where it stumbles over the text, and both are able to account for most of the texts (which, depending on the angle from which they are viewed, can assume the gestalt of either a substantial rabbit or an austere duck). My concern here will not be to referee such a dispute about Frege, but rather only to advance a claim about Wittgenstein and how he read Frege: namely, in a way that assigns to each of these readings half of the truth about Frege.

19. The positivist interpretation is all for showing that some sentences are non-sensical, but it wants no truck with the idea of philosophically illuminating nonsense. It wants to hold onto the substantial conception of nonsense (the idea that metaphysical nonsense arises through violations of logical syntax), while eschewing the idea that there are things that can be “shown” but not said.

20. I distinguish between these two variants because proponents of the substantial conception tend to present themselves as *prima facie* distinct in this respect. As we shall see, however, these variants cannot in the end be clearly distinguished from one another in the manner I am here pretending that they can be.

21. In fact, the interpretative options available in connection with this dimension of Frege’s thought perfectly parallel those available in connection with the *Tractatus*. Some commentators have ascribed to Frege the positivist variant of the substantial conception (e.g., Dummett), others (as mentioned in the previous note but two) the ineffability variant (e.g., Geach) or the austere conception (e.g., Diamond). I repeat: this paper is agnostic as to which of these readings represents the true Frege.

22. Contrary to the assumption implicit in most of the secondary literature on it, the *Tractatus* itself scrupulously marks this distinction (between what I misleadingly refer to here as two senses of ‘show’) by reserving *zeigen* to refer only to the first notion and using *erläutern* to refer to the second. Both of these notions are, in turn, to be distinguished from the notion of “showing” that figures in the ineffability interpretation (see note 4).

23. The widespread assumption in the scholarly literature that this distinction (between saying [*sagen*] and showing [*zeigen*]) is crucial for understanding §6.54 arises from the conflation of the two distinctions I am trying to disentangle here. The distinction between *sagen* and *zeigen* has no application to *Unsinn*. A proposition that is *sinnvoll* says what is the case and shows its sense (§§4.021–4.022). A proposition that is *sinnlos* shows that it says nothing (§4.461). A “proposition” that is *unsinnig* (contrary to the ineffability interpretation) neither says nor shows anything (which is not to say that it cannot elucidate). Section 6.54 is concerned with those sentences of the work that are (to be recognized as) *unsinnig*.

24. I speak here of different “*kinds* of use of language”—instead merely of different “uses of language”—to note a distinction that must be respected if we are to avoid confusion later when we turn to the topic of what §3.326 of the *Tractatus* calls “significant use.” To distinguish (what I here call) “kinds of use” is to distinguish the different sorts of things one can *do* with language over and above putting it to the use of *saying* something. Later on in this essay, when I turn to the point of §3.326, I will employ the expression “uses of language” to discriminate *within* the (primary) field of the assertoric employment of language different ways to use language to say things. Whenever the *Tractatus* itself speaks of the “use” (*Gebrauch*) of a sign, it is always in this latter sense.

25. I borrow this useful term from Austin, 1962.

26. The early Wittgenstein did not think these two kinds of use of language (the constative and the elucidatory) constituted an exhaustive classification—he thought there was also a distinct *ethical* employment of language. (What Wittgenstein thinks comprises an ethical use of language needs to be understood before one can approach the question of what Wittgenstein means when he says that “the point of the *Tractatus* is ethical”; see Diamond, 1991b, chap. 8; 1991a, 1996 and Conant, 2002b.) There is therefore good reason to be wary of an oft-repeated textbook platitude concerning the fundamental difference between the thought of the early and the later Wittgenstein:

that early Wittgenstein thought that language can only be put to *one* kind of use, whereas later Wittgenstein demolished his earlier doctrine by pointing out that language has a multiplicity of kinds of use. A way to put what is sound in the textbook platitude would be to say: for early Wittgenstein, nonconstative kinds of use of language (1) come in only two flavors (elucidatory and ethical), and (2) are not, properly speaking, employments of *language per se* (see §§4–4.001) but rather employments of *language-like* structures; whereas for later Wittgenstein, the category of nonconstative kinds of use (1) subtends many more kinds of use than ever dreamed of in the philosophy of early Wittgenstein (expressive uses of language, performative uses of language, etc.), and (2) represents not a mutually exclusive alternative to the constative employment of language, but rather a pervasive dimension of all language use.

27. The kind of *showing* that is at issue in the first half of this passage (one according to which logical category distinctions show themselves in a well-constructed formalized language) is a kind of showing that the Tractarian notion of *zeigen* aims to accommodate (though in this sense of ‘show’, according to the *Tractatus*, logical category distinctions only show themselves in *sinnvolle* sentences). Geach speaks of the nonsensical “sentences” that form the subject of the latter half of this passage (sentences in the vernacular that “are logically improper and admit of no translation into well-formed formulas of symbolic logic”) as seeking to “convey” these same distinctions of logical category. Such sentences, according to Geach, seek to convey something that cannot be said. The idea that the latter sort of “sentences” intend to *convey* what the former sort *show* might invite the idea that it ought to be possible to formulate a more inclusive notion of “showing”—one that construes as a single sort of activity something that logically proper sentences (of either a natural language or a well-constructed formalized language) and certain logically improper sentences (of ordinary language which admit of no translation into a well-constructed formalized language) are both able to engage in. Some commentators on the *Tractatus* employ the term ‘showing’ in this (by my lights, hybrid) way to encompass both these sorts of cases. Most commentators on the *Tractatus*, however, seem to have only the latter sort of case in view when they employ the term. Geach himself, however, is careful to employ the term to refer only to the former sort of case.

28. See, for example, Wittgenstein, 1980b, §42.

29. For further discussion of why logic, for Frege, does not stand in need of psychology, see Ricketts, 1986; and Weiner, 1990, chap. 2. For further discussion of why logic, for Frege, does not stand in need of metaphysics, see Ricketts, unpublished; and Weiner, 1995a, 1995b.

30. What Frege has done for Kerry (not to mention Schubart, Thomae, and others) brings to mind Heinrich Heine’s remark (from Part 2 of *Religion and Philosophy in Germany*) à propos Lessing’s polemics against Götze, Reimarus, and others: “He has snatched many a name from a well-deserved oblivion . . . and preserved it for posterity like an insect trapped in amber.”

31. This charge is expressed more emphatically in the unpublished version of “On Concept and Object”: “In my view the reason for the sorry state of affairs we find in Kerry, where the distinctions between concept and object, characteristic mark and property, are effaced is that logical and psychological questions and viewpoints are scrambled together. . . . He will speak now of a concept, then of the idea of a concept, now of an object, then of the idea of it, without its ever being wholly clear whether it is one or the other that is in question, whether we are engaged in a logical or psychological inquiry. . . . Here Kerry has simply succumbed to a widespread sickness” (Frege, 1979, pp. 104–5). Frege and early Wittgenstein both use the expression ‘concept’ in the same (“strictly logical,” non-surface-grammatical) way. What Frege calls a concept (“in the strictly logical sense”) is what Wittgenstein in *The Blue Book* calls a nongrammatical kind (Wittgenstein, 1968, p. 19). Frege and Wittgenstein differ, however, about which kinds are nongrammatical kinds—e.g., “number” is for Frege a nongrammatical kind; for (both early and later) Wittgenstein it is merely a grammatical kind.

32. In *The Foundations of Arithmetic*, Frege takes his second principle (or context principle) to apply to (what he, in this work, calls) either the *Inhalt* or the *Bedeutung* of an expression, failing to distinguish—as he did from 1891 on—between *Sinn* and *Bedeutung*. Some commentators have argued that once this distinction is drawn, the context principle breaks down, others that the context principle applies only to the *Sinn* of an expression. I disagree with these commentators. I take it that the context principle continues to figure in Frege's later work as a pair of principles: one for *Sinn* and one for *Bedeutung*. Once these principles are separately articulated, however, it is not clear what their relations of dependence are. One could argue that first comes (1) the context principle for *Sinn*, then (2) the principle that *Sinn* determines *Bedeutung* (“It is via a *Sinn*, and only via a *Sinn*, that a proper name is related to an object”; (Frege, 1979, p. 124), and that (1) and (2) together imply (3), a context principle for *Bedeutung*. (See, e.g., Reck, 1997, p. 142n for such a reading of Frege.) Alternatively, it is possible to argue that, at least with respect to the expressions of Frege's symbolic language in the *Grundgesetze*, the relation of dependence between the two principles is the reverse. (One could argue that in *Grundgesetze*, Frege first introduces (in vol. 1, §§10, 29, 31) what is in effect a context principle for *Bedeutung* and then (in §32) claims that the *Sinn* of every expression of his symbolic language is fixed by the stipulations specifying its *Bedeutung*.) But the *Grundgesetze* procedure for fixing *Sinn* requires that a *Bedeutung* for every expression of the language has already been secured. It might therefore appear to have no bearing on Frege's views concerning natural language, since there are passages in Frege which suggest that he holds that there are expressions of natural language that have *Sinn* but lack *Bedeutung*. Perhaps most telling, in this regard, are those passages in which Frege affirms the existence of sentences that lack *Bedeutung* but nonetheless express a thought (“without *Bedeutung*, we could indeed have thought, . . . without a *Sinn*, we could have no thought”; Frege, 1980, p. 80), and those in which he affirms, moreover, that “the thought remains the same” whether or not the proper name which such a sentence contains has *Bedeutung* (Frege, 1984, p. 163; see also Frege, 1979, p. 191). But, such passages notwithstanding, it can be argued that this is either not Frege's considered view or at least not the view that some of his own doctrines, resolutely thought through, commit him to. If a *Sinn* is the mode of presentation of a *Bedeutung*, how can there be a *Sinn* if there is no “it” presented? (Frege also sometimes suggests that the identity of a thought depends on its inferential relations, which in turn depend on its having *Bedeutung*; see, e.g., Frege, 1980, p. 70). Moreover, Frege characterizes a proper name lacking *Bedeutung* as a “mock proper name” and the Sätze that contain such mock proper names as expressing “mock thoughts” [*Scheingedanken*]” (see Frege, 1979, p. 130)—thus suggesting that we are only apparently able to grasp the *Sinne* of such Sätze. This appears to imply that (what we take to be) the constituents of such a Satz only make an apparent contribution to the determination of its *Sinn*. This opens the door for a Tractarian reading of Frege on *Scheingedanken*, parallel to the *Tractatus* on *Scheinsätze*: *Scheinsätze* are Sätze which have no *Sinn* but which occasion the illusion that one is able to grasp their *Sinn*. (For such a reading of Frege, see Evans, 1982, pp. 14–30.) I believe that Wittgenstein saw a profound tension in Frege's views here concerning the degree of independence that *Sinn* can attain from *Bedeutung*. (Evans's interpretation of Frege, followed up in McDowell, 1977, 1982, 1984, points in the direction in which the *Tractatus* itself goes on to resolve this tension.) The *Tractatus* resolves it in the direction of denying that “without *Bedeutung*, we could indeed have thought.” The *Tractatus* argues for a mutual interdependence of sentential *Sinn* and subsentential *Bedeutung*, holding (1) that a Satz only has *Sinn* if its expressions have *Bedeutung* (§§5.473, 5.4733, 6.53), and (2) that a subsentential expression only has *Bedeutung* in the context of a sinnvoller Satz (§3.3). (Though, since Wittgenstein accepts Russell's theory of descriptions, some of Frege's examples of names lacking *Bedeutung* would not count for the *Tractatus* as examples of terms that require *Bedeutung* in order for the *Satz* in which they occur to have *Sinn*.)

33. In conformity with this doctrine of the primacy of judgment, Frege's concept-script forbids the isolated occurrence of designations for the various possible components of a judgment:

Instead of putting a judgement together out of an individual as subject and an already previously formed concept as predicate, we do the opposite and arrive at a concept by splitting up the content of possible judgement. . . . But it doesn't follow from this that the ideas of these properties and relations are formed apart from their objects: on the contrary they arise simultaneously with the first judgement in which they are ascribed to things. Hence in the concept-script their designations never occur on their own, but always in combinations which express contents of possible judgement. . . . A sign for a property never appears without a thing to which it might belong being at least indicated, a designation of a relation never without indication of the things which might stand in it. (Frege, 1979, pp. 15–17)

34. Frege does, of course, speak of a thought's having "parts" out of which it is "built up" (see, e.g., Frege, 1979), and of how we can "distinguish parts in the thought corresponding to parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought" (Frege, 1984, p. 390). But Frege immediately follows this latter remark with the observation: "To be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the analogy is so ready to hand and so generally appropriate that we are hardly bothered by the hitches that occur from time to time" (Frege, 1984, p. 390). What kind of hitches? Hitches, for example, of the sort Kerry fails to notice when he imagines that he can get hold of a concept merely by employing an expression that elsewhere, in its usual employment, is able to symbolize a concept. Frege thus worries that the all but unavoidable (and in itself potentially innocent) locution of a thought's having "parts" or "components" will mislead one into attributing a false independence to the parts of a thought—so that we imagine that the parts could retain their identity apart from their participation in a whole of the appropriate structure: "But the words 'made up of,' 'consist of,' 'component,' 'part' may lead to our looking at it the wrong way. If we choose to speak of parts in this connection, all the same these parts are not mutually independent in the way that we are elsewhere used to find when we have parts of a whole" (Frege, 1984, p. 386). Frege's context principle—and the correlative doctrine of the primacy of judgment (which refuses to allow that the parts of the whole are "mutually independent in the way that we are elsewhere used to find when we have parts of a whole")—in thus insisting upon the *unity* of a thought or a proposition, in no way denies the compositionality of either thought or language. It insists only upon the mutual interdependence of compositionality and contextuality. (Diego Marconi [unpublished] nicely summarizes the position in the slogan "Understanding without contextuality is blind; understanding without compositionality is empty.") Frege's view of natural language—upon which the *Tractatus* builds its "understanding of the logic of language"—affirms both (1) that it is in virtue of their contributions to the senses of the whole that we identify the logical "parts" of propositions, and (2) that it is in virtue of an identification of each "part" as that which occurs in other propositional wholes that we segment the whole into its constituent parts (see note 37).

35. Gilbert Ryle attempted to summarize this "difficult but crucial point" of Frege's by saying that the meanings of words "are not proposition components but propositional differences": "Frege's difficult but crucial point . . . [is] that the unitary something that is *said* in a sentence or the unitary sense that it expresses is not an assemblage of detachable sense atoms, of, that is, parts enjoying separate existence and separate thinkability, and yet that one truth or falsehood may have discernible, countable, and classifiable similarities to and dissimilarities from other truths and falsehoods. Word meanings or concepts are not proposition components but propositional

differences. They are distinguishables, not detachables; abstractables, not extractables" (Ryle, 1971, p. 58). As this essay goes on, it will prove to be a matter of some interest that Gilbert Ryle—the man who made the notion of a category-mistake famous as a term of philosophical criticism—should have (at least occasionally) had such a firm grip on "Frege's difficult but crucial point."

36. It has been thought by some commentators that Frege's claim that objects—unlike concepts—are "self-subsistent" should be interpreted to mean that the context principle does not apply to object-expressions: that object-expressions mean—or name—objects prior to and apart from any contribution they make to the sense of (whole) propositions. Frege explicitly repudiates such an interpretation: "The self-subsistence which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes" (Frege, 1968, p. 72). For a thorough vindication of the claim that Frege's context principle is meant to apply to object-expressions, not just concept-expressions, see Reck, 1997.

37. How do we find this out? What determines the logical segmentation of a sentence, for Frege, are the inferential relations that obtain between the judgment the sentence expresses and other judgments. Identifying an expression as a logical unit and determining its logical role consequently turn on appreciating the inferential relations that obtain between the judgment in which the expression occurs and other judgments.

38. The sorts of "working parts of the proposition" at issue here cannot be identified as the sorts of parts they are simply through reference to a theory of the workings of (the grammar of) the natural language to which they belong. For Frege, we may not take the surface grammar of an ordinary-language sentence to be a strictly reliable guide to the logical structure of the proposition expressed by it. "Instead of following grammar blindly the logician ought to see his task as that of freeing us from the fetters of language" (Frege, 1979, p. 143). Frege is, of course, interested in formulating generally reliable guides to the logical structure of ordinary-language sentences (based on the surface structure of ordinary language)—guides such as "the singular definite article indicates an object," "the indefinite article indicates a concept," etc. But these remarks are intended to formulate rules of thumb, not sets of necessary and sufficient conditions. The surface-grammatical rules of thumb that Frege furnishes ("the singular definite article indicates an object," "the indefinite article indicates a concept," etc.) are intended to help his audience latch onto the logical distinctions he is after—they are themselves intended as elucidatory hints. Such rules of thumb are available for any language in which there is "a good accord between the linguistic distinction and the real one" (Frege, 1984, p. 185), but the linguistic distinction cannot serve as a substitute, and must not be mistaken, for the "real" logical distinction that Frege is after—not merely because of the differences between natural languages (different languages employ different linguistic markers for parallel kinds of logical structure; Japanese has no indefinite article, etc.), but because of the grammatical plasticity within any natural language (any natural language is sufficiently *unbegiffsschriftlich* to permit the same proposition to be expressed through distinct sorts of grammatical form and the same grammatical form to express propositions of distinct logical structure). As regards the rule "the indefinite article indicates a concept," Frege goes so far as to venture that "there are probably no exceptions" to it, but the efficacy of his elucidation of his use of the term 'concept' does not depend on the claim that his surface-grammatical rule for the German language has no exceptions (nor does it depend on the claim that there is some depth-grammatical rule of the natural language in question that has no exceptions); and he readily admits that in the case of the rule "the singular definite article indicates an object," indeed, "the matter is not so simple"—there are exceptions—but these do not impair "the value of the rule" (p. 185). The value of the rule lies in its capacity to lead us to grasp the real logical distinction in question. Once we have grasped this distinction, we will have the ability to discern cases

in which the surface grammar of ordinary language misleads us into treating sentences of like surface grammar as having like logical structure (thus helping, as Frege puts it in the preface to *Begriffsschrift*, "to break the domination of the word over the spirit.") Surface grammar is never the *sole* guide to the logical segmentation of a sentence of ordinary language for Frege. In a number of places, for example, Frege takes which inferences a given sentence licenses and which propositions it itself can be inferred from as a more reliable guide to logical structure than surface grammar. (This is evidenced, for example, by his treatment of the example of "The horse is a four-legged animal" (1984, p. 185).)

39. The only way to refer to a concept, for Frege, is to *use* a concept expression: i.e., to employ it predicatively within the context of a judgment. Thus his argument against Kerry can be rephrased as a substitutional argument. Two expressions mean the same thing (have the same *Bedeutung*) only if the new expression can be substituted for the original expression without changing the truth-value of any judgment in which the original expression occurred. Whenever we attempt, however, to substitute an object-expression (such as 'the concept horse') for a concept-expression (such as '— is a horse'), not only do we not get a new sentence with the same truth-value, we get nonsense. See Weiner, 1990, pp. 251ff., for an excellent discussion of this point.

40. The sign that such a Fregean elucidation has been successful—that the desired "meeting of minds" between the elucidator and his audience has been achieved—is that the other person is able to go on as a user of *Begriffsschrift* on his or her own in the right way. Frege therefore has an answer to an obvious objection (voiced by some commentators on the *Tractatus*) to the doctrine that there are fundamental logical distinctions that underlie but cannot be expressed in language. The objection goes as follows: there is no way to adjudicate the success of an attempt to communicate such distinctions—for there is no way for someone who has grasped such a distinction to exhibit his mastery of the distinction. But Frege furnishes a touchstone of success: the sign that we have grasped his elucidations is that we emerge masters of his symbolism. A reader can be said to have grasped one of Frege's elucidations (e.g., his elucidation of the distinction between concept and object) if he is able to employ the appropriate elements of the symbolism (the symbol for an object only if an object is denoted, etc.) when segmenting judgments and translating them from ordinary language into *Begriffsschrift*. His segmentation of the judgment can, in turn, be checked by making sure that the translation of the judgment into *Begriffsschrift* preserves the appropriate inference and substitution licenses between the judgment in question and other judgments.

41. This and related aspects of Frege's conception of elucidation are discussed in illuminating detail in the final chapter of Weiner, 1990.

42. Geach is one of the few commentators who sees a connection between this moment in Frege's work and the concerns of both the *Tractatus* and Wittgenstein's later work:

One thing I learned from Wittgenstein, in part from the *Tractatus* but still more from personal contact, is that philosophical mistakes are often not refutable falsehoods but confusions; similarly the contrary insights cannot be conveyed in proper propositions with a truth-value. I offer as [an] instance . . . of such [an] insight . . . Frege's distinction between concept and object ("No concept is an object" has no translation into a well-constructed symbolism). . . . Such insights cannot be demonstrated as theses, but only conveyed dialectically; the dialectic process largely consists in the art, whose practice I have perhaps learned in some measure from Wittgenstein, of reducing to patent nonsense the buried nonsense that is found in attempts to reject these insights. We cannot refute nonsense by a straightforward process; as Frege said, logic cannot deal with nonsense, but only characterize it as being nonsense.

Having come to Frege by way of the *Tractatus*, I could see that his difficulties in expressing himself about function, concept, and object were not from a muddled self-bemusement but from the nature of the case. (Geach, 1991, pp. 13–14, 16)

It is remarkable, with a few notable exceptions (Anscombe, Diamond, Geach, Ricketts, and Weiner) how little of the secondary literature on the *Tractatus* has interested itself in this moment in Frege's thought. The contrast Max Black draws in the following passage between Frege (who has no use for the idea that nonsense can be illuminating) and Wittgenstein (who does have a use for the idea) is typical of the sort of contrast between Frege and Wittgenstein one finds throughout the secondary literature on the *Tractatus*:

Is the *Tractatus* self-defeating? Nothing in the book has aroused more interest or provoked more scandal than its concluding remarks. . . . The very words in which our predecessors' errors were castigated have to be acknowledged as nonsensical. . . . With what relish Frege would have assaulted this position. One can imagine him smacking his lips over the deliciously absurd notion that "nonsense" can be understood. "If we understand the conclusion," he might well have said, "then it *cannot* be nonsensical. Since it implies its own lack of sense it must at best be false. For, if it were true, it would have to be nonsensical, and hence without truth-value, which is a contradiction. So, the supposed conclusion is at best *necessarily* false. But all this is unnecessary—we can't begin to take seriously a statement that claims to imply its own absurdity." (Black, 1982, pp. 378–79)

This stock contrast of Frege and Wittgenstein renders the topic of this essay invisible: namely, how Wittgenstein's notion of elucidation inherits, reshapes, and ultimately repudiates Frege's notion of elucidation.

43. Henceforth, whenever it is employed in connection with Frege, the term 'elucidation' will be used only to refer to the species of elucidation at issue in "On Concept and Object." (In their original context, some of the passages from Frege's work I cite below are concerned in the first instance with the broader genus, but pertain a fortiori to the species, and are adduced below solely to illuminate the nature of the species.)

44. Contrary to what some commentators have claimed, this is an oft-repeated refrain in Frege's work. Here are some representative passages:

The question arises what it is that we are here calling an object. *I regard a regular definition as impossible*, since we have here something too simple to admit of logical analysis. (Frege, 1984, p. 147)

It is not possible to give a definition of what a function is, because we have here to do with something simple and unanalysable. (Frege, 1979, p. 235, emphasis in the orginal)

If . . . the meaning to be assigned is logically simple, then one cannot give a proper definition. (Frege, 1980, p. 37)

To this difference in the signs there of course corresponds an analogous one in the realm of meanings: to the proper name there corresponds the object; to the predicative part, something I call a concept. *This is not supposed to be a definition*; for the decomposition into a saturated and an unsaturated part must be considered to be a logically primitive phenomenon which must simply be accepted and cannot be reduced to something simpler. (Frege, 1984, pp. 281–82, my emphasis)

The peculiarity of functional signs, which we here called "unsaturatedness," naturally has something answering to it in the functions themselves. They too may be called "unsaturated" . . . Of course *this is no*

definition; but likewise none is here possible. (Frege, 1984, p. 292, my emphasis)

It is indeed not the least of the logician's tasks to indicate the pitfalls laid by language in the way of the thinker. After refuting errors, it may be useful to trace the sources from which they have flowed. One source, I think, in this case is the desire to give definitions of the concepts one means to employ. It is certainly praiseworthy to try to make clear to oneself as far as possible the sense one associates with a word. But here we must not forget that *not everything can be defined*. If we insist at any price on defining what is essentially indefinable, we readily fasten upon inessential accessories, and thus start the inquiry on a wrong track at the very outset. (Frege, 1984, p. 381, my emphasis)

This difference between the signs must correspond to a difference in the realm of meanings The analysis of the proposition corresponds to an analysis of the thought, and this in turn to something in the realm of meanings, and I should like to call this a primitive logical fact. This is precisely why *no proper definition is possible here*. (Frege, 1980, pp. 141–42, my emphasis)

45. The word “categories” will not really do here. But, as we shall see, there is, according to Frege, no word that will do. I shall continue, throughout the rest of this essay, to finesse this problem by pretending that talk of “logical categories” is able to possess greater referential powers than Frege thinks it can.

46. Here, again, are some representative passages:

The question arises what it is that we are here calling an object. . . . It is only possible to gesture towards [*hinzudeuten*] what is meant. (Frege, 1984, p. 147)

If . . . the meaning to be assigned is logically simple, then one . . . must confine oneself to warding off the unwanted meanings among those that occur in linguistic usage and to pointing to the wanted one, and here one must always rely on being met half-way by an intelligent guess. (Frege, 1980, p. 37)

It is not possible to give a definition of what a function is. . . . It is only possible to hint at what is meant and to make it clearer by relating it to what is known. Instead of a definition we must provide elucidations [*Erläuterungen*]; here of course we must count on a meeting of minds. (Frege, 1979, p. 235, emphasis in the original)

I must confine myself [in attempting to explain the “unsaturatedness” of functional signs] to hinting at what I have in mind by means of a metaphorical expression, and here I must rely on my reader's meeting me half-way. (Frege, 1984, p. 292)

47. “I am well aware that expressions like ‘saturated’ and ‘unsaturated’ are metaphorical and only serve to indicate what is meant—whereby one must always count on the co-operative understanding of the reader” (Frege, 1984, pp. 281–82).

48. It is only this species of elucidation that Frege thinks compels us to traffic in nonsense. Within the broader genus of elucidation, elucidations will generally take the form of perfectly meaningful propositions (such as, e.g., elucidations of geometrically primitive terms). It is worth noting, however, that a parallel distinction between a generic and a specific notion of elucidation must also be drawn if one seeks to understand the different occurrences of the term *Erläuterung* in the *Tractatus*. In §3.263 what is at issue is the species of the genus pertaining to the elucidation of primitive signs

(which I will not explore further here, other than to remark that perfectly meaningful propositions can serve as elucidations of this sort), whereas an understanding of §6.54 is unattainable apart from an understanding of what is peculiar to that species of the genus that aims to elucidate “philosophic matters” (and which proceeds through the employment of Sätze that the reader is to recognize as *Unsinn*).

49. Closer inspection of this passage and its context reveals that on *CP*, p. 300, Frege speaks of elucidation in connection with the Bedeutung of a term, and on *CP*, p. 301, in connection with its Sinn. I agree with Weiner (see 1990 p. 235 n) that, as this passage suggests, Frege wants elucidations to secure both the Sinn and the Bedeutung of a term.

50. Here are two representative passages:

We must admit logically primitive elements that are indefinable. Even here there seems to be a need to make sure that we designate the same thing by the same sign (word). . . . Since definitions are not possible for primitive elements, something else must enter in. I call it elucidation [*Erläuterung*]. It is this, therefore, that serves the purpose of mutual understanding among investigators. . . . We may relegate it to a propaedeutic. It has no place in the system of a science; in the latter, no conclusions are based on it. (Frege, 1984, pp. 300–301, translation emended)

I should like to divide up the totality of mathematical propositions into definitions and all the remaining propositions (axioms, fundamental laws, theorems). . . . One can also recognize a third kind of proposition, elucidatory propositions, but I would not want to count them as part of mathematics itself but refer them to the antechamber, the propaedeutics. (Frege, 1980, p. 37)

51. “We cannot come to an understanding with one another apart from language, and so in the end we must always rely on other people’s understanding words, inflections, and sentence-construction in essentially the same way as ourselves. As I said before, I was not trying to give a definition, but only hints; and to this end I appealed to the general feeling for the German language” (Frege, 1984, pp. 184–85).

52. The final hedge here—first he says “impossible” and then “senseless”—occurs frequently in Frege’s discussion of this topic and can be taken to be indicative of a profound ambivalence on his part. The ambivalence is tied to the tension the *Tractatus* discerns in Frege’s thought: a tension between (a) wanting to say that there are inexpressible *thoughts* which certain forms of words attempt to express, and (b) wanting to say that the distinction between what can and what cannot be rendered in *Begriffsschrift* provides a precise logical demarcation of what is and what is not a thought (and hence that there is *no* thought expressed by forms of words which cannot be so rendered). The differences in various interpretations of Frege can be traced in part to how this hedge is resolved—whether the accent is placed on “impossible” or on “senseless” (and if the former, then that accordingly determines how the notion of “senselessness” in play here ends up being construed)—which, in turn, determines whether one takes Frege to be committed (as Geach, Weiner, and Dummett, in their very different ways, do) to a substantial conception or (as Diamond does) to an austere conception of nonsense.

53. In order to avoid a possible confusion, I should remark that although I have followed the practice of translating Frege’s term *sinnlos* as “senseless,” I think it could equally well be rendered as “nonsense.” In similar contexts, Frege sometimes employs the term *unsinnig* instead in order to make the same sort of point. Unlike Wittgenstein in the *Tractatus*, in his alternating between these terms, Frege does not have any systematic distinction in view.

54. Throughout Frege’s corpus we find numerous remarks that appear to be defective in just the way he takes Kerry’s remark to be—remarks in which Frege employs

expressions of the form ‘the concept *X*’ and in which he wants to put forward this or that claim either about the nature of concepts *überhaupt* or about some particular concept (most famously, for example, the concept of number). Frege seems to be committed to the claim that these remarks (in which expressions of the form ‘the concept *X*’ figure) have the status of elucidations. I only mean here to be pointing out an apparent consequence of Frege’s doctrines. (I am not committed to defending the claim that Frege himself faced up to this implication of his views.) See Weiner, 1990, chap. 6, for a spirited defense of the claim that Frege’s doctrines do indeed have this consequence (though Weiner herself is careful to insist that the views that she thus attributes to Frege—on the grounds that they are the only views that she thinks can make sense of the relevant portions of Frege’s writings—may well be views that “Frege-the-historical-person” would have disavowed).

55. There is that hedge again: “it requires something contradictory, . . . or perhaps better still, . . . something nonsensical.”

56. Frege actually goes so far as to argue that the terms ‘function’ and ‘concept’ even when they occur predicatively are defective (because they function in ordinary language as names of first-level functions rather than themselves ranging over first-level functions) and thus “should properly speaking be rejected”: “The words ‘function’ and ‘concept’ should properly speaking be rejected. Logically, they should be names of second-level functions; but they present themselves linguistically as names of first-level functions. It is therefore not surprising that we run into difficulties in using them” (Frege, 1980, pp. 141–42).

57. The claim that “concepts cannot stand in the same relations as objects” might strike one as false. What about the relation of identity, cannot both concepts and objects stand in that relation? Frege thinks not: “The relation of equality between objects cannot be conceived as holding between concepts too, but . . . there is a corresponding relation for concepts. It follows that the word ‘the same’ that is used to designate the former relation between objects cannot properly be used to designate the latter relation as well. If we try to use it to do this, the only recourse we really have is to say ‘the concept ϕ is the same as the concept Ψ ’ and in saying this we have of course named a relation between objects, where what is intended is a relation between concepts” (Frege, 1984, pp. 121–22). If by “the relation of identity” we mean a relation in which objects can stand to one another, then it is not a relation in which concepts can stand to one another. We can of course say that this object is “the same” as that one; and we can also say that this concept is “the same” as that one. But Frege’s thinks there is no univocal notion of “sameness” here. We are misled by the fact that in ordinary language we use the same sign to express two logically distinct kinds of relation into thinking that there is some overarching mode of relation into which both concepts and objects can enter. The difference between these two cases is rendered manifest in a proper *Begriffsschrift*: a different arrangement of signs expresses each of these distinct kinds of logical relation—in modern logical notation: $x = y$ and $(\forall x)(Fx \leftrightarrow Gx)$. There is no way in a proper *Begriffsschrift* to express the (by Frege’s lights, philosophically confused) thought that these two logically distinct kinds of relation are both species of a single genus (say, the genus *ways of being the same*). An attempt to express such a (pseudo-)thought in a proper *Begriffsschrift* can help to make manifest the confusion that (its apparent expressibility in) ordinary-language disguises. This is a nice example of the feature of a proper *Begriffsschrift* that interests early Wittgenstein most: its potential as a tool for making latent nonsense patent.

58. The *Tractatus* will seek to press the question: To what extent is Frege, by his own lights, entitled to look upon that which his words here intend (but fail) to express as a *thought*?

59. The conclusion of this passage—the idea that what matters in such cases (where we are forced to make use of such inappropriate forms of expression) is that “we know we are doing it and how it happens”—parallels the conclusion of “On Concept and

Object": "Over the question of what it is that is called a function in Analysis, we come up against the same obstacle; and on thorough investigation it will be found that the obstacle is essential, and founded on the nature of language; that we cannot avoid a certain inappropriateness of linguistic expression; and that there is nothing for it but to realize this and always take it into account" (Frege, 1984, p. 194).

60. "Something" is, again, a weasel word here. *Tractatus*, §§4.126–4.1272 rework the same sort of example which figures in Frege's correspondence with Russell:

We can speak of formal concepts. . . . I introduce the expression in order to make clear the confusion of formal concepts with proper concepts. . . .

That anything falls under a formal concept as an object belonging to it, cannot be expressed by a proposition. But it is shown in the symbol for the object itself. (The name shows that it signifies an object, the numerical sign that it signifies a number, etc.) (§4.126)

So the variable name "x" is the proper sign of the pseudo-concept *object*.

Wherever the word "object" ("thing," "entity," etc.) is rightly used, it is expressed in logical symbolism by the variable name.

For example in the proposition "there are two objects which. . . , " by "(\exists x,y). . . .".

Wherever it is used otherwise, i.e., as a proper concept word, there arise nonsensical pseudo-propositions.

So one cannot, e.g. say "There are objects" as one says "There are books."

The same holds of the words "Complex," "Fact," "Function," "Number," etc.

They all signify formal concepts and are presented in logical symbolism by variables. (§4.1272)

Wittgenstein's way of putting the point in this passage (about 'X is an object') appears at first blush to parallel Frege's discussion of 'X is a function': what an object is can only be "shown in the symbol for the object itself," and if we try to say what an object is by employing the word 'object' as a proper concept word, then "there arise nonsensical pseudo-propositions." Thus Peter Hacker, for example, summarizes the point of the passage in a way that parallels Geach's reading of Frege: "An attempt to describe the essence of things will unavoidably violate the bounds of sense . . . and produce nonsense. Thus, for example, that A is or is not an *object* cannot be said because 'object' is a formal concept" (Hacker, 1986, p. 21). On Hacker's view, "A is an object" is nonsense, but we know *what* it is trying to say and we know that "it" cannot be said.

There is this much of a disanalogy between Geach's reading of Frege and Hacker's reading of Wittgenstein: Hacker talks as if the invocation of "formal concepts" allowed for the introduction of a device for *saying* why the sentences in question are nonsense (as opposed to one that simply enables the production of more nonsense). On Geach's reading, to grasp the teaching of the *Tractatus* as a whole is to grasp why a passage such as §4.126 is nonsense. (Geach's view is nicely summarized by Anscombe's remark in her book on the *Tractatus*: "Sentences . . . cannot represent, and nothing in them can stand for, 'the logic of the facts': they can only reproduce it. An attempt to say what they so reproduce leads to stammering"; Anscombe, 1971, p. 164.) On Hacker's reading, on the other hand, §4.126 seems to succeed in *saying* why certain subsequent passages in the book are nonsense by specifying the "it" which cannot be spoken about. But this (by Geach's lights) is to miss the point of §4.1272. If it were possible thus to refer to that which allegedly cannot be spoken about, then there would be no problem about putting "it" into words. Hacker's reading threatens to leave Wittgenstein in the

position of a fool—one who first says that there are these things that cannot be spoken about and who then proceeds to *tell* what they are.

61. How one reads this pivotal passage of “On Concept and Object” will, of course, be a function of how one reads the rest of the essay and the rest of Frege. I am offering here a reading of the passage—in line with my overall exposition of Frege on elucidation—in the spirit of Geach (on the hypothesis that such a reading parallels Wittgenstein’s own reading of Frege on these matters). Such a reading of the passage, however, would appear to presuppose a (very un-Wittgensteinian, and in certain ways un-Fregean) separation of the conditions of thought from the conditions of language. That is, it would appear to presuppose a conception of thought according to which a thought can figure as a target one is able to keep in one’s sights without the aid of any linguistic medium—a target that one can hit or miss, and which one can tell one is hitting or missing, without reliance on language. More drastically still, it looks as if a thought can figure as the target of one’s thinking, and that there is something that can count as one’s having taken aim at *that* thought, though there is nothing in one’s speaking that can ever count as one’s having hit the target. On such a reading of the passage, Frege—when he says “by a kind of *necessity* of language, my expressions, taken literally, . . . *miss* my thought; I mention an object when what I *intend* is a concept”—takes himself to be able to take *aim* at (to “intend”) the thought, even though his words *necessarily* “miss” the target. What Frege says here (that his words miss his thought if “taken literally”) suggests that it is by taking his words nonliterally that we are to be *guided* to his thought; but it would seem that no such (“nonliteral”) way of taking his words can ever, by his own lights, be taken to count as an *expression* of his thought (as a way of taking the words which hits the target). Rather his words, which taken literally miss his thought, are to *evoke* in us the thought which they are necessarily unable to express. We are, in our thinking, asked to hit the target which his ways of speaking necessarily miss. Thus Frege, in this passage, provides a paradigmatic example of someone trying “to use language to get outside language” (Wittgenstein, 1975 §6).

62. I hear a reader grumbling: “What about Russell? Isn’t the *Tractatus* as much a response to Russell as to Frege?” Of course. But the relation to Frege is more instructive for seeing how the *Tractatus* is (and especially for seeing how it is not) to be read—for seeing, that is, what the method of the *Tractatus* is. I have thus confined myself in this essay to showing how the problematic of the *Tractatus* can be seen to parallel the problematic of elucidation in Frege’s work. I take the (for Wittgenstein) generous acknowledgment to Frege in the preface to the *Tractatus* to suggest—and to invite the reader to look for—some parallel of this general sort. That acknowledgment leaves no doubt that Wittgenstein thought (1) that his work was indebted to the works of Frege, and (2) that he thought those works were “great”; and it would seem to imply (3) that what is of most value in the *Tractatus* is especially indebted to what is comparatively “great” about “the great works of Frege.” But even if one grants (1), (2), and (3), this still leaves wide open the question to what extent the presence of such a parallel is *directly a product of Frege’s influence* on the *Tractatus* and to what extent it is the product of Wittgenstein’s *having worked through the stages of a dialectic* common to both Frege and Russell to the point where Wittgenstein’s path through these issues converges for a stretch with Frege’s. (For further discussion of Wittgenstein’s respective debts to Frege and Russell, see Conant 2002a.)

63. And, surely, it is right to think a viable reading of §6.54 requires such a distinction. The question is: How is it to be drawn?

64. That their account of Unsinn should be thus distributed over these two variants is, as we shall see, unsurprising. It is not uncommon, however, for commentators to hover between the variants even within their characterizations of misleading and illuminating nonsense, respectively. Consequently, my working criterion for what qualifies a commentator on the *Tractatus* as a proponent of the ineffability interpretation is merely that the commentator at least so hover—i.e., that he at least intermit-

tently evince a hospitable attitude toward the ineffability variant of the substantial conception (and not that he elaborate it in a consistent manner).

65. Few proponents of the ineffability interpretation succeed in resolutely adhering to such an account of misleading nonsense. They tend to stray into maintaining that when the metaphysician comes out with misleading nonsense he is not merely mouthing (linguistically defective) words, but also giving voice to a kind of (logically flawed) thought. But as soon as such a description of misleading nonsense is allowed, a problem arises for such readings of the *Tractatus*: How are we in the end to distinguish between misleading and illuminating nonsense? The usual answer (when the problem is faced) is to stick to the ineffability variant for both kinds of nonsense and to say that there is no difference (in the kinds of things that are respectively thought or said by a speaker of misleading nonsense and by a speaker of illuminating nonsense). The difference lies merely in the speaker's self-conscious appreciation of the logical character of that which he is trying to say: the speaker of illuminating nonsense is fully aware that that which he is thinking is unsayable, the speaker of misleading nonsense (while thinking the same thought and coming out with the same words) is not thus aware. The problem with this answer is that it reduces Wittgenstein's objection to philosophical nonsense merely to an objection to the spirit in which one comes out with certain "philosophical" utterances. There is no longer anything wrong with the metaphysician *thinking* a nonsensical metaphysical thought—just with his believing that such a thought is (correctly classified as belonging to the species of thought which is) expressible.

66. This will not deter Hacker and many other commentators from saying that they agree that, for the *Tractatus*, "the limits of language are the limits of thought." They may attempt to remove the apparent contradiction by explaining that what is thus *meant* or *intended* by nonsense is not, strictly speaking, a "thought"—and thus is not, strictly speaking, "meant" or "intended" either. On the use, on the part of commentators, of such devices for begging the question, see Conant, 1991a, pp. 154–55.

67. Some proponents of the ineffability interpretation advance readings of the *Tractatus* according to which it would appear that there is really very little difficulty, at least if you are a commentator on the *Tractatus*, in just coming out and *saying* what it is that Tractarian nonsense seeks to "show"; they thus implicitly answer this question (of how we are to discern the presence of meaning in the absence of meaning) with the straightforward proposal that we understand Tractarian (so-called) nonsense simply in virtue of understanding what it says! John Koethe is that rare example of a commentator who is willing to be explicit about this implication of such an exegetical procedure: "Showing, in my view, . . . is a kind of second-rate saying" (Koethe, 1996, p. 39). If such an admission remains out in the open, then, of course, it no longer remains appropriate to characterize the commentator who makes it as a proponent of an *ineffability* interpretation.

68. Thus Hacker: "Categorial necessities are reflected in the formation-rules of language. Any attempt to express them involves . . . the violation of rules of logical syntax" (Hacker, 1986, p. 106). The idea that the *Tractatus* seeks to specify "violations of logical syntax" (either effable or ineffable) will be taken up later when we look more closely at what is meant by "logical syntax" in the *Tractatus*.

69. I use this term here in accordance with the definition provided in note 4. Hacker himself does not speak of "showing" in this connection, but rather of (the illuminatingly nonsensical sentences of the *Tractatus*) "hinting" or "gesturing" at ineffable truths. (Hacker, quite properly, reserves "showing" as a translation for the Tractarian notion of *zeigen* introduced in §4.022 and §§4.121–4.126.)

70. More precisely: the parallel is with Frege's conception of what is involved in the elucidation of the meaning of a term that denotes something logically primitive.

71. Dummett himself never, in his discussion of Frege on nonsense, makes an explicit connection between the conception of nonsense he ascribes to Frege and the

doctrine that there are things that can be “shown” but not said. But, Dummett’s remarks elsewhere (in particular, his responses to related aspects of Geach’s work on Frege, his vehement attribution to Frege of the thesis of the priority of thought over language, and his occasional asides about the “self-refuting” character of “the Tractarian doctrine” that there are inexpressible thoughts (see, e.g., Dummett, 1992) leave little doubt that he would not favor the attribution of an ineffability variant of the substantial conception to Frege.

72. The ensuing discussion of this example is indebted to Diamond, 1991, chap. 2.

73. As, e.g., in §51 of *The Foundations of Arithmetic*: “With a concept the question is always whether anything, and if so what, falls under it. With a proper name such questions make no sense. We should not be deceived by the fact that language makes use of proper names, for instance Moon, as concept words, and vice versa; this does not affect the distinction between the two” (Frege, 1968, p. 64).

74. This is not to say that, in general, any proposal that yields a possible segmentation of a string is equally tenable. In real-life cases of interpretation, we are obliged, on the one hand, to make sense of the way a sentence occurs within a larger stretch of discourse. (“Understanding without contextuality is blind.”) To commit oneself to a segmentation of the string, on the other hand, is to commit oneself to patterns of inference (see note 88) that are a function of how these words (of which the string is composed) occur in other propositions. (“Understanding without compositionality is empty.”) The attribution of the endorsement of inferences of certain patterns to a speaker is governed by those considerations of charity and relevance that govern all aspects of interpretation. These considerations generally uniquely determine a segmentation (and, where not, they at least severely constrain the range of reasonable proposals).

75. For purposes of simplifying the exposition, I have restricted my definition to (what the *Tractatus* calls) “written signs”—the *Tractatus* explicitly allows for “sound signs” (see §3.321) and implicitly for other sorts.

76. My self-defeating exposition of the alleged distinction between the two variants of the substantial conception mirrors, albeit in a highly summary fashion, the first half of the elucidatory strategy of the *Tractatus*. Half of the central point of the *Tractatus*, on my reading, is to show that once one has bought into the substantial conception, one has implicitly committed oneself to a conception, on which there are ineffable thoughts—thoughts which we can gesture at (with the aid of nonsensical language) but cannot express in language. (A central part of the interest of Frege’s work for Wittgenstein, as he read him, is that Frege recognized and drew this consequence.) The second half of the point of the work is to show that the way to escape this consequence is to abandon the substantial conception of nonsense altogether (not, according to Wittgenstein, an easy thing to do). As will become clear, my exposition of the alleged distinction between the substantial and austere conceptions of nonsense aims to mirror, in equally summary fashion, this second (and largely unnoticed) half of the elucidatory strategy of the *Tractatus*.

77. I say “reformulation of Frege’s second principle” (rather than restatement of it) because the *Tractatus* is concerned to refashion Frege’s distinction between *Sinn* and *Bedeutung* (for reasons touched on in note 32). §3.3 is worded as it is precisely to mark a departure from Frege in this regard. Just what sort of departure from Frege is here being marked, however, is far less clear (at least to me). At the level of the proposition, the departure from Frege is presumably tied to a further consideration (which Wittgenstein thought Frege failed to appreciate): the essential bipolarity of the proposition. In the “Notes on Logic,” Wittgenstein equates a proposition’s being “essentially true-false” with “the *sense* of a proposition” (which, in turn, is apparently meant to contrast with “the *meaning* of a proposition” which he equates with “the fact that actually corresponds to it”; Wittgenstein, 1979a, p. 94). In Friedrich Waismann’s *Thesen* (which is an attempt to furnish the members of the Vienna Circle with an overview of the main ideas of the *Tractatus*, based on detailed conversations with Wittgenstein), we find the following: “A proposition has *Sinn*, a word has *Bedeutung*”

(Waismann, 1979, p. 237). Should this be taken to mean that words do not have Sinn or that propositions do not have Bedeutung? Enigmatic as this remark may seem, it is straightforward compared with anything to be found anywhere in the *Tractatus* itself on the subject. Section 3.3 (along with §3.144) does appear to seek to exclude the applicability of Sinn to any kind of symbol other than a Satz. When read in the light of §3.3, a number of earlier passages (§§3.142, 3.144, 3.203, 3.22) also appear to be worded in a manner suggesting that the overall doctrine of the work indeed is that (at least) *names*—i.e., the constituent parts of a fully analyzed sentence—do not have Sinn. The corresponding principle in regard to Bedeutung does not obviously hold, however: the application of Bedeutung in the *Tractatus* does not appear to be restricted (as the passage from Waismann's *Thesen* might seem to imply) to the subjudgmental components of propositions. Throughout the *Tractatus*, the term *Bedeutung* is employed in a (relatively nontechnical) manner so as to suggest that any sign (including a Satz, i.e., a propositional sign) with a determinate linguistic function can be said to have a Bedeutung (see, e.g., §5.451 for the claim that the negation sign has a Bedeutung), and, as such, is to be contrasted only with a sign that has no Bedeutung or (as the *Tractatus* prefers to say) to which no Bedeutung has been given (see, e.g., §§5.4733, 6.53). What *Tractatus* §3.3 is concerned to withhold endorsement from is—not the bare idea that Sätze can be said to have Bedeutungen, but rather—“Frege's theory of the Bedeutung of Sätze and Funktionen” (§5.02), i.e., Frege's assimilation of sentences and functions to the category of proper names (and especially his doctrine that the *truth-value* of a sentence is its Bedeutung). For useful discussion touching on this extraordinarily obscure region of the *Tractatus*, see Diamond, forthcoming; and Hylton, 1997.

78. A number of commentators have attributed to the *Tractatus* the view that a special mental act (of intending to mean a particular object by a particular word) is what endows a name with meaning (see, e.g., Hacker, 1986, pp. 73–80; Black, 1982, pp. 114–22; Malcolm, 1986, pp. 63–82). If textual support for this attribution is adduced at all, it is usually through appeal to texts outside of the *Tractatus*—usually passages from the *Notebooks* (Wittgenstein, 1979a; e.g., pp. 33–4, 99, 129–30), or corresponding passages from Wittgenstein's correspondence with Russell. (When a passage from the *Tractatus* is adduced, it usually requires (i) a contrived song and dance to explain how it is supposed to support the attribution, and (ii) reliance on the assumption that the *Tractatus* is in no way concerned to criticize or distance itself from the doctrines of the *Notebooks*. For discussion of (i), see Winch, 1987; for discussion of (ii), see Kremer, 1997. According to commentators who argue for the attribution, the *Tractatus* holds that the connection between a name and its meaning can only be fixed by a mental act that confers upon an initially semantically impotent sign the power to signify the object “one has in mind.” To think that one can fix the meanings of names by means of such an act just is to think that one can fix their meanings prior to and independently of their use in propositions; and it is just this psychologistic conception of meaning that Frege's and early Wittgenstein's respective versions of the context principle are concerned to repudiate.

79. Although the notion of *Satz* that figures in the context principle (only the Satz has sense; only in the context of a Satz has a name meaning) is of a certain kind of a symbol, the term ‘*Satz*’ in the *Tractatus* floats between meaning (1) a propositional symbol (as, e.g., in §§3.3ff. and §§4ff.) and (2) a propositional sign (as, e.g., in §§5.473 and §6.54). It is important to the method of the *Tractatus* that the recognition that certain apparent cases of (1) are merely cases of (2) be a recognition that the reader achieve on his own. Consequently, at certain junctures, the method of the *Tractatus* requires that the reference of ‘*Satz*’ remain provisionally neutral as between (1) and (2). At the corresponding junctures in my own discussion, I leave ‘*Satz*’ untranslated.

80. Wittgenstein's distinction between *propositional sign* and *propositional symbol* parallels the distinction between *string of words* and *proposition* that Geach draws in the following passage: “Recognizing repeated occurrences of the same proposition is not merely mechanical; the identity of a proposition is not the identity of a string of

words. The proposition ‘Socrates was bald’ occurs over again in ‘Socrates, who taught Plato, was bald,’ but does not occur in ‘A philosopher whose teacher was Socrates was bald’” (Geach, 1979; p. 221–22). A version of this distinction (between sign [*Zeichen*] and symbol [*Symbol*]) is implicit in Frege’s work; for example, in his “Introduction to Logic,” Frege writes: “The same thought cannot be true at one time, false at another The reason . . . [people] believe the thought to be the same is that in such cases what is the same is the form of words; the form of words [which is said to both true and false] will then be a counterfeit (non-genuine) proposition [*wird dann ein uneigentlicher Satz sein*]. We do not always adequately distinguish the sign [*Zeichen*] from what it expresses” (Frege, 1979, p. 186, translation emended). Wittgenstein’s notion of an expression or symbol (that which is common to a set of propositions)—as opposed to a sign (that which is common to, what Frege here calls, forms of words)—builds on Frege’s idea that what determines the logical segmentation of a sentence are the inferential relations that obtain between the judgment that the sentence expresses and other judgments. *Language (Sprache)* is Wittgenstein’s term for the totality of such propositional symbols; and *logical space* is his term for the resulting overall network of inferential relations within which each of these propositional symbols has its life. Sections 4–4.001 build on the notion of *Satz* qua *propositional symbol* developed in §§3.31ff. (“The thought is the *sinnvolle Satz*. The totality of *Sätze* is the language.”) *Language (Sprache)* in the *Tractatus* refers to the totality of possible propositional symbols. One might think of this as Wittgenstein’s attempting to follow Frege’s example (in his exchange with Kerry about concepts) by “keeping to the strictly logical use” of the word ‘language’. It is trivially true, if one employs this idiom, that *there is only one language*—though there are, of course, countless alternative systems of signs that may differ widely from one another in their respective expressive powers (and thus in how much and which aspects of *die Sprache* they are each able to express).

81. The ensuing exposition of this example only really works if we assume all the letters of the sentence to be capitalized so that we have no orthographic clues as to when the expression ‘GREEN’ is being used as the proper name of a person and when as a concept-expression.

82. The sequence of (a), (b), and (c) nicely brings out a further asymmetry between sign and symbol. In the rendition of (b) into logical notation, we might think of the sign ‘=’ as corresponding to the sign ‘is’ in the ordinary-language version of (b); i.e., we might think of these two signs (‘=’, ‘is’) as symbolizing the same relation (the relation of identity). But in the rendition of (a) into logical notation, there is no candidate for a sign that corresponds to ‘is’—there is here nothing which is *the* sign which symbolizes the copula. The *Tractatus* draws five morals from this: (M1) a method of symbolizing is not simply a matter of a sign *naming* an item of a particular logical category; (M2) a symbol is expressed not simply through a sign but through *a mode of arrangement* of signs; (M3) not every logically significant aspect of a mode of arrangement of signs corresponds to an argument place (into which a different sign can be substituted); (M4) it is not the case that each method of symbolizing requires the employment of a distinct sign to express the method of symbolizing (a method of symbolizing can be expressed through a mode of arrangement of signs, such as the method of symbolizing the copula in modern logical notation); (M5) for certain methods of symbolizing the employment of a distinct sign is required.

(M4) is of great importance. The *Tractatus* distinguishes between kinds of symbol by distinguishing degrees of “dispensability” of signs for different kinds of symbol. The degree of the “dispensability” of a sign depends on how easy it is to express the symbolic function of the sign while making the sign itself (as the *Tractatus* puts it) “disappear.” (My appreciation of the importance of this point for the *Tractatus* is indebted to discussion with Michael Kremer.) (M4) sets up two further doctrines that play a central role in the *Tractatus*: (i) that any sign which symbolizes a *relation* can in principle be dispensed with and expressed instead through a mode of arrangement

of signs (§§3.1431–3.1432); (ii) that this shows us something about such symbols: they are not (in the Tractarian sense) *names* (§§3.1432–3.22).

In ascribing these two doctrines to the *Tractatus*, I am here parting company with highly entrenched assumptions in the secondary literature (and am here, to some extent, following the lead of Sellars, 1962). I therefore should make three of my own interpretative assumptions in this connection explicit: (1) *name* and *relation* refer, in the idiom of the *Tractatus*, to distinct modes of symbolizing: what is common to “*aRb*” and “*cRd*” symbolizes a relation, but in these propositions ‘*R*’ does not name anything, and thus (pace Copi, 1958, and others) in “*aRb*” there occur only two names; (2) only those symbols that are absolutely indispensable (in the sense sketched above) count in the idiom of the *Tractatus* as names; (3) not all non-name symbols are relations—logical constants, for example, are neither names nor relations.

In connection with (3), it should be noted that signs for logical constants—or, as the *Tractatus* calls them, *logische Operationszeichen*—count for the *Tractatus* only in a very degenerate sense as symbols. The *Tractatus* says that a symbol characterizes everything essential to their Sinn that propositions can have in common (§3.31), and that the occurrence of a logical operation does not characterize the Sinn of a proposition (§§4.0621, 5.25). The failure of *Operationszeichen* to characterize the Sinn of a proposition is connected with their being dispensable in a yet more radical sense than signs denoting relations. Section 5.4611 puts *Operationszeichen* in a box with punctuation marks. Such signs can easily be made to “vanish” (*verschwinden*) (§§5.254, 5.441). Their function can be taken over by (something which is undisguisedly akin to) punctuation. The negation sign is Wittgenstein’s favorite candidate for replacement in this connection: it could be replaced by a convention governing the manner in which (negated) propositions are written (e.g., in boldface).

83. This is not to claim that it is possible to understand a sentence, if *none* of its constituent signs symbolize in the same manner in which they symbolize in other sentences. (Hence *Tractatus*, 4.03: “A proposition must use old expressions to communicate new senses.”) It is only to claim that not *all* of the constituent signs must symbolize in a precedented fashion. But an unprecedented usage of a sign will only be intelligible if the constituent signs that symbolize in the “old” manner determine a possible segmentation of the propositional sign—where such a segmentation specifies both the logical role of the sign that symbolizes in an unprecedented manner and the position of the resulting propositional symbol in logical space (see note 88).

84. In the absence of any familiarity with the way words (signs) ordinarily occur (symbolize) in propositions, we would have no basis upon which to fashion possible segmentations of propositional signs, and hence no way to *recognize* (rather than simply fantasize) the symbol in the sign. (This is the situation we find ourselves in when faced with a sentence of a language which we do not know and which does not in the least resemble any which we do know.)

85. So, on this reading of Dummett’s example, the sentence might mean something like “The kind of exemplary statesmanship Chairman Mao exhibited is rare.”

86. The second reading is more readily available in this case than it might otherwise be for a reason to which Dummett is strangely oblivious: there is already an established English usage in which ‘rare’ expresses a first-level function (as in “That piece of meat is rare!”). Admittedly, it still requires a bit of a stretch to bring Chairman Mao under *that* concept. But one might try to prepare the way for such a use with: “Chairman Mao is going to get a terrible sunburn [i.e., will soon be well-done] if he doesn’t come in out of the sun soon!”

87. Our familiarity with previous occurrences of the expressions ‘Chairman Mao’ and ‘____ is rare’ furnishes two alternative, equally natural and equally awkward proposals for conferring sense upon the propositional sign ‘Chairman Mao is rare’; but, according to the *Tractatus*, we only determine the sense of these expressions in a particular occurrence of the propositional sign ‘Chairman Mao is rare’ when we adopt

one of these proposals for determining a *possible* method of logically segmenting the string. Thus, as they stand, our established conventions for employing signs *underdetermine* the segmentation of the propositional sign ‘Chairman Mao is rare’: there is no *single* reading that our established conventions (for employing the signs ‘Chairman,’ ‘Mao,’ ‘is,’ and ‘rare’) naturally favor. That our established conventions favor to an equal degree two alternative readings (based on two logically distinct segmentations) plays a crucial role in the Tractarian account of what is (not logically, but) psychologically distinctive about the sort of phenomenology of meaning we undergo in (attempting to understand) cases of apparently substantial nonsense.

88. The segmentation of a propositional sign, for Frege and Wittgenstein, is a function of its position (or better: the position of the proposition it symbolizes) in a network of inferential relations—its position in (what the *Tractatus* calls) logical space. To fix the position of a proposition in logical space is to fix how its logical constituents occur in other propositions. To segment ‘Chairman Mao is rare’ in accordance with the first proposal is to take it to express a judgment that licenses certain inferences of certain patterns; e.g., the inference from the conjunction of (1) “Chairman Mao is rare” and (2) “The sort of politician that Dan Quayle is (an example of) is not rare” to (3) “Dan Quayle is no Chairman Mao.” To segment ‘Chairman Mao is rare’ in accordance with the second proposal is, again, to take it to express a judgment that licenses certain inferences of certain patterns; e.g., the inference from the conjunction of (1') “Chairman Mao is rare” and (2') “This steak is rare” to (3') “There are (at least) two things that are rare!.” The conjunction of (1') and (2), on the other hand, is logically inert: it licenses no inference because these two propositions have no symbol in common. All (1) and (2) have in common are the signs ‘is’ and ‘rare’. The ‘is’ of (1') is the “is” of predication; the ‘is’ of (2) is the ‘is’ of (conceptual) subordination. The ‘rare’ of (1') is a first-level function; the ‘rare’ of (2) is a second-level function. In most cases, recourse to the hypothesis that constituents of a propositional sign are employed in a logically unprecedented fashion yields a segmentation of the string that renders the proposition (which the sign symbolizes) logically inert with respect to its context. When a propositional sign (such as ‘Chairman Mao is rare’) admits of more than one natural segmentation, its context within a larger stretch of discourse should specify which of its possible segmentations is the logically relevant one (as was originally the case, for example, with all of the newspaper headlines in note 116).

89. Or, to put the point in a way that brings out the incoherence in question more vividly—in Frege’s idiom: there isn’t anything which is a proposition’s simultaneously standing in two logically distinct sets of inferential relations with respect to other propositions—in the idiom of the *Tractatus*: there isn’t anything which is a proposition’s occupying two different positions in logical space at the same time. “The proposition determines a place in logical space: the existence of such a place is secured through the existence of its constituent parts alone, through the existence of the significant [*sinnvollen*] proposition” (§3.4). The determination of the logical segmentation of a propositional sign (and thus the conferral of a method of symbolizing on each of its constituent signs) is the specification of a determinate position in logical space. If the “proposition” in question is not *sinnvoll*, then it determines no place in logical space. Thus one way of putting the illusion that underlies the substantial conception would be to say that it imagines that logical segmentation can proceed outside logical space.

90. Both the positivist and the ineffability readings of the *Tractatus* require that these two forms of recognition be mutually compatible: that we be able to recognize the symbol in the sign *and* that we recognize his propositions as nonsensical because the symbols clash with one another.

91. We can now begin to see how misleading is the standard attribution to early Wittgenstein of (what gets called) a “logical atomist theory of meaning.” It is just such a theory that is under indictment in passages such as §§3.3, 3.314, 3.341, and 3.344. Gilbert Ryle noticed that already early Wittgenstein had been concerned to attack

Russell's atomism; and he offered a rather eloquent summary of Wittgenstein's criticisms of an atomistic theory of meaning:

It was . . . Wittgenstein who, developing arguments of Frege, showed that the sense of a sentence is not, what had hitherto been tacitly assumed, a whole of which the meanings of the words in it are independently thinkable parts but, on the contrary, that the meanings of the parts of a sentence are abstractible differences and similarities between the unitary sense of that sentence and the unitary senses of other sentences which have something but not everything in common with that given sentence. To put it in epistemological terms, we do not begin with the possession of concepts and then go on to coagulate them into thoughts. We begin and end with thoughts, and by comparative analysis we can discriminate ways in which something is constant *vis-à-vis* what else is varied between different unitary things we think. . . . [A]n assertion is not a molecule of which the meanings of the words in which it is worded are the atoms. . . . Concepts are not things that are there crystallized in splendid isolation; they are discriminable features, but not detachable atoms, of what is integrally said or integrally thought. They are not detachable parts of, but distinguishable contributions to, the unitary senses of completed sentences. To examine them is to examine the live force of things we actually say. It is to examine them not in retirement, but doing their co-operative work. (Ryle, 1971, pp. 184–85)

Aside from a few notable exceptions, hardly anyone writing on the *Tractatus* over the subsequent several decades seems to have either noticed early Wittgenstein's repudiation of an atomist theory of meaning or noticed that Ryle noticed it.

92. This is the point of the 'only' in "If [a proposition] has no sense this can only be because we have given no meaning to some of its constituent parts" (§5.4733, my emphasis). Most commentary on the *Tractatus*, in attributing to that work the substantial conception of nonsense, leaves that 'only' here looking as if it must be a slip of the pen. Hans-Johann Glock is exceptional in allowing the 'only' some weight here; but, once having done so, it is difficult to see how he can go and say what he does. In the entry entitled "Nonsense" in *A Wittgenstein Dictionary*, Glock writes:

The *Tractatus* features two accounts of nonsense. One is that the nonsensicality of . . . "Socrates is identical" is a matter of deprivation, that is, due to the fact that we have failed to give 'identical' an adjectival meaning. . . . If a proposition "has no sense this can only be because we have given no meaning to some of its constituent parts" (§5.4733). . . . At the same time, the *Tractatus* espouses a form of compositionism: the sense of elementary propositions is determined by the meanings of their constituent names, that is, by what objects they stand for. In the case of a meaningful proposition, to grasp the meaning and logical form of its names is to grasp the possible combination of objects it depicts . . . , while in the case of . . . "Point x,y is C-flat" it is to grasp that this combination of names does not depict a possible combination of objects precisely because the constituents have incompatible meanings. (Glock, 1996, pp. 259–60)

Glock quotes §5.4733 as evidence of the first of Wittgenstein's two "accounts" of nonsense. But the presence of the 'only' would seem explicitly to preclude the second account. I take it that this is Glock's point in speaking of two *accounts* of nonsense—rather than of two *kinds* of nonsensical pseudostatement as, say, Carnap does (see, e.g., the quotation from Carnap in note 100). Rather than taking austere and substantial *Unsinn* to represent two *species* of nonsense (as most standard readings of the *Tractatus* do), Glock seems to take the *Tractatus* to be committed to offering a unitary

account of the genus *Unsinn*. So far, so good, by my lights. This way of reading Glock accords well with the opening sentence of his next paragraph: “Wittgenstein’s later work undermines both sides of this antinomy” (p. 260). “Antinomy,” again, suggests that what must be at issue is two conflicting accounts of the *entire* genus (rather than two mutually compatible species). It looks as if the only way Glock can see how to acknowledge the force of the ‘only’ is to conclude that §5.4733 introduces an independent “account of nonsense” that is at odds with the one advanced elsewhere in the book. This adds up to a very unflattering reading of the book. Are we to imagine that the author simply failed to notice that he was offering two such flagrantly incompatible accounts of nonsense? (This is supposed to be the result of all those years of hard reflection that went into the *Tractatus*?) One would want very compelling textual grounds for concluding that an author of such a short book is so completely unable to keep track of his own views. The evidence licensing the attribution of the first “account” to the *Tractatus* is clear enough and, moreover, is supplied in the paragraph from Glock above. What is the textual evidence for the second account?

93. In order to count as sinnvoll, a Satz has to be able to serve as a vehicle of communication: it has to make a statement about how things are—it has to assert what is the case (*der sinnvolle Satz sagt etwas aus*) (§6.1264). Such a Satz is characterized by both a form (*Form*) and a content (*Inhalt*) (§3.31). A Satz that is sinnlos possesses a (logical) form but no content. *Unsinn*, on the other hand, possesses neither a form nor a content.

A Satz that is sinnlos does not make a claim on reality; it has no bearing on how things are. There is no need to consult how things stand in order to determine its truth-value—mere “inspection of the sign” is sufficient to determine its truth-value. The *Tractatus* therefore distinguishes between the broader genus of Sätze (sinnlos or sinnvoll) characterized by a logical form (i.e., in which we can recognize the symbol in the sign) and the narrower genus of (genuine [*eigentliche*]) Sätze. The latter sort of Satz asserts, “This is how things stand” (*Es verhält sich so und so*) and thus is characterized by “the general form of a proposition” (cf. §4.5)—where this latter phrase should be understood to mean: “the general form of a genuine proposition.” In saying that a “proposition” of logic is sinnlos, the *Tractatus* is identifying it as belonging to a degenerate species (or “limiting case”; cf. §4.466) of the genus proposition—it has the logical form of a proposition without its being *gehaltvoll* (§6.111): “the representational relations it subtends cancel one another out, so that it does not stand in any representational relation to reality” (§4.462).

To say of a Satz (a propositional sign) that it is *Unsinn* is to say that it is a mere sign: no determinate method of symbolizing has yet been conferred on it. Whereas to say of it that it is sinnlos is to affirm that a method of symbolizing has been conferred on it, but that the method of symbolizing in question fails to yield a proper proposition. A Satz that is sinnlos is unlike a genuine proposition (and like *Unsinn*), in that it fails to express a thought (it does not restrict reality to a yes or no and hence does not represent a state of affairs): it says nothing. Yet it is like a genuine proposition (and unlike *Unsinn*), in that we are able to recognize the symbol in the sign and hence are able to express it in a Begriffsschrift—it forms, as the *Tractatus* puts it, “part of the symbolism” (§4.4611). Thus what logic is, for the *Tractatus*, is internal to what it is to say something; and hence which Sätze are logical Sätze (and thus form part of the symbolism) only shows itself (*zeigt sich*) in language—i.e., in the meaningful employment we already make of (what the *Tractatus* calls) “our everyday language” (*unsere Umgangssprache*).

According to a widely accepted reading of the *Tractatus*, the so-called propositions of logic represent a set of a priori “conditions on the possibility of thought”—a set of requirements laid down in advance on what can and cannot be said. Yet it is, in fact, just such a Fregean/Russellian conception of the “substantiality” of logic that is under indictment in the *Tractatus* on the grounds that (a) the so-called truths of logic are not only not prior to, but rather parasitic on ordinary garden-variety truths; (b) logic

therefore cannot be abstracted from language so as to form a body of independently thinkable or assertable truths; (c) the “propositions” of logic (because they are void of content [*inhaltsleer*]) cannot be construed as forming a body of truths at all (let alone, as Frege and early Russell would have it, a body of maximally general truths); and (d) (because they say nothing) they cannot require anything and hence cannot be construed as “laws of thought”; so (e) there is no (Fregean/Russellian) science of logic. For more on (c)-(e), see Conant, 1991a.

94. I have occasionally amended Pap's translations. References to the original German are to Carnap, 1932, pp. 219–41. Carnap's views underwent a considerable and rapid evolution, as did also his understanding of the views of the *Tractatus*. My references to Carnap pertain only to his views in this essay, unless otherwise noted. For a more extended treatment of the differences between Wittgenstein and Carnap, see Conant 2001a.

95. Indeed, it is largely in honor of Carnap's comparatively lucid account of it that I have dubbed the variant in question as “the positivist variant.” The label is in one respect misleading, inasmuch as many who qualify by my lights as (at least irresolute) proponents of the variant in question favor interpretations of the *Tractatus* that stress Wittgenstein's hostility to other aspects of logical positivism.

96. Carnap thus, interestingly, seizes upon and takes up into the title of his essay the very word (from the closing lines of the *Tractatus*) that—once translated into English or French—has often been seized upon and made much of by proponents of the diametrically opposed reading of the *Tractatus*. Pears and McGuinness translate *überwinden* as “transcend,” thus inviting the ineffability interpretation—an invitation that is reinforced through their translation of *schweigen* in the next sentence (which calls merely for silence) as an injunction to the reader to “to pass over [something] in silence.” These mistranslations are mirrored in Gilles-Gaston Granger's French translation of the *Tractatus* (Wittgenstein, 1993), in which *überwinden* is rendered *dépasser* (to go beyond) and *schweigen* is rendered *garder le silence* (to keep silent [in the sense of observing a rule of silence]). These translations are philosophically consequential: talk of “transcending” or “going beyond” only makes sense where there is a *beyond*, and talk of “passing [something] over in silence” or “guarding one's silence [with respect to something]” only makes sense where *breaking* one's silence is a possibility.

97. As intimated earlier, it is instructive to look at Carnap for a second reason as well: he is not someone who wishes to have any truck with the idea that there are ineffable thoughts (that can be “shown” but not said); nonetheless, he, too, can be seen to adhere to a version of the substantial conception of nonsense.

98. That Carnap took his views on what metaphysics is and how it is to be overcome to be influenced by Wittgenstein is evident from his generous references and acknowledgments to the *Tractatus* both before and after he wrote Carnap, 1932. (See, e.g., Carnap, 1967, esp. pp. 290–92, 297–98; 1937, esp. pp. 282–84; 1963; esp. pp. 24–29, 45).

99. That Wittgenstein thought Carnap repeatedly and grossly misunderstood the *Tractatus* is evident from his irate correspondence with Schlick about Carnap's efforts to build on his ideas and from his brief correspondence with Carnap himself on the subject in 1932. (See, e.g., the letters reprinted in Nedo and Ranchetti, 1983, pp. 254–55, 381–82.) Passing remarks sprinkled throughout Carnap's letters and papers at the University of Pittsburgh Archives for Scientific Philosophy bear witness to Carnap's continued (and eminently justified) frustration concerning both the obscurity and the harshness of Wittgenstein's complaints about Carnap's (mis)appropriation of his work.

100. Here is Carnap on the two kinds of pseudostatement: “There are . . . those pseudo-statements which contain a meaningless word. But there is also a second kind of pseudo-statement. They consist of meaningful words, but the words are put together in such a way that nevertheless no meaning results. The syntax of a language specifies which combination of words are admissible and which inadmissible. The gram-

matical syntax of natural languages, however, does not fulfill the task of elimination of senseless combinations of words in all cases" (Carnap, 1959, p. 67).

101. Carnap is not of the view that *all* discourse involving the term 'God' is of this sort. He distinguishes (1959, pp. 66–67) between four sorts of usage: (1) type (i) nonsense; (2) the *mythological* usage—in which 'God' has a determinate meaning, occurs in empirically verifiable statements, and refers to a kind of physical being with specifiable properties whose possibility and existence is a possible topic of scientific inquiry; (3) the *theological* usage, which involves an oscillation between uses (1) and (2); and (4) cases in which a definition of 'God' is furnished but involves type (ii) nonsense.

102. The example is in the present context in one respect an unhappy one: Frege would have regarded it as not nonsensical, but simply false.

103. Carnap, 1959, pp. 67–68

104. I have (except for capitalizing the word "Being") reproduced Pap's translation of this passage. It should be noted, however, that Heidegger's last sentence contains a neologism, and thus, for reasons that will become clear, would be more faithfully rendered: "The nothing itself noths."

105. And which, moreover, do not occur consecutively in Heidegger's essay.

106. Carnap's argument here still rests on the claim that Heidegger's intentions fix the meaning of the word 'nothing' in his sentences—only now the appeal is to a very different sort of intention on Heidegger's part: an intention to violate the logical structure of language on purpose. The postulation of this more baroque intention saves Carnap from Frege's objections to Kerry by casting Heidegger in the role of someone who shares the linguistic intentions of (Geach's) Frege in "On Concept in Object"; but it still leaves Carnap open to the *Tractatus*'s objections to Frege.

107. Someone might object that the claim that Heidegger intends to speak nonsense does not play a weight-bearing role in the argument of Carnap's essay, and that I am attaching too much significance to Carnap's observation (1959, pp. 71–72) that Heidegger does so intend. It is true that the claim is not represented by Carnap as playing an important role in his argument. But his argument nonetheless requires it. Faced with the choice of (1) attributing to someone the intention to fail to make sense, or (2) attributing to him the intention to use words in an unprecedented but potentially intelligible manner, any sound theory of interpretation will prescribe that we settle for (1) only if we have excellent grounds for preferring it over (2). If Carnap is unable to rule out the possibility of a more charitable construal of Heidegger's sentences, then his entire analysis stands under threat of failing to make contact with Heidegger's text. He therefore needs an argument for why we should go with (1).

108. The evidence that Heidegger means to speak nonsense is drawn from the same essay of Heidegger's from which Carnap's original exhibit is drawn. This evidence, Carnap claims, shows that "the author of the treatise is clearly aware of the conflict between his questions and statements and logic" (1959, p. 72). But we have already seen Frege acutely aware of such a conflict in his writings; and such a conflict lies just below the surface of much of Russell's work (see Conant 2002a), and bubbles increasingly to the surface in the period 1913–14; consider, e.g., the following remarks from Russell's 1914 lectures at Harvard: "A fact is not a thing. When I say that I am talking nonsense. Nevertheless, I want you to take it as a profound truth" (Russell, 1914, pp. 4–5). Indeed, a careful reading of Heidegger's essay suggests that it is, in fact, for considerations not altogether dissimilar from those that move Frege and Russell to query the logical integrity of some of their sentences that Heidegger pauses over the logical standing of remarks, such as his own, which seek to turn (that which corresponds to the meaning of the word) "nothing" into a logical subject. Thus, though Carnap's concluding observations about Heidegger (e.g., "the metaphysician himself states that his questions and answers are irreconcilable with logic"; Carnap, 1959, p. 71) are evidently meant to be scathing, they are equally applicable to moments that occur in the writings of his heroes, Frege and Russell.

109. The following two excerpts from §§5.473–5.4733 are potentially misleading and might appear to conflict with what I say about the point of the passage:

- (1) “‘Socrates is identical’ means nothing because there is no property which is called ‘identical.’” (§5.473)
- (2) “‘Socrates is identical’ says nothing, because we have given *no* meaning to the word ‘identical’ as *adjective*.” (§5.4733)

One might think that the reason Wittgenstein thinks that his example ‘Socrates is identical’ is not a case of the symbol being “in itself impermissible” is merely because he (somehow) knows that ‘identical’ in this context is functioning as an adjective. (Though this still leaves the question: How does he know this?) The problem with this way of taking what Wittgenstein means (when he says “we have given no meaning to the word ‘identical’ as adjective”) is that it leaves it a mystery why the example is taken by Wittgenstein to illustrate the general point of the whole passage: i.e., “we *cannot* give a sign the wrong sense” (§5.4732).

So what are we to make of Wittgenstein’s talk of “‘identical’ as *adjective*” and of there being “no property which is called ‘identical’”?

In ordinary contexts (ones in which we are able to see the symbol in the sign), the sign “identical” symbolizes a relation of identity between two objects. Given the absence of a second (candidate for an) object-expression, the present example does not admit of such a reading. Our ordinary strategy for seeing the symbol in the sign ‘identical’ comes to grief in the case of “Socrates is identical.” The point of remark (1)—about “identical” naming an unspecified property—is to offer an alternative reading of the string: a suggestion intended to enable us, based on the surface grammar of this peculiar string, to find a way to begin to see a symbol in the sign. There is an invitation present in the pattern of ordinary language for us to try to read the sign in this way (on the model of “Socrates is happy”). But we can only go so far in this direction. We can assimilate “Socrates is identical” to an established pattern (and thereby recognize the symbol in the sign); but we still do not yet know *what* the sentence says, because there is no established use of “identical” as a concept-expression. On such a reading of the string (which, without further contextual clues, is as far as we are able to go in trying to make sense of it), we know that we are predicating some concept of Socrates, but not *which* concept: “because there is no property which is called ‘identical’.”

But this only clears up half the mystery: What does Wittgenstein mean by the term “adjective” in remark (2)? When Wittgenstein talks in remark (1) about a property, he is talking about a method of *symbolizing*. When he talks in remark (2) about “identical” as adjective, he is referring to a feature of the “external form” (§4.002) of certain sentences—a grammatical surface pattern—of ordinary language (a certain sort of configuration of signs). Recall §3.323: “In the language of everyday life it very often happens that the same word signifies in two different ways—and therefore belongs to two different symbols—or that two words, which signify in different ways, are apparently applied in the same way in the proposition. Thus the word ‘is’ appears as the copula, as the sign of equality, and as the expression of existence; ‘to exist’ as an intransitive verb like ‘to go’; ‘identical’ as an adjective; we speak of something but also of the fact of something happening.” The term “adjective” here and in §5.473 refers to a feature of the surface grammar (the sign structure) of ordinary language—not a proper *logical* category. The point here is about the sign ‘identical’, not the symbol. What does Wittgenstein mean here by saying that ‘identical’ sometimes *appears* as an adjective? The point is that this sign figures in sentences of ordinary language in such a way as to mimic the surface grammar of signs that symbolize concepts. Consider sentences (a) and (b):

- (a) Socrates and the teacher of Plato are identical.
- (b) Socrates and the teacher of Aristotle are happy.

The words ‘happy’ and ‘identical’ as they occur in sentences (a) and (b) provide an example of the point of the above passage: how “two words, which signify in differ-

ent ways, are apparently applied in the same way.” These two sentences appear to have the same logical form. But, if we reflect upon our understanding of what each of them says and what sorts of inferences each of them licenses, it becomes immediately apparent that this appearance is deceptive. Consider the following inferences:

(a) <i>Socrates and Aristotle are happy.</i>	(b) <i>Socrates and the teacher of Plato are identical.</i>
(a') <i>Socrates is happy.</i>	(b') <i>Socrates is identical.</i>

The inference from (a) to (a') is a felicitous one, the pseudo-inference from (b) to (b')—though (as indicated by the italicized portions) superficially patterned on that from (a) to (a')—is from a meaningful statement to a piece of nonsense. ‘Identical,’ as it occurs in sentence (b), appears to symbolize in the same way as ‘happy,’ as it occurs in sentence (a), though it does not do so, and that it does not do so would be rendered perspicuous by a proper *Begriffsschrift*. As it occurs in sentence (b), ‘identical’ has the same surface grammar as an adjective such as “happy.” This is what Wittgenstein means when he says that “‘identical’ sometimes appears as an adjective.”

110. In conversation with Carnap and other members of the Vienna Circle, Wittgenstein remarked: “To be sure, I can imagine what Heidegger means by Being [*Sein*] and Anxiety [*Angst*]” (1979b, p. 68). Wittgenstein’s response to Heidegger’s remarks—in contrast with Carnap’s—is to attempt to *imagine* what Heidegger might mean by his words. The task of philosophical elucidation, for Wittgenstein, always begins with such an attempt.

111. The *Tractatus* works through in (its characteristically compressed) detail a wide variety of such cases (of hovering between determinate possibilities of use) as they arise in connection with the philosophical employment of *Scheinbegriffe* such as “world,” “fact,” “essence,” “logical form,” “representation,” “language,” “thought,” “concept,” “object,” “generality,” etc. The opening sentences of §4.1272 furnish a relatively straightforward example of such a case: the “use” of (the philosophical *Scheinbegriff*) “object” wavers between wanting ‘object’ to symbolize a variable and wanting it to symbolize a *Begriff*. For a nice discussion of such wavering in connection with this example (following up §4.1272), see Diamond, 1997, esp. pp. 79–80; and in connection with “generality” (following up §4.0411), see Kremer, 1992, esp. pp. 411–16.

112. Thus Wittgenstein says in §6.53: the aim is to demonstrate to the metaphysically inclined speaker that he has given “*no meaning* to certain signs in his *Sätze*.” If the standard reading of the *Tractatus* were correct, this is not what Wittgenstein should be saying here. The complaint should be directed not at the (mere) *signs* in the metaphysician’s *Sätze* (on the grounds that no meaning—i.e., no method of symbolizing—has been conferred upon them), but at the impermissible character of the propositional *symbols* which the metaphysician employs. The reason most commentators on the *Tractatus* fail to be struck by such textual anomalies is that they themselves run roughshod over the distinction between symbol and sign. One finds a great deal of this sort of thing: “For early Wittgenstein . . . logic is concerned with the . . . question what strings of signs are propositions capable of representing reality at all” (Glock, 1996, p. 258).

113. There is therefore an asymmetry in the attitude of the *Tractatus* toward these two sorts of proscription. The latter sort (i.e., the proscription of combinations of symbols) rests on philosophical confusion; the former does not. The *Tractatus* clearly thinks it is desirable for certain purposes (and for systems of notation that facilitate those purposes) to introduce principles that proscribe combinations of signs (as, e.g., a *Begriffsschrift* does). But there is reason to think the *Tractatus* would not look favorably upon a general reform of natural language based on principles that sought to proscribe sequences of natural language icons. (§§4.002 and 5.5563 taken in conjunction yield: “Our everyday language is part of the human organism and no less complicated than it . . . and in perfect logical order, just as it is.”) It is important that natural lan-

guages be able to tolerate the sorts of innovative use of signs exemplified in a mild way by Frege's example about 'Vienna,' with a vengeance by Heidegger's employment of 'nothing,' and by Wittgenstein's own remarks (in §§5.473, 5.4733) about the possibility of giving 'identical' an adjectival use.

114. Carnap goes on to cite *Tractatus*, §3.33 as evidence that he and Wittgenstein are in agreement on this point. For an excellent discussion of what Wittgenstein does not and Carnap does think logic is, see Friedman, 1997.

115. This is not obviously what "logical syntax" means in Carnap, 1959 (most of what Carnap says about Heidegger's employment of *nothing* makes no sense if he is only concerned with the sign 'nothing'); but it is not altogether clear what "logical syntax" means in this essay. The closest he comes in Carnap, 1959, to a definition of logical syntax is to say: "The syntax of a language specifies which combination of words are admissible and which inadmissible" (p. 67). This reference throughout Carnap's discussion to admissible and inadmissible combinations of *words* allows Carnap, from the point of view of the *Tractatus*, systematically to conflate questions concerning admissible combinations of signs with questions concerning admissible combinations of symbols. Carnap wavers, in this essay, between characterizing the principles of "logical syntax" as principles that govern combinations of symbols (as he needs to, if they are to isolate a flawed *Sinn* in the utterances of the metaphysician) and characterizing them as principles that govern the combinations of signs (as he would need to, if he wanted to maintain—as he does in his later work—that logical syntax is only concerned with "language" qua purely formal combinatorial syntactic object).

116. The following newspaper headlines offer examples of cross-category equivocation—cases in which there is an ambiguity (not just in the Bedeutungen of the words, but in the logical syntax of the string and thus) in the logical category of the symbol we should see in the sign:

- (a) British Left Waffles on Falkland Islands
- (b) British Push Bottles Up German Rear
- (c) Potential Witness to Murder Drunk
- (d) Legal Aid Advocates Worry
- (e) Crowds Rushing to See Pope Trample 6 to Death
- (f) Beating Witness Provides Names
- (g) Nixon Stands Pat on Watergate Tapes
- (h) University Studies Mushroom
- (i) Carter Plans Swell Deficit

117. Some version of this thought—and, with it, the insight that this might be the way out of the problems that plagued Russell's philosophy (see Conant 2002a)—came to Wittgenstein remarkably early. Already in January 1913, he was writing Russell as follows: "Every theory of types must be rendered superfluous by a proper theory of the symbolism. . . . What I am *most* certain of is not . . . the correctness of my present way of analysis, but of the fact that all theory of types must be done away with by a theory of symbolism showing that what seem to be *different kinds of things* are symbolized by different kinds of symbols which *cannot* possibly be substituted in one another's places" (Wittgenstein, 1995, pp. 24–25). This way of attempting to formulate his insight still involves Wittgenstein in attempting to say what the "it" is here that "*cannot* possibly" be. The road from the early insight ("every theory of types" must be rendered superfluous by a proper theory of the symbolism") to the later insight (the task of "a proper theory of symbolism" is to self-destruct in a manner that shows *all theories of symbolism* to be "superfluous") is not a short one. One might think of it as the distance that needs to be traveled by a reader of the *Tractatus* who imagines he *understands the propositions* in §§3.331–3.334 but has not yet *understood the author* in the manner called for in §6.54. One can see the subsequent five years of Wittgenstein's work, culminating in §6.54 of the *Tractatus*, as a process of gradually climbing up this ladder, by teasing out the implications of his early insight, to the

point where the illusion that there is an “it” (whose impossibility needs to be shown) vanishes—and, along with it, the necessity of formulating (something properly described as) “a theory of symbolism” which demonstrates that such things cannot possibly be. Such forms of “theory” give way to the “activity” of elucidation (§4.112).

118. It is worth noting that what the *Tractatus* thus rejects has become, quite literally, the textbook account of its own project. The idiom of “drawing [or fixing] the bounds of sense” has come to be an indispensable shorthand in thumbnail sketches of the project of the *Tractatus*. Thus we find in the “Wittgenstein” entry of Blackwell’s *A Companion to Epistemology*: “In the *Tractatus* a general theory of language is used to fix the bounds of sense” (Pears, 1992, p. 524). And in *A Wittgenstein Dictionary*, we find: “The early Wittgenstein . . . is concerned with the . . . question what strings of signs are . . . capable of representing reality. . . . He combined this with Kant’s idea that philosophy is a critical activity which draws the bounds between legitimate discourse . . . and illegitimate speculation. . . . While Kant draws limits to knowledge, Wittgenstein draws limits to meaningful discourse” (Glock, 1996, p. 258).

119. Wittgenstein’s point in devising alternative logical notations in which certain signs (e.g., logical connectives [*logische Operationszeichen*]) are made to disappear is to devise a language that suits his elucidatory purposes in philosophy. Wittgenstein’s aim is to free us from the philosophical confusions (which the outward form of our language leads us into) by showing us that we can dispense with such signs. It is not to encourage us, outside the context of philosophical elucidation, to prefer a language that dispenses with such signs. On the contrary, according to the *Tractatus*, the outward form of our language is already exquisitely tailored to suit our everyday purposes in communication (see §4.002).

120. Ogden’s translation, as published, has: “The result of philosophy is not a number of ‘philosophical propositions,’ but to make propositions clear.” This came about as a response to Wittgenstein’s initial suggestion that *das Klarwerden von Sätzen* be rendered (instead of “the clarification of propositions”) as “the getting clear of propositions” (Wittgenstein, 1973, p. 28). Ogden, having convinced Wittgenstein that this is not much help, tries “to make propositions clear.” Wittgenstein (in his annotations of Ogden’s revisions) changes this to “that propositions become clear.” But Ogden still finds this unclear and awkward English to boot, thus prompting Wittgenstein’s more illuminating (though even more awkward) suggestion on page 49.

121. The *Tractatus* articulates what Robert Brandom calls an *expressivist* conception of logic (Brandom, 1986, 1988, 1994), insofar as it conceives of logical syntax as an instrument for (1) explicating the logical structure of thought and thus enabling (what the *Tractatus* calls) *das Klarwerden von Sätzen*, (2) revealing specifically logical vocabulary (such as the logical constants) to be linguistically optional and thus subject to possible “disappearance” (see note 82), and (3) perspicuously representing the inferential relations between thoughts (see note 124).

122. John Koethe, for example, writes: “[T]o try to read . . . the *Tractatus* as urging to adopt an ideal language analogous to Frege’s *Begriffsschrift* . . . seems at odds with Wittgenstein’s insistence that ‘all the propositions of our everyday language, just as they stand, are in perfect logical order’” (Koethe, 1996, p. 39).

123. This, of course, does not mean that language itself prevents us from ever making “logical mistakes” in the ordinary (nonphilosophical) sense of the expression “logical mistake”—i.e., that it keeps us from ever contradicting ourselves! Indeed, the possibility of forming contradictions is, according to the *Tractatus*, a constitutive feature of any symbolism (which, for the *Tractatus*, means any system capable of expressing thought). What this passage refers to, rather, is the prevention of the possibility of the (peculiarly philosophical) sort of “logical mistake” that Russell’s theory of types or Carnap’s theory of logical syntax sought to exclude. This latter notion of “a violation of logic” depends upon a philosophical *theory* (which seeks to draw a limit to the sorts of thoughts that are so much as possible).

124. The *Tractatus* sacrifices all the other ends to which Frege and Russell sought to put a *Begriffsschrift* to the sole end of notational perspicuity. Early Wittgenstein champions a logical syntax that avoids a plurality of logical constants because such a plurality frustrates the sole application which the *Tractatus* seeks to make of a logical syntax: to allow the logical form of propositions to appear with “complete clarity.” A plurality of logical constants frustrates this end in two ways: it permits the same thought to be rendered in diverse ways, and it obscures the logical relations between propositions. The first of these points is introduced in §3.325: the logical symbolisms of Frege and Russell do not sufficiently minimize the possibility of the same propositional symbol being expressed by two distinct propositional signs. (§6.1203 offers an ingenious partial notation that seeks to render maximally perspicuous certain sorts of cases in which distinct propositional signs express the same symbol.) The second of these points is first touched on in §§5.13–5.1311. Consider the following inferences:

- (a) $(p \vee q) \wedge \neg p$, therefore: q
- (b) $\neg(\neg p \wedge \neg q) \wedge \neg p$, therefore: q

Statements (a) and (b) appear here to be two distinct inferences, although they are logically equivalent. A notation system with three different connectives (negation, disjunction, and conjunction) allows what, for the *Tractatus*, is one logical operation (see §§5.21–5.23) to appear here as two distinct operations, thereby obscuring the manner in which “the structures of propositions stand in internal relations to one another” (§5.2) and thus the lines along which each individual proposition (if it is to license inferences of the appropriate patterns) is to be segmented. This is an example of the kind of thing Wittgenstein says a “proper logical syntax” (i.e., one that exclusively serves his idiosyncratic elucidatory purpose!) ought to exclude. A helpful discussion of this and related matters is to be found in Mounce, 1981.

125. See also §3.03 and §5.4731.

126. Here is my favorite example: “Even if it were admissible to introduce ‘nothing’ as a name or description of an entity, still the existence of this entity *would* be denied in its very definition This sentence therefore *would* be contradictory, hence nonsensical [*unsinnig*] were it not already senseless [*sinnlos*]” (Carnap, 1959, p. 65, my emphases; Carnap, 1932, p. 231; I have emended Pap’s translation).

127. The context of the quotation is as follows:

Different kinds of nonsense. Though it is nonsense to say “I feel his pain,” this is different from inserting into an English sentence a meaningless word, say “abracadabra” (compare Moore last year on “Scott kept a runcible at Abbotsford”) and from saying a string of nonsense words. Every word in this sentence is English, and we shall be inclined to say that the sentence has a meaning. The sentence with the nonsense word or the string of nonsense words can be discarded from our language, but if we discard from our language “I feel Smith’s toothache” that is quite different. The second seems nonsense, we are tempted to say, because of some truth about the nature of things or the nature of the world. We have discovered in some way that pains and personality do not fit together in such a way that I can feel his pain. —The task will be to show that there is in fact no difference between these two cases of nonsense, though there is a psychological distinction, in that we are inclined to say the one and be puzzled by it and not the other. We constantly hover between regarding it as sense and regarding it as nonsense, and hence the trouble arises. (Wittgenstein, unpublished)

Although this passage is from 1935, in its insistence upon an austere conception of nonsense, it is equally pertinent to an understanding of Wittgenstein’s early and his later teaching. For further discussion, see Conant 2001b.

128. Except that proponents of the ineffability interpretation of the *Tractatus* pretend, unlike Geach, to be able to see how to go on and adapt the guiding idea of this interpretation—i.e., that attempts to formulate propositions that violate the logical structure of language are able to convey insights into *logical* features of reality—so that it extends to the possibility of conveying additional insights into other apparently quite different, yet equally ineffable (usually ethical, aesthetic, and/or religious) features of reality.

129. Or to put the same point differently: it aims to show us that we cannot use language as Geach reads Frege as supposing we can and as proponents of the ineffability interpretation read the *Tractatus* as supposing we can. For example, Peter Hacker, as we saw in note 60, takes §§4.126 and 4.1272 of the *Tractatus* to be concerned with showing how a certain sort of attempt to “violate the bounds of sense”—in the case in question, the violation (allegedly) incurred by a certain employment of the expression ‘object’—enables us to hint at something that cannot be said. Hacker and I agree that these sections of the *Tractatus* do not succeed in *saying* anything. But Hacker takes these passages of the book to be *trying* to say what cannot be said but only “shown.” He implicitly attributes to the *Tractatus* the doctrines (a) that there is something which is a piece of nonsense’s trying but failing to say something, and (b) that there is something which can count as one’s knowing what the nonsense in question would be saying if it were something that could be said. Thus, on Hacker’s interpretation, the whole point of the book is to show us how to employ language (or at least language-like structures) to get outside language (to what cannot be said but only “shown”). On my interpretation, the whole point of the *Tractatus* is—not to get us to see the truth of (a) and (b), but rather—to get us to see that (a) and (b) rest upon the (only apparently intelligible) notion that nonsense can so much as try to say something.

130. I am here borrowing a phrase of John McDowell’s (1981, 1994).

131. “The only strictly correct method” of philosophy described in §6.53 is quite different from the one actually practiced in the *Tractatus*. The practitioner of the strictly correct method eschews nonsense, confining himself to displaying what can be said and to pointing out where the other has failed to give a meaning to one of his signs; whereas the elucidatory method of the *Tractatus* involves the production of vast quantities of nonsense. The former method depends on the elucidator always being able to speak second; the latter attempts to achieve the aims of the former but in a situation in which the interlocutor is not present. The actual method of the *Tractatus* is thus a literary surrogate for the strictly correct method—one in which the text invites the reader alternately to adopt the roles played by each of the parties to the dialogue in the strictly correct method. As the addressees of this surrogate form of elucidation, we are furnished with a series of “propositions” whose attractiveness we are asked both to feel and to round on. This raises the question: Which of the Sätze of which the work is composed are *really* nonsense and which not? Which belong to the voice of temptation and which to the voice of correctness? The question is based on a confusion—on the idea that Sinn is the sort of property a Satz can possess on its own steam, apart from any relation that we, as users of it, enter into with it. (See note 135.)

132. *Manuscript 110, Handschriftlicher Nachlass*, p. 239. When the aim of a work is “to place an illusion before one’s eyes,” the task of offering an exegesis of the work becomes a delicate one. Much of what proponents of the ineffability interpretation write often amounts to little more than a paraphrase of things Wittgenstein himself (apparently) says in the *Tractatus*. How can a commentator who furnishes us with a seemingly faithful paraphrase of Wittgenstein’s own words be leading himself or his readers astray as to the point of the passage in question? Well, it depends on the sort of use to which one wants to put such a paraphrase. It depends on whether the paraphrase is adduced as a transitional remark (whose sense is subsequently to be queried) or as an explanation of the meaning of the passage. What is it to exemplify an understanding of the point of those passages from the *Tractatus* which the reader is to recognize as *Unsinn*? Here are two possible answers: (a) one exemplifies one’s

understanding of the passages in question through a faithful paraphrase of them, where what one says makes explicit what these passages (are at least trying to) say; (b) one exemplifies one's understanding of the passages in question by bringing out how they are to serve as expressions of philosophical temptations which are eventually to be recognized as *Unsinn* and to be thrown away. Answer (a) is quite properly presupposed in most expositions of most philosophical works; but to presuppose (a) in an exposition of the point of the relevant passages from the *Tractatus* is inevitably to fall into the very confusions which the passages in question seek to expose. An undue confidence on the part of a commentator in the reliability of paraphrase as a method of explicating the point of a passage will lead to a complete missing of its point if the point is to carry the reader along a movement of thought that culminates in an undermining of its credentials as thought (if it is latent nonsense which is to be recognized as patent nonsense). To think that one can faithfully exhibit an understanding of those passages of the *Tractatus* that are to be recognized by the reader as *Unsinn* by offering (what one takes to be) a faithful paraphrase of them is to fail (to do what §6.54 calls upon the reader to do: namely) to understand the author of the book and the character of the project of elucidation in which he is engaged.

133. "The book will, therefore, draw a limit . . . not to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought)" (*Tractatus*, preface).

134. "The limit can, therefore, only be drawn in language and what lies on the other side of the limit will be *simply nonsense*" (*Tractatus*, preface, my emphasis).

135. In §6.54, Wittgenstein draws the reader's attention to a kind of employment of linguistic signs that occurs within the body of the work. Commentators fail to notice that what Wittgenstein says in §6.54 is not: "all of my sentences are nonsensical" (thus giving rise to the self-defeating problematic Geach has nicely dubbed *Ludwig's self-mate* (1980, p. 265)). Section 6.54 characterizes the way in which those of his propositions which serve as elucidations elucidate. He says: "my sentences serve as elucidations in the following way: he who understands me recognizes them as nonsensical"; or better still, to quote from the English translation of §6.54 that Wittgenstein himself proposed to Ogden: "my propositions elucidate—whatever they do elucidate—in this way, he who understands me recognizes them as nonsensical" (Wittgenstein, 1973, p. 51). The aim of the passage is (not to propose a single all-encompassing category into which the diverse sorts of propositions that comprise the work are all to be shoehorned, but rather) to explicate how those passages of the work that succeed in bearing its elucidatory burden are meant to work their medicine on the reader.

Question: Which sentences are (to be recognized as) nonsensical? Answer: those that elucidate. Section 4.112 does not say: "A philosophical work consists entirely of elucidations." It says: "A philosophical work consists essentially of elucidations." Not every sentence of the work is (to be recognized as) nonsense. For not every sentence serves as an elucidation. Some sentences subserve the elucidatory aim of the work by providing the framework within which the activity of elucidation takes place. Some of them do this by saying things about the work as a whole (and offering instructions for how the work is to be read); others by saying things with the aim of helping us to see what is going on in some part of the work (i.e., within a particular stretch of elucidation). Many of the sections of the *Tractatus* to which this essay has devoted most attention—e.g., the preface, §§3.32–3.326, 4–4.003, 4.111–4.112, 6.53–6.54—belong to the frame of the work and are only able to impart their instructions concerning the nature of the elucidatory aim and method of the work if recognized as *sinnvoll*. (Indeed, what I have just done in this note is offer a partial explanation of what §4.112 and §6.54 say.)

Question: What determines whether a remark belongs to the frame of the work (preparing the way for those remarks that do serve as elucidations) or to the (elucidatory) body of the work? Answer: its role within the work. The distinction between what is

part of the frame and what is part of the body of the work is not, as some commentators have thought, simply a function of where in the work a remark occurs (say, near the beginning or the end of the book). Rather, it is a function of how it occurs.

Question: How are we to tell this? What criteria govern whether a given remark is Unsinn or not? This question presupposes that certain strings of signs are intrinsically either cases of Unsinn or cases of Sinn. But the *Tractatus* teaches that this depends on us: on our managing (or failing) to perceive (*erkennen*) a symbol in the sign. There can be no fixed answer to the question what kind of work a given remark within the text accomplishes. It will depend on the kind of sense a reader of the text will (be tempted to) make of it. Many of the remarks are carefully designed to tempt a reader to find a (substantially) "nonsensical sense" in them. In order to ascend the ladder, a reader must yield to (at least some of) these temptations.

Certain remarks in the *Tractatus* can be seen to have a triple-aspect structure: liable to flip-flop between (1) (apparently) substantial nonsense, (2) mere Sinn, and (3) (what the *Tractatus* calls) Unsinn—i.e., between (1) a remark in which the reader (imagines she) is able to perceive a symbol in each sign but is unable to attach Sinn to the resulting combination; (2) a remark in which the reader is able to perceive a logically unproblematic proposition in the propositional sign; and (3) a remark in which the reader perceives (*erkennt*) a mere string of signs upon which no determinate method of symbolizing has been conferred. Some remarks—including the final remark (read, e.g., as the tautology: "We must be silent [i.e., say nothing] where there is nothing to say.")—can present yet a fourth aspect: that of *Sinnlosigkeit*. What sort of foothold(s) a given remark provide(s) a given reader in her progress up the ladder thus depend(s) upon the sort(s) of aspect it presents to her, and that will depend on her—on the use(s) to which she is drawn to put it in the course of her ascent.

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